

6. FUNCTIONS-3

Example 1: Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by

$$a) f(x) = e^{-x^2} - \cos(\sin 5x) \quad \dots\dots \quad b) f(x) = \tan x - \sqrt[3]{x^3 - x}$$

Determine whether f is even or odd function.

Solution:

a) Since $f(-x) = e^{-(-x)^2} - \cos(\sin(-5x)) = e^{-x^2} - \cos(\sin 5x) = f(x)$, f is an even function.

b) Since $f(-x) = \tan(-x) - \sqrt[3]{(-x)^3 - (-x)} = -\tan x + \sqrt[3]{x^3 - x} = -f(x)$, f is an odd function.

Example 2: Prove that

- i) If f is an increasing function, $-f$ is a decreasing function.
- ii) If f is a decreasing function, $-f$ is an increasing function.

Solution:

i) Let f is an increasing function. Then we have

$$\begin{aligned} x_1 < x_2 &\Rightarrow f(x_1) < f(x_2) \\ &\Rightarrow -f(x_1) > -f(x_2) \\ &\Rightarrow (-f)(x_1) > (-f)(x_2) \end{aligned}$$

which shows that $-f$ is a decreasing function.

ii) Homework.