Time varying fields and Maxwell's equations

Static electric fields are produced by electric charges,

Static magnetic fields are produced by charges in motion or by steady current.

Static electric field is a conservative field and has no curl:

$$\nabla \times \vec{E} = 0$$

The static magnetic field is continuous and its divergence is zero.

$$\nabla \cdot \vec{D} = \rho_v$$

For a linear and isotropic medium,

$$\vec{D} = \varepsilon \vec{E}$$

$$\nabla . \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{J}$$

$$\vec{B} = \mu \vec{H}$$

Faraday’s Law of Induction

Two scientists are given credit for the discovery of electromagnetic induction: the Englishman Michael Faraday (1791 – 1867) and the American Joseph Henry (1797 – 1878). Henry was the first to observe electromagnetic induction, but Faraday investigated it in more detail and published his findings first. Michael Faraday, in 1831 discovered experimentally that a current was induced in a conducting loop when the magnetic flux linking the loop changed Hence the law of induction bears Faraday’s name.

Whenever there is a change in magnetic flux through a loop of wire, an electromotive force emf E is induced in the loop. The quantitative relation between the induced emf (the voltage that arises from conductors moving in a magnetic field or from changing magnetic fields) and the rate of change of flux linkage developed based on experimental observation is known as Faraday's law. The induced emf is equal to the negative of the rate of change of magnetic flux multiplied by the number of turns N in the loop:

$\mathscr{E} = -N\dfrac{\Delta\Phi}{\Delta t}$ where ϕ is the flux linkage over the closed path.

The unit of induced emf is the *volt*: $\frac{\mathrm{T\cdot m^2}}{\mathrm{s}}=\frac{\mathrm{N\cdot m^2/(A\cdot m)}}{\mathrm{s}}=\frac{\mathrm{N\cdot m}}{\mathrm{A\cdot s}}=\frac{\mathrm{J}}{\mathrm{C}}=volts$.

A non zero $\frac{d\phi}{dt}$ may result due to any of the following:

(a) time changing flux linkage a stationary closed path.

(b) relative motion between a steady flux a closed path.

(c) a combination of the above two cases.

The negative sign was introduced by Lenz in order to comply with the polarity of the induced emf. The negative sign implies that the induced emf will cause a current flow in the closed loop in such a direction so as to oppose the change in the linking magnetic flux which produces it. (It may be noted that as far as the induced emf is concerned, the closed path forming a loop does not necessarily have to be conductive).

If the closed path is in the form of N tightly wound turns of a coil, the change in the magnetic flux linking the coil induces an emf in each turn of the coil and total emf is the sum of the induced emfs of the individual turns, i.e.,

$$Emf = -N\frac{d\phi}{dt} \quad Volts$$

By defining the total flux linkage as

$$\lambda = N\phi$$

The emf can be written as

$$\text{Emf} = -\frac{d\lambda}{dt}$$

Over a closed contour '*C*' we can write

$$\text{Emf} = \oint_C \vec{E}.d\vec{l}$$

where $\vec{E}$ is the induced electric field on the conductor to sustain the current.

Further, total flux enclosed by the contour 'C ' is given by

$$\phi = \int_S \vec{B}.d\vec{s}$$

Where S is the surface for which 'C' is the contour.

$$\oint_C \vec{E}.d\vec{l} = -\frac{\partial}{\partial t}\oint_s \vec{B}.d\vec{s}$$

By applying stokes theorem

$$\int_s \nabla\times\vec{E}.d\vec{s} = -\int_s \frac{\partial\vec{B}}{\partial t}.d\vec{s}$$

Therefore, we can write

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

which is the Faraday's law in the point form

We have said that non zero $\frac{d\phi}{dt}$ can be produced in a several ways. One particular case is when a time varying flux linking a stationary closed path induces an emf. The emf induced in a stationary closed path by a time varying magnetic field is called a transformer emf**.**

From Principle of Charge Conservation:

$$I = -\frac{dQ}{dt} = -\frac{d}{dt}\int_v (\rho_v\, dv) = -\int_v \frac{d\rho_v}{dt}dv$$

$$I = \oint_S \vec{J}\bullet\vec{dS} = \int_v \nabla\bullet\vec{J}\,dV$$

$$-\int_v \frac{d\rho_v}{dt}dv = \int_v \nabla\bullet\vec{J}\,dV$$

$$-\frac{d\rho_v}{dt} = \nabla\bullet\vec{J}$$ Continuity Equation

$$\nabla\times\vec{H} = \vec{J}+\vec{X}$$

$$\nabla\bullet\nabla\times\vec{H} = \nabla\bullet(\vec{J}+\vec{X})$$

$$\nabla\bullet\nabla\times\vec{H} = \nabla\bullet\vec{J}+\nabla\bullet\vec{X}$$

$$\nabla\bullet\nabla\times\vec{H}=0$$

$$\nabla \bullet \vec{J} + \nabla \bullet \vec{X} = 0$$

$$-\frac{d\rho_v}{dt} + \nabla \bullet \vec{X} = 0$$

$$-\frac{d\nabla \bullet \vec{D}}{dt} + \nabla \bullet \vec{X} = 0$$

$$-\nabla \bullet \frac{d\vec{D}}{dt} + \nabla \bullet \vec{X} = 0$$

$$\nabla \bullet \left(-\frac{d\vec{D}}{dt} + \vec{X}\right) = 0$$

$$-\frac{d\vec{D}}{dt} + \vec{X} = 0$$

$$\vec{X} = \frac{d\vec{D}}{dt}$$

$$\nabla \times \vec{H} = \vec{J} + \vec{X}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$$

Maxwell's Equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \bullet \vec{D} = \rho_v$$

$$\nabla \bullet \vec{B} = 0$$