Static Field	$A(x,y,z)$
Time-varying field	$A(x,y,z;t)$
Time harmonic	$A(x,y,z;t)$ is in the form $\cos(\omega t+\theta_0)$

Phasor representation:
Any sinusodial signal can be represented by three parameters:

I_0: Amplitude
ω_0:Angular frequency (rad/sec)
ϕ_0: Phase (Radians)

$$i(t) = I_0 \cos(\omega_0 t + \phi_0)$$

Expressing the signal in phasor form simplifies the mathematical operations

Phasor expression of a Time-harmonic signal:

$$\vec{E}(x,y,z;t) = Re\{\vec{E}(x,y,z)e^{j\omega t}\}$$

$$\vec{E}(x,y,z) = E_R + jE_I$$

$$e^{j\omega t} = \cos(\omega t) + jsin(\omega t)$$

$$\vec{E}(x,y,z;t) = Re\{\vec{E}(x,y,z)\,(\cos(\omega t) + jsin(\omega t))\}$$

$$\vec{E}(x,y,z;t) = Re\{(E_R + jE_I)\,(\cos(\omega t) + jsin(\omega t))\}$$

$$\vec{E}(x,y,z;t) = Re\{(E_R\cos(\omega t) - E_I sin(\omega t)) + j(E_R\sin(\omega t) + E_I cos(\omega t))\}$$

$$\vec{E}(x,y,z;t) = E_R\cos(\omega t) - E_I sin(\omega t)$$

$$\vec{E}(x,y,z) = E_R + jE_I = |\vec{E}|e^{j\emptyset_E}$$

$$\vec{E}(x,y,z;t) = Re\{\vec{E}(x,y,z)e^{j\omega t}\}$$

$$\vec{E}(x,y,z;t) = Re\{|\vec{E}|e^{j\emptyset_E}e^{j\omega t}\}$$

$$\vec{E}(x,y,z;t) = Re\{|\vec{E}|e^{j(\omega t+\emptyset_E)}\}$$

$$\vec{E}(x,y,z;t) = |\vec{E}|\cos(\omega t + \emptyset_E)$$

Note the difference of notation between time-harmonic signal and phasor representation of the same signal:

Time-harmonic field: $\vec{E}(x,y,z;t)$
Phasor representation of the same signal: $\vec{E}(x,y,z)$

$$\frac{d}{dt}(e^{j\omega t}) = j\omega(e^{j\omega t})$$

$$\int (e^{j\omega t})dt = \frac{1}{j\omega}(e^{j\omega t})$$

As a result:

$$\frac{\partial \vec{E}(x,y,z;t)}{\partial t} \leftrightarrow j\omega \vec{E}(x,y,z)$$

$$\int \vec{E}(x,y,z;t)dt \leftrightarrow \frac{\vec{E}(x,y,z)}{j\omega}$$

which simplifies the mathematical operations.

Time Domain Maxwell Equations:

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	Faraday's Law
$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	Ampere's Law
$\nabla \bullet \vec{D} = \rho_v$	Gauss' Law
$\nabla \bullet \vec{B} = 0$	No isolated magnetic charge exists

Phasor Domain Maxwell Equations:

$\nabla \times \vec{E} = -j\omega \vec{B}$	Faraday's Law
$\nabla \times \vec{H} = \vec{J} + j\omega \vec{D}$	Ampere's Law
$\nabla \bullet \vec{D} = \rho_v$	Gauss' Law
$\nabla \bullet \vec{B} = 0$	No isolated magnetic charge exists

Using Faraday's Law, taking the curl:

$$\nabla \times \nabla \times \vec{E} = \nabla(\nabla \bullet \vec{E}) - \nabla^2 \vec{E}$$

$$\nabla \times \nabla \times \vec{E} = \nabla(\nabla \bullet \vec{E}) - \nabla^2 \vec{E} = \nabla \times (-j\omega \vec{B})$$

$$\nabla(\nabla \bullet \vec{E}) - \nabla^2 \vec{E} = (-\nabla \times j\omega \vec{B})$$

$$\nabla(\nabla \bullet \vec{E}) - \nabla^2 \vec{E} = (-j\omega \nabla \times \vec{B})$$

$$\vec{B} = \mu \vec{H}$$

$$\nabla(\nabla \bullet \vec{E}) - \nabla^2 \vec{E} = (-j\omega \nabla \times \mu \vec{H})$$

$$\nabla(\nabla \bullet \vec{E}) - \nabla^2 \vec{E} = (-j\omega \mu \nabla \times \vec{H})$$

$$\nabla(\nabla \bullet \vec{E}) - \nabla^2 \vec{E} = -\mu(j\omega(\nabla \times \vec{H}))$$

$$\nabla(\nabla \bullet \vec{E}) - \nabla^2 \vec{E} = -\mu(j\omega(\vec{J} + j\omega \frac{\partial \vec{D}}{\partial t}))$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\nabla \bullet \vec{D} = \rho_v$$

$$\nabla \bullet \varepsilon \vec{E} = \rho_v$$

$$\varepsilon \nabla \bullet \vec{E} = \rho_v$$

$$\nabla \bullet \vec{E} = \frac{\rho_v}{\varepsilon}$$

$$\nabla(\nabla \bullet \vec{E}) - \nabla^2 \vec{E} = -\mu(j\omega(\vec{J} + j\omega \varepsilon \vec{E}))$$

$$\nabla(\nabla \bullet \vec{E}) - \nabla^2 \vec{E} = -\mu(j\omega(\vec{J} + j\omega \varepsilon \vec{E}))$$

$$\nabla(\nabla \bullet \vec{E}) - \nabla^2 \vec{E} = -\mu(j\omega \vec{J} + j\omega(\varepsilon j\omega \vec{E}))$$

$$\nabla(\nabla \bullet \vec{E}) - \nabla^2 \vec{E} = -\mu j\omega \vec{J} + \mu \varepsilon \omega^2 \vec{E}$$

$$\nabla(\frac{\rho_v}{\varepsilon}) - \nabla^2 \vec{E} = -\mu j\omega \vec{J} + \mu \varepsilon \omega^2 \vec{E}$$

$$\nabla^2 \vec{E} + \mu \varepsilon \omega^2 \vec{E} = \mu j\omega \vec{J} + \frac{\nabla(\rho_v)}{\varepsilon}$$

In the source free region $\vec{J} = 0$ and $\rho_v = 0$

$$\nabla^2\vec{E} + k^2\vec{E}=0$$

$$k = \omega\sqrt{\mu\varepsilon}$$

This equation is in the same form with the wave equation that we have defined in the beginning. As a result, $\vec{E}$ satisfies the wave equation.

Using Ampere's Law, taking the curl:

$$\nabla \times \nabla \times \vec{H} = \nabla(\nabla \bullet \vec{H})\text{-}\ \nabla^2\vec{H}$$

$$\nabla \times \nabla \times \vec{H} = \nabla(\nabla \bullet \vec{H})\text{-}\ \nabla^2\vec{H}=\nabla \times (\vec{J} + j\omega\vec{D})$$

$$\nabla(\nabla \bullet \vec{H})\text{-}\ \nabla^2\vec{H}=\nabla \times \vec{J} + \nabla \times (j\omega\varepsilon\vec{E})$$

$$\nabla(\nabla \bullet \vec{H})\text{-}\ \nabla^2\vec{H}=\nabla \times \vec{J} + j\omega\varepsilon(-j\omega\vec{B})$$

$$\vec{B} = \mu\vec{H}$$

$$\nabla(\nabla \bullet \vec{H})\text{-}\ \nabla^2\vec{H}=\nabla \times \vec{J} + j\omega\varepsilon(-j\omega\mu\vec{H})$$

$$\nabla(\nabla \bullet \vec{H})\text{-}\ \nabla^2\vec{H}=\nabla \times \vec{J} + \mu\varepsilon\omega^2\vec{H}$$

$$\nabla(\nabla \bullet \frac{\vec{B}}{\mu})\text{-}\ \nabla^2\vec{H}=\nabla \times \vec{J} + \mu\varepsilon\omega^2\vec{H}$$

$$\nabla(\frac{1}{\mu}\nabla \bullet \vec{B})\text{-}\ \nabla^2\vec{H}=\nabla \times \vec{J} + \mu\varepsilon\omega^2\vec{H}$$

$$\nabla^2\vec{H} + \mu\varepsilon\omega^2\vec{H} =\text{-}\ \nabla \times \vec{J}$$

In the source free region $\vec{J} = 0$ and $\rho_v = 0$

$$\nabla^2\vec{H} + k^2\vec{H}=0$$

$$k = \omega\sqrt{\mu\varepsilon}$$