Specific heat below freezing point:

- **Only the difference due to water** is taken into consideration.

↓

- Other than water, the specific heats of the other contituents of food do not change with the temperature of food.

$$C_p = \mathbf{0.5}\, m_s + 0.2\,(1 - m_s)$$

$$C_p = 0.5\, m_s + 0.2 - 0.2\, m_s$$

$$C_p = 0.3\, m_s + 0.2$$

Arranging in SI unit system;

$$C_p = 1.256\ m_s + 0.83736$$

Above freezing point, for fat containing foods;

$C_p = 0.4\ F + 0.2\ NFS + \mathbf{0.5}\ M$ (İngiliz)

$C_p = 1.67472\ F + 0.83736\ NFS + 2.934\ W$ (SI)

Taking consideration of carbohydrate, protein, ash and so on;

- $C_p = 1.424\,\mathbf{C} + 1.549\,\mathbf{P} + 1.675\,\mathbf{F} + 0.837\,\mathbf{A} + \mathbf{2.0934\,M}$

- **Example 4.11**: Calculate the specific heat of strawberries having a brix of 9 below freezing point in **SI** unit system.

Answer

- $C_{\text{strawberries}}$ = 0.473 kcal/(kg °C)
- **$C_{\text{strawberries}}$ = 1980 J/(kg K)**

- **Example 4.13:** Calculate the heat required to raise the temperature of a 4.535 kg roast containing 15% protein, 20% fat and 65% water from 4.44°C to 65.55°C. Express this energy in:

a) BTUs,
b) kJ's,
c) W h.

Answer

a) Q = 836 BTU
b) Q = 882 kJ
c) Q = 245 W h

C_p of various foods

Product	Water (%)	Cp (J/kg K)
Butter	14	2050
Milk, skim	91	4000
Chicken	74	3310
Egg white	87	3850
Egg yolk	48	2810
Apples	75	3370
Cucumber	97	4103
Spinach	87	3800
Bread	44	2720
Flour	13	1800

There are two weak points of Siebel equation!!

0.2
BTU/lbm °F

1) Siebel’s equation assumes that all types of nonfat solids have the same specific heat. This is not correct.
2) Below the freezing point, Siebel’s equation for specific heat assumes that all the water is frozen. **This is most inaccurate.**

Choi and Okos (1987) equation for specific heat of foods at any temp. above freezing

- Used for solids and liquids.
- Takes into account of all food constituents.

- Specific heat is calculated **at any temperature above freezing point.**

- **This equation is more accurate at low moisture contents (M < 0.7) and for fat containing foods**.

- Values for C_p calculated using Choi and Okos' equation are generally higher than those calculated using Siebel's equations.

Siebel equation for C_p of foods

- Siebel's equations have been found to agree closely with experimental values **when $M > 0.7$ and when no fat is present**.
- The simplicity of Siebel's equations appeals to most users.

C_p of a food at any temp. (Choi and Okos equation)

Protein : $C_p = 2008.2 + 1208.9\ 10^{-3}\ T - 1312.9\ 10^{-6}\ T^2$

Fat : $C_y = 1984.2 + 1473.3\ 10^{-3}\ T - 4800.8\ 10^{-6}\ T^2$

Carboh.: $C_k = 1548.8 + 1962.5\ 10^{-3}\ T - 5939.9\ 10^{-6}\ T^2$

Fiber: $C_l = 1845.9 + 1930.6\ 10^{-3}\ T - 4650.9\ 10^{-6}\ T^2$

Ash : $C_{kü} = 1092.6 + 1889.6\ 10^{-3}\ T - 3681.7\ 10^{-6}\ T^2$

Water: $C_s = 4176.2 - 9.0862\ 10^{-5}\ T + 5473.1\ 10^{-6}\ T^2$

C_p → J/(kg K) Temperature → °C

Choi and Okos (1987) equation for C_p of foods above freezing

$$C_p(\textbf{food}) = Cp\,(P) + Cf\,(F) + Cc\,(C) + Cfib\,(Fib) + Ca\,(A) + Cm\,(M)$$

- **Example 4.14**: Calculate the specific heat of a formulated food product which contains 15% protein, 20% starch, 1% fiber, 0.5% ash, 20% fat and 43.5% water at 25°C.

Answer

Protein: $C_{pp} = 2008.2 + 1208.9 \times 10^{-3}\ (25) - 1312.9 \times 10^{-6}\ (25)^2$

$$C_{pp} = 2037.6\ \text{J/(kg K)}$$

Answer

C_{pavg} = 2865 J/(kg K) **at 25ºC**

C_p calculation over the range of temp.s (Choi and Okos equation)

- Mean specific heat (C*) of a food over a temperature range from T_1 to T_2,

$$C^*_{pp} = 1/\delta \int_{T_1}^{T_2} c_p \, dT$$

δ: Tempereture difference ($\Delta T = T_2 - T_1$)

$$\delta = (T_2 - T_1)$$
$$\delta^2 = (T_2^2 - T_1^2)$$
$$\delta^3 = (T_2^3 - T_1^3)$$

Protein: $C^*_{pp} = (1/\delta)\,[2008.2\,(\delta) + 0.6045\,(\delta^2) - 437.6 \times 10^{-6}\,(\delta^3)]$

Fat: $C_{pf} = (1/\delta)\,[1984.2\,(\delta) + 0.7367\,(\delta^2) - 1600 \times 10^{-6}\,(\delta^3)]$

Carboh: $C_{pc} = (1/\delta)\,[1548.8\,(\delta) + 0.9812\,(\delta^2) - 1980 \times 10^{-6}\,(\delta^3)]$

Fiber: $C_{pfi} = (1/\delta)\,[1845.9\,(\delta) + 0.9653\,(\delta^2) - 1500 \times 10^{-6}\,(\delta^3)]$

Ash: $C_{pa} = (1/\delta)\,[1092.6\,(\delta) + 0.9448\,(\delta^2) - 1227 \times 10^{-6}\,(\delta^3)]$

Water: $C_{pm} = (1/\delta)\,[4176.2\,(\delta) + 4.543 \times 10^{-5}\,(\delta^2) - 1824 \times 10^{-6}\,(\delta^3)]$

$$\boxed{C^*_{avg} = P\,(C^*_{pp}) + F\,(C^*_{pf}) + C\,(C^*_{pc}) + Fi\,(C^*_{pfi}) + A\,(C^*_{pa}) + M\,(C^*_{pm})}$$

- **Example 4.15:** Calculate the mean specific heat of the formulated food product in Example 4.14 in the temperature range of 25°C and 100°C.

Answer

- δ = 75°C, δ^2 = 9375°C ad δ^3 = 984,375.

Answer

Protein:

$$C^*_{pp} = (1/75)\ [2008.2\ (75) + 0.6045\ (9375) - 437.6 \times 10^{-6}\ (984{,}375)]$$

$$C^*_{pp} = 2078\ \mathrm{J/(kg\ K)}$$

Answer

C_{pavg} = 2904 J/(kg K) **between 25^o-100^{o}C**

Enthalpy calculation during freezing

- Not all water in a food turns into ice at freezing point. Some unfrozen water exists below freezing point. Therefore, **Siebel's equations for specific heat below freezing point are very inaccurate**.

- Best method for determining the amount of heat to be removed during freezing is to calculate **enthalpy change**.

- One method for calculating enthalpy change below freezing point (**good only for moisture contents between 73 and 94%**) is the procedure of **Chang and Tao (1981)**.

- In this calculations, all water is asumed to be frozen at **227.6 K** (–45.4°C).

Entalphy calculation for foods containing 73–94% moisture below freezing point

Process steps:

- **1)** The freezing point (T_f) of food in **K** is:

Meat : $T_f = 271.18 \times 1.47 M$

Fruits and vegetables: $T_f = 287.56 - 49.19 M + 37.07 M^2$

Juices: $T_f = 120.47 + 327.35 M - 176.49 M^2$

2) A reduced temp. (T_r) is calculated:

$$\boxed{T_r = \frac{T - 227.6}{T_f - 227.6}}$$

T_f: Freezing point,

T: Final temperature below freezing point

3) ***a*** and ***b*** parameters are calculated as a function of the mass fraction of moisture in the product:

Meat :

$a = 0.316 - 0.247\,(M - 0.73) - 0.688\,(M - 0.73)^2$

$b = 22.95 + 54.68\,(a - 0.28) - 5589.03\,(a - 0.28)^2$

Fruit and vegetables, and their juices:

$a = 0.362 - 0.0498(M - 0.73) - 3.465\,(M - 0.73)^2$

$b = 27.2 + 129.04\,(a - 0.23) - 481.46\,(a - 0.23)^2$

4) Entalphy (H_f) of food at freezing point is calculated in «**J/kg**»:

$$H_f = 9792.46 + 405{,}096\ M$$

5) Entalphy of food (H_f) at “T” temp. (final temp. below freezing point) is calculated in «**J/kg**»:

$$H = H_f\,[a\,T_r + (1 - a)\,T_r^{\,b}]$$

6) Entalphy needed to be removed to cool the food from T_f to the “T” temp. is calculated from the following equation:

$$\Delta H = H - H_f$$

- **Example 4.16:** Calculate the amount of heat which must be removed in order to freeze 1 kg of grape juice containing 25% solids from the freezing point to –30°C.

Answer

- $T_f = 266.7$ K
- $T_r = 0.394$

Answer

- $a = 0.3616$
- $b = 1.879$

Answer

- $H = 79{,}457$ J/kg
- $H_f = 313{,}614$ J/kg

Answer

ΔH = -234 157 J/kg

Specific heats of gases and vapors

- Specific heat of gases depends upon whether the process is carried out at constant pressure or at constant volume.
- Specific heat at constant pressure is designated by C_p.
- Specific heat of gases is tabulated (listed in tables

- Heat required to raise the temperature of a gas with mass *m* at constant pressure equals the change in enthalpy, ΔH.

- **Enthalpy change** associated with a change in temperature from a reference temperature T_o to T_2 is:

Change in entalphy;

$$\boxed{\begin{gathered}\Delta H = m \int_{T_o}^{T_2} C_p \, dT \\[2em] \Delta H = m \; C_{pm} \, (T_2 - T_o)\end{gathered}}$$

C_{pm} : mean specific heat from the
reference temperature T_o to T_2.

Heating a gas from T_1 temp. to T_2 temp.;

$$\boxed{\Delta H = m\,C_{pm}\,(T_2 - T_o) - m\,C'_{pm}\,(T_1 - T_o)}$$

C'_{pm} : mean specific heat from the reference temperature T_o to T_1.

- **Example 4.17**: Calculate the heating requirement for drying the apples in an air drier that uses 2000 ft^3/min air at 1 atm and 170°F if ambient air at 70°F is heated to 170°F for use in the process.

$R = 1545\ lb_f\ ft/(lb_{mole}\ R)$

Answer

$$\Delta H = 3039.8 \ \ \text{BTU/min}$$

- **Example 4.18**: How much heat would be required to raise the temperature of 10 m^3/s air at 50°C to 120°C at 1 atm?

  $R = 8.206$ atm $m^3/(kg_{mole}\ K)$

Answer

$$\Delta H = 7758.7 \text{ J/s}$$