1. SETS

Definition: In mathematics, a well-defined collection of distinct objects is called a set. We generally use capital numbers to denote sets such as A, B, X, Y... An object from a set is called an element of the set. We generally use lowercase letters to denote elements such as a, b, x, y...

If x is an element of the set A then we write $x \in A$. The set which does not have any element is called the empty set. We dente the empty set with $\varnothing$.

Definition: Let X and Y be two sets. If for any $x \in X$ we have $y \in Y$ then we say that X is a subset of Y or Y contains X. I this case we write $X \subset Y$. Note that empty set is the subset of any set.

Set operations: Let X be a set and let $A, B \subset X$.

i) The set which consists of all common elements of A and B is called the intersection of A and B. This set is denoted by $A \cap B$.

$$A \cap B := \{x \in X : x \in A \wedge x \in B\}$$

ii) The set which consists of all elements of A and B is called the union of A and B. This set is denoted by $A \cup B$.

$$A \cup B := \{x \in X : x \in A \vee x \in B\}$$

iii) The set that consists of the elements which do not belong A is called the complement of A and denoted by A^c.

Example 1: Prove that

$$\begin{array}{rl} i) & (A \cup B)^c = A^c \cap B^c \\ ii) & (A \cap B)^c = A^c \cup B^c. \end{array}$$

Solution:

$$\begin{array}{lllll} \mathbf{i}) & x \in (A \cup B)^c & \Leftrightarrow & x \notin A \cup B \\ & & \Leftrightarrow & x \notin A \; and \; x \notin B \\ & & \Leftrightarrow & x \in A^c \; and \; x \in B^c \\ & & \Leftrightarrow & x \in A^c \cap B^c \\ \mathbf{ii}) & x \in (A \cap B)^c & \Leftrightarrow & x \notin A \cap B \\ & & \Leftrightarrow & x \notin A \; or \; x \notin B \\ & & \Leftrightarrow & x \in A^c \; or \; x \in B^c \\ & & \Leftrightarrow & x \in A^c \cup B^c \end{array}$$

Example 2: Prove that

$$(A \cap B)\text{x}C \quad = \quad (A\text{x}C) \cap (B\text{x}C)\,.$$

Solution:

$$\begin{aligned}
x \in (A \cap B)\text{x}C &\Leftrightarrow x \in A \cap B \textit{ and } y \in C \\
&\Leftrightarrow (x \in A \textit{ and } x \in B) \textit{ and } y \in C \\
&\Leftrightarrow (x \in A \textit{ and } y \in C) \textit{ and } (x \in B \textit{ and } y \in C) \\
&\Leftrightarrow (x, y) \in A\text{x}C \textit{ and } (x, y) \in B\text{x}C \\
&\Leftrightarrow (x, y) \in (A\text{x}C) \cap (B\text{x}C)
\end{aligned}$$