In general, the following equation is called as a Wave Equation:

$$\frac{\partial^2 f}{\partial u^2} + \frac{1}{v^2}\frac{\partial^2 f}{\partial u^2} = 0$$

A scalar or vectorel function $f(u,t)$ satisfying the above equation is a wave. It can be shown that $f(u,t) = f(u \pm vt)$ satisfies the wave equation.

Maxwell's Equations can be used to show that time varying electric and time-varying magnetic field satisfies the wae equation:

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	Faraday's Law
$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	Ampere's Law
$\nabla \bullet \vec{D} = \rho_v$	Gauss' Law
$\nabla \bullet \vec{B} = 0$	No isolated magnetic charge exists

$$\nabla \times \nabla \times \vec{E} = \nabla(\nabla \bullet \vec{E})\text{-}\ \nabla^2 \vec{E}$$

$$\nabla \times \nabla \times \vec{E} = \nabla(\nabla \bullet \vec{E})\text{-}\ \nabla^2 \vec{E} = \nabla \times (-\frac{\partial \vec{B}}{\partial t})$$

$$\nabla(\nabla \bullet \vec{E})\text{-}\ \nabla^2 \vec{E} = (-\nabla \times \frac{\partial \vec{B}}{\partial t})$$

$$\nabla(\nabla \bullet \vec{E})\text{-}\ \nabla^2 \vec{E} = (-\frac{\partial}{\partial t}\nabla \times \vec{B})$$

$$\vec{B} = \mu \vec{H}$$

$$\nabla(\nabla \bullet \vec{E})\text{-}\ \nabla^2 \vec{E} = (-\frac{\partial}{\partial t}\nabla \times \mu \vec{H})$$

$$\nabla(\nabla \bullet \vec{E})\text{-}\ \nabla^2 \vec{E} = (-\frac{\partial}{\partial t}\mu \nabla \times \vec{H})$$

$$\nabla(\nabla \bullet \vec{E})\text{-}\ \nabla^2 \vec{E} = -\mu(\frac{\partial}{\partial t}(\nabla \times \vec{H}))$$

$$\nabla(\nabla \bullet \vec{E})\text{-}\ \nabla^2 \vec{E} = -\mu(\frac{\partial}{\partial t}(\vec{J} + \frac{\partial \vec{D}}{\partial t}))$$

$$\vec{D} = \varepsilon\vec{E}$$

$$\nabla \bullet \vec{D} = \rho_v$$

$$\nabla \bullet \varepsilon\vec{E} = \rho_v$$

$$\varepsilon\nabla \bullet \vec{E} = \rho_v$$

$$\nabla \bullet \vec{E} = \frac{\rho_v}{\varepsilon}$$

$$\nabla(\nabla \bullet \vec{E}) - \nabla^2\vec{E} = -\mu\left(\frac{\partial}{\partial t}\left(\vec{J} + \frac{\partial\varepsilon\vec{E}}{\partial t}\right)\right)$$

$$\nabla(\nabla \bullet \vec{E}) - \nabla^2\vec{E} = -\mu\left(\frac{\partial}{\partial t}\left(\vec{J} + \varepsilon\frac{\partial\vec{E}}{\partial t}\right)\right)$$

$$\nabla(\nabla \bullet \vec{E}) - \nabla^2\vec{E} = -\mu\left(\frac{\partial}{\partial t}\vec{J} + \left(\varepsilon\frac{\partial\vec{E}}{\partial t}\right)\right)$$

$$\nabla(\nabla \bullet \vec{E}) - \nabla^2\vec{E} = -\mu\frac{\partial\vec{J}}{\partial t} - \mu\frac{\partial}{\partial t}\left(\varepsilon\frac{\partial\vec{E}}{\partial t}\right)$$

$$\nabla(\nabla \bullet \vec{E}) - \nabla^2\vec{E} = -\mu\frac{\partial\vec{J}}{\partial t} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\nabla\left(\frac{\rho_v}{\varepsilon}\right) - \nabla^2\vec{E} = -\mu\frac{\partial\vec{J}}{\partial t} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\frac{\nabla(\rho_v)}{\varepsilon} - \nabla^2\vec{E} = -\mu\frac{\partial\vec{J}}{\partial t} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\nabla^2\vec{E} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = \mu\frac{\partial\vec{J}}{\partial t} + \frac{\nabla(\rho_v)}{\varepsilon}$$

In the source free region $\vec{J} = 0$ and $\rho_v = 0$

$$\nabla^2\vec{E} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0$$

This equation is in the same form with the wa ve equation that we ha ve defined in the beginning. As a result, $\vec{E}$ satisfies the wa ve equation.

Again starting from Maxwell's Equations:

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	Faraday's Law
$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	Ampere's Law
$\nabla \bullet \vec{D} = \rho_v$	Gauss' Law
$\nabla \bullet \vec{B} = 0$	No isolated magnetic charge exists

$$\nabla \times \nabla \times \vec{H} = \nabla(\nabla \bullet \vec{H}) - \nabla^2 \vec{H}$$

$$\nabla \times \nabla \times \vec{H} = \nabla(\nabla \bullet \vec{H}) - \nabla^2 \vec{H} = \nabla \times (\vec{J} + \frac{\partial \vec{D}}{\partial t})$$

$$\nabla(\nabla \bullet \vec{H}) - \nabla^2 \vec{H} = \nabla \times \vec{J} + \nabla \times (\frac{\partial \vec{D}}{\partial t})$$

$$\nabla(\nabla \bullet \vec{H}) - \nabla^2 \vec{H} = \nabla \times \vec{J} + (\frac{\partial}{\partial t} \nabla \times \vec{D})$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\nabla(\nabla \bullet \vec{H}) - \nabla^2 \vec{H} = \nabla \times \vec{J} + \frac{\partial}{\partial t} (\nabla \times \varepsilon \vec{E})$$

$$\nabla(\nabla \bullet \vec{H}) - \nabla^2 \vec{H} = \nabla \times \vec{J} + \frac{\partial}{\partial t} \varepsilon (\nabla \times \vec{E})$$

$$\nabla(\nabla \bullet \vec{H}) - \nabla^2 \vec{H} = \nabla \times \vec{J} + \varepsilon \frac{\partial}{\partial t} (\nabla \times \vec{E})$$

$$\nabla(\nabla \bullet \vec{H}) - \nabla^2 \vec{H} = \nabla \times \vec{J} + \varepsilon \frac{\partial}{\partial t} (-\frac{\partial \vec{B}}{\partial t})$$

$$\nabla(\nabla \bullet \vec{H}) - \nabla^2 \vec{H} = \nabla \times \vec{J} - \varepsilon \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\vec{B} = \mu \vec{H}$$

$$\vec{H} = \frac{\vec{B}}{\mu}$$

$$\nabla(\nabla \bullet (\frac{\vec{B}}{\mu}))\text{-}\ \nabla^2\vec{H}=\nabla \times \vec{J} - \varepsilon\frac{\partial^2\vec{B}}{\partial t^2}$$

$$\nabla(\frac{1}{\mu}\left(\nabla \bullet \vec{B}\right))\text{-}\ \nabla^2\vec{H}=\nabla \times \vec{J} - \varepsilon\frac{\partial^2\vec{B}}{\partial t^2}$$

$$-\ \nabla^2\vec{H}=\nabla \times \vec{J} - \varepsilon\frac{\partial^2(\mu\vec{H})}{\partial t^2}$$

$$-\ \nabla^2\vec{H}=\nabla \times \vec{J} - \mu\varepsilon\frac{\partial^2\vec{H}}{\partial t^2}$$

$$\nabla^2\vec{H} - \mu\varepsilon\frac{\partial^2\vec{H}}{\partial t^2}=-\nabla \times \vec{J}$$

In the source free region $\vec{J} = 0$

$$\nabla^2\vec{H} - \mu\varepsilon\frac{\partial^2\vec{H}}{\partial t^2}=0$$

This equation is in the same form with the wa ve equation that we ha ve defined in the beginning. As a result, $\vec{H}$ satisfies the wa ve equation.