Potential Functions

$$\vec{B} = \nabla \times \vec{A}$$

From Faraday'a Law

$$\nabla \times \vec{E} = -\frac{\partial(\nabla \times \vec{A})}{\partial t}$$

$$\nabla \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t}\right) = 0$$

$$\vec{E} + \frac{\partial \vec{A}}{\partial t} = -\nabla V$$

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$

$$\nabla \times \nabla \times \vec{A} = \mu \vec{J} + \mu\varepsilon \frac{\partial}{\partial t}(-\nabla V - \frac{\partial \vec{A}}{\partial t})$$

$$\nabla(\nabla \bullet \vec{A})\text{-}\ \nabla^2 \vec{A} = \ \mu \vec{J} - \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} - \mu\varepsilon \frac{\partial}{\partial t} \nabla V$$

$$\nabla(\nabla \bullet \vec{A})\text{-}\ \nabla^2 \vec{A} = \ \mu \vec{J} - \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} - \mu\varepsilon \nabla \frac{\partial}{\partial t} V$$

$$\nabla \bullet \vec{A} + \mu\varepsilon \frac{\partial}{\partial t} V = 0$$

$$\nabla^2 \vec{A} - \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} = \ -\mu \vec{J} - \nabla(\nabla \bullet \vec{A} + \mu\varepsilon \frac{\partial}{\partial t} V)$$

$$\nabla^2 \vec{A} - \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} = \ -\mu \vec{J}$$

$$\nabla \bullet \vec{D} = \rho_v$$

$$\nabla \bullet \varepsilon \vec{E} = \rho_v$$

$$\varepsilon \nabla \bullet \vec{E} = \rho_v$$

$$\varepsilon\nabla \bullet (-\nabla V - \frac{\partial \vec{A}}{\partial t}) = \rho_v$$

$$-\varepsilon\nabla \bullet \nabla V - \varepsilon\nabla \bullet (\frac{\partial \vec{A}}{\partial t}) = \rho_v$$

$$\nabla^2 V + \frac{\partial}{\partial t}(\nabla \bullet \vec{A}) = -\frac{\rho_v}{\varepsilon}$$

$$\nabla \bullet \vec{A} + \mu\varepsilon \frac{\partial}{\partial t} V = \frac{\rho_v}{\varepsilon}$$

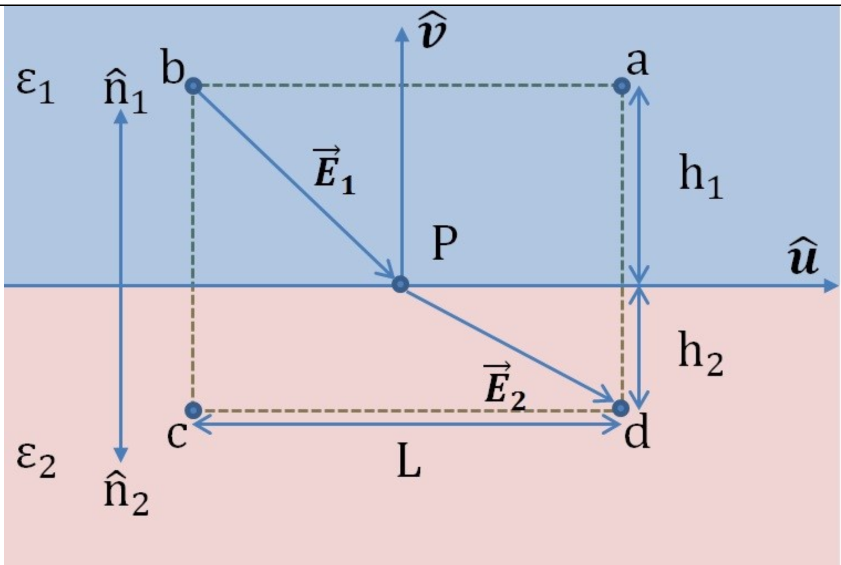
$$\nabla \bullet \vec{A} = -\mu\varepsilon \frac{\partial V}{\partial t}$$

$$\nabla^2 V + \frac{\partial}{\partial t}(-\mu\varepsilon \frac{\partial V}{\partial t}) = -\frac{\rho_v}{\varepsilon}$$

$$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho_v}{\varepsilon}$$

Boundary conditions for time-varying electromagnetics fields

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	
$\int_S \nabla \times \vec{E} \bullet \overrightarrow{dS} = \oint_C \vec{E} \bullet \overrightarrow{dl} = \int_S -\frac{\partial \vec{B}}{\partial t} \bullet \overrightarrow{dS}$	
$\int_S \nabla \times \vec{E} \bullet \overrightarrow{dS} = \oint_C \vec{E} \bullet \overrightarrow{dl} = -\frac{\partial}{\partial t}\int_S \vec{B} \bullet \overrightarrow{dS} = 0$	

	
$\oint_C \vec{E} \bullet \overrightarrow{dl} = \sum_{i=1}^{4} \int_{C_i} \vec{E} \bullet \overrightarrow{dl}$	
$\oint_C \vec{E} \bullet \overrightarrow{dl} = \int_a^b \vec{E} \bullet \overrightarrow{dl} + \int_b^c \vec{E} \bullet \overrightarrow{dl} + \int_c^d \vec{E} \bullet \overrightarrow{dl} + \int_d^a \vec{E} \bullet \overrightarrow{dl} +$	
$\int_b^c \vec{E} \bullet \overrightarrow{dl} = \int_d^a \vec{E} \bullet \overrightarrow{dl} = 0$	
$\int_a^b \vec{E}_1 \bullet \overrightarrow{dl} + \int_c^d \vec{E}_2 \bullet \overrightarrow{dl} = 0$	
$E_{1//}(-L) + E_{2//}(-L) = 0$	
$E_{1//} = E_{2//}$	
Tangential component of $\vec{E}_1$ is equal to tangential component of $\vec{E}_2$	

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<tr><td colspan="2">Boundary Condition 2</td><td></td></tr>
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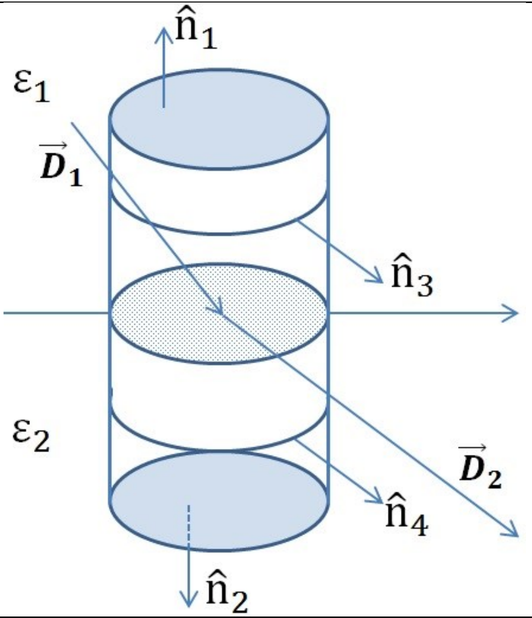

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$$\oint_S \vec{E} \bullet \overrightarrow{dS} = \int_V \frac{\rho_v}{\varepsilon_0} dV$$

$$\oint_S \vec{E} \bullet \overrightarrow{dS} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\oint_S \vec{D} \bullet \overrightarrow{dS} = Q_{enclosed}$$

$$= \rho_S S$$

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$$\oint_S \vec{D} \bullet \overrightarrow{dS} = \int_{S_1} \vec{D}_1 \bullet \overrightarrow{dS} + \int_{S_2} \vec{D}_2 \bullet \overrightarrow{dS} + \int_{S_3} \vec{D}_1 \bullet \overrightarrow{dS}$$

$$+ \int_{S_4} \vec{D}_2 \bullet \overrightarrow{dS}$$

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$$\int_{S_3} \vec{D}_1 \bullet \overrightarrow{dS} \rightarrow 0$$

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$$\int_{S_4} \vec{D}_2 \bullet \overrightarrow{dS} \rightarrow 0$$

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$$\oint_S \vec{D} \bullet \overrightarrow{dS} = \rho_S S = \int_{S_1} \vec{D}_1 \bullet \overrightarrow{dS} + \int_{S_3} \vec{D}_3 \bullet \overrightarrow{dS}$$

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$$D_{1\perp}(-S) + D_{2\perp}(+S) = \rho_S S$$

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$$=> D_{2\perp} - D_{1\perp} = \rho_S$$

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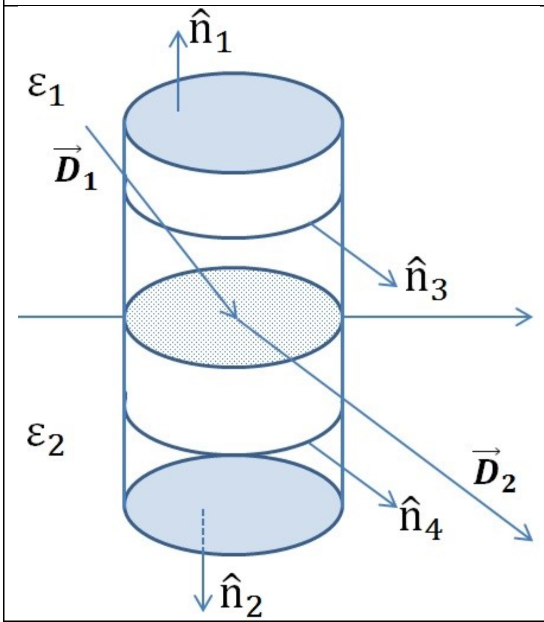
$$=> \hat{n}_1 \bullet \left(\vec{D}_2 - \vec{D}_1\right) = \rho_S$$

$$=> \hat{n}_2 \bullet \left(\vec{D}_1 - \vec{D}_2\right) = \rho_S$$

$$E_{\perp} = \frac{\rho_S}{\varepsilon} \text{ on } E_{\perp} = S$$

Boundary Condition for a conductor surface



$$\oint_S \vec{E} \bullet \overrightarrow{dS} = \int_V \frac{\rho_v}{\varepsilon_0} dV$$

$$\oint_S \vec{E} \bullet \overrightarrow{dS} = \frac{Q_{enclosed}}{\varepsilon_0}$$

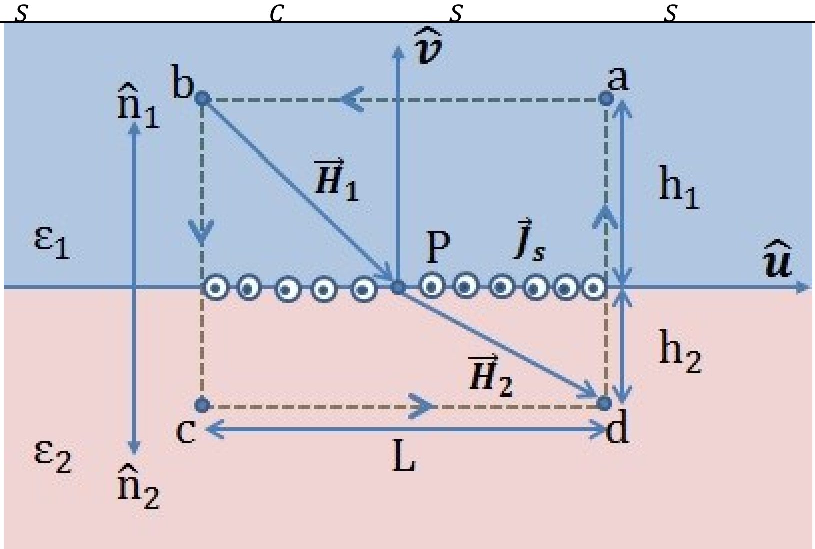
$$\oint_S \vec{E} \bullet \overrightarrow{dS} = \int_{S_1} \vec{E}_1 \bullet \overrightarrow{dS} + \int_{S_2} \vec{E}_2 \bullet \overrightarrow{dS} + \int_{S_3} \vec{E}_3 \bullet \overrightarrow{dS} + \int_{S_4} \vec{E}_4 \bullet \overrightarrow{dS} +$$

$\vec{E}_2$=$\vec{E}_4$=0 (due to fact 1)

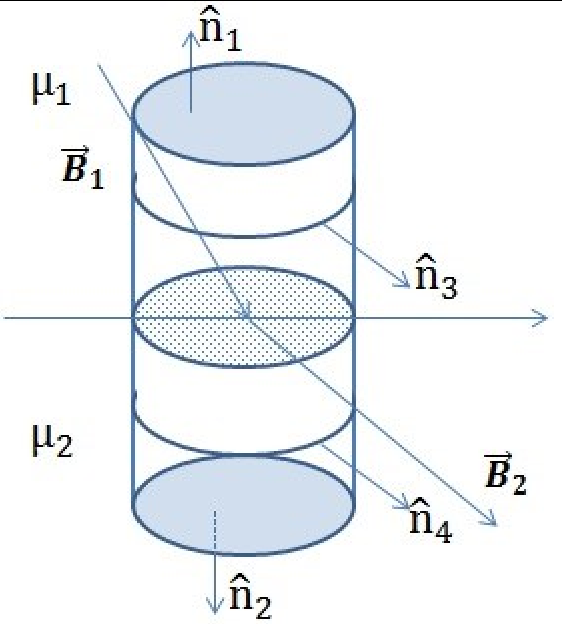
$$\oint_S \vec{E} \bullet \overrightarrow{dS} = \int_{S_1} \vec{E}_1 \bullet \overrightarrow{dS} + \int_{S_3} \vec{E}_3 \bullet \overrightarrow{dS}$$

$$S_3 \to 0 => \int_{S_3} \vec{E}_3 \bullet \overrightarrow{dS} \to 0$$		
$$=> \oint_S \vec{E} \bullet \overrightarrow{dS} = \lim_{h\to 0} E_{1\perp} S_1 = \frac{Q_{enclosed}}{\varepsilon_0}$$		
$$E_{1\perp} S_1 = \frac{S_1 . \rho_s}{\varepsilon_0}$$		
$E_\perp = \frac{\rho_s}{\varepsilon}$ on $E_\perp = S$		

Boundary conditions for time-varying electromagnetics fields

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$	
$$\int_S \nabla \times \vec{H} \bullet \overrightarrow{dS} = \oint_C \vec{H} \bullet \overrightarrow{dl} = \int_S \vec{J} + \frac{\partial \vec{D}}{\partial t} \bullet \overrightarrow{dS}$$	
$$\int_S \nabla \times \vec{H} \bullet \overrightarrow{dS} = \oint_C \vec{H} \bullet \overrightarrow{dl} = \int_S \vec{J} \bullet \overrightarrow{dS} + \frac{\partial}{\partial t}\int_S \vec{D} \bullet \overrightarrow{dS}$$	
	
$$\oint_C \vec{H} \bullet \overrightarrow{dl} = \sum_{i=1}^{4} \int_{C_i} \vec{H} \bullet \overrightarrow{dl}$$	

$\oint_C \vec{H} \bullet \vec{dl} = \int_a^b \vec{H} \bullet \vec{dl} + \int_b^c \vec{H} \bullet \vec{dl} + \int_c^d \vec{H} \bullet \vec{dl} + \int_d^a \vec{H} \bullet \vec{dl}$	
$\int_b^c \vec{H} \bullet \vec{dl} = \int_d^a \vec{H} \bullet \vec{dl} = 0$	
$\int_a^b \vec{H}_1 \bullet \vec{dl} + \int_c^d \vec{H}_2 \bullet \vec{dl} = J_s.L$	
$H_{1//}(-L) + H_{2//}(-L) = J_s.L$	
$\hat{n}_2 \times \left(\vec{H}_1 - \vec{H}_2\right) = \vec{J}_s$	

Boundary Condition 2		
	$\nabla \bullet \vec{B} = 0$ $\oint_S \vec{B} \bullet \vec{dS} = \int_V 0 dV$	
$\oint_S \vec{B} \bullet \vec{dS} = \int_{S_1} \vec{B}_1 \bullet \vec{dS} + \int_{S_2} \vec{B}_2 \bullet \vec{dS} + \int_{S_3} \vec{B}_1 \bullet \vec{dS} + \int_{S_4} \vec{B}_2 \bullet \vec{dS}$		
$\int_{S_3} \vec{B}_1 \bullet \vec{dS} \to 0$		
$\int_{S_4} \vec{B}_2 \bullet \vec{dS} \to 0$		

$\oint_{S} \vec{B} \bullet \overrightarrow{dS} = 0 = \int_{S_1} \vec{B}_1 \bullet \overrightarrow{dS} + \int_{S_3} \vec{B}_3 \bullet \overrightarrow{dS}$	
$B_{1\perp}(-S) + B_{2\perp}(+S) = 0S$	
$=> B_{2\perp} - B_{1\perp} = 0$	
$=> \hat{n}_1 \bullet (\vec{B}_2 - \vec{B}_1) = 0$	
$=> \hat{n}_2 \bullet (\vec{B}_1 - \vec{B}_2) = 0$	