The wavelength λ is defined as the distance between two successive maxima (or minima or any other reference points):

$$(\omega t - kz) - \left[\omega t - k(z+\lambda)\right] = 2\pi$$

$$k\lambda = 2\pi$$

$$\lambda = \frac{2\pi}{k}$$

Substituting $k = \dfrac{\omega}{v_P}$

$$\lambda = \frac{2\pi v_P}{2\pi f} = \frac{v_P}{f}$$

$$\lambda f = v_P$$

Thus, wavelength λ also represents the distance covered in one period of the wave.

Similarly, $E_x^-(z,t) = E_0^- \cos(\omega t + kz)$represents a plane wave traveling in the –z direction.

The associated magnetic field can be found as follows:

$$E_x^+(z) = E_0^+ e^{-jkz}$$

$$\vec{H}(R) = -\frac{1}{j\omega\mu}\nabla \times \vec{E}$$

$$= -\frac{1}{j\omega\mu}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_0^+ e^{-jkz} & 0 & 0 \end{vmatrix}$$

$$= \frac{k}{\omega\mu} E_0^+ e^{-jkz}\hat{y}$$

$$= \frac{E_0^+}{\eta} e^{-jkz}\hat{y}$$

$$= H_0^+ e^{-jkz}\hat{y}$$

where $\eta = \frac{\omega\mu}{k} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}}$ is the intrinsic impedance of the medium.

When the wave travels in free space

$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cong 120\pi = 377\Omega$ is the intrinsic impedance of the free space.

$$= H_0^+ e^{-jkz}\hat{y}$$

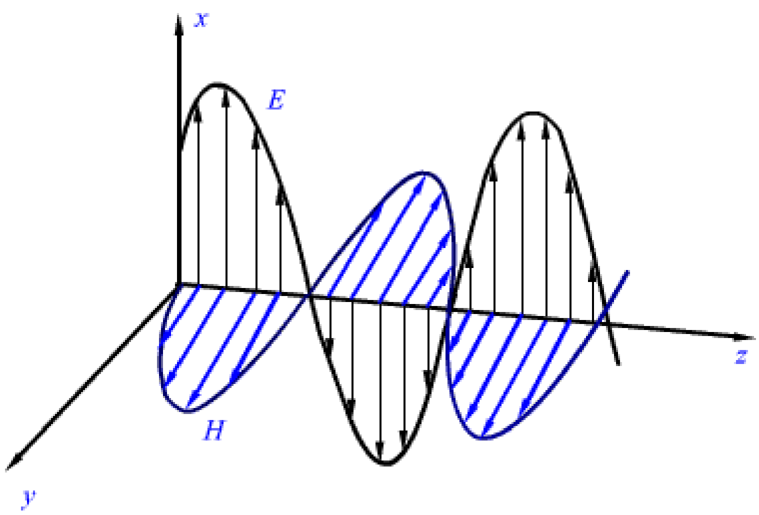
$$H_x^+(z,t) = \frac{E_0^+}{\eta}\cos(\omega t - kz)$$

which represents the magnetic field of the wave traveling in the +z direction.

For the negative traveling wave,

$$H_x^-(z,t) = \frac{E_0^-}{\eta}\cos(\omega t + kz)$$

For the plane waves described, both the *E* & *H* fields are perpendicular to the direction of propagation, and these waves are called TEM (transverse electromagnetic) waves. The *E* & *H* field components of a TEM wave is shown bwlow:



TEM Waves:

For a uniform plane wave propagating in z-direction

$$\boldsymbol{E}(z) = \boldsymbol{E}_0 e^{-jkz}, \ E_0 \text{ is a constant vector}$$

For a wave propagating in any arbitrary direction that doesn't necessarily coincide any axis, the more general form of the above equation is

$$\boldsymbol{E}(x, y, z) = \boldsymbol{E}_0 e^{-jk_x x - jk_y y - jk_z z}$$

This equation satisfies Helmholtz's equation $\nabla^2 \boldsymbol{E} + k^2 \boldsymbol{E} = 0$ provided,

$$k_x^{\ 2} + k_y^{\ 2} + k_z^{\ 2} = k^2 = \omega^2 \mu \varepsilon$$

We define wave number vector :

$$\vec{k} = k_x \hat{x} + k_y \hat{y} + k_z \hat{z}$$

And radius vector from the origin

$$\vec{R} = x\hat{x} + y\hat{y} + z\hat{z}$$

Therefore we can write

$$\vec{E}(R) = \vec{E}_0 e^{-j\vec{k}\cdot\vec{R}}$$

Here $\vec{k} \cdot \vec{R}$=constant is a plane of constant phase and uniform amplitude just in the case of $\boldsymbol{E}(z) = \boldsymbol{E}_0 e^{-jkz}$, z = constant denotes a plane of constant phase and uniform amplitude.

If the region is charge free,

$$\nabla \bullet \vec{E} = 0.$$

$$\nabla \bullet E_0 e^{-j\vec{k}\cdot\vec{R}} = 0$$

Using the vector identity $\nabla.(f\boldsymbol{A}) = \boldsymbol{A}.\nabla f + f\nabla.\boldsymbol{A}$ and noting that $\vec{E}_0$ is constant we can write,

$$\vec{E}_0 \bullet \nabla e^{-j\vec{k}\cdot\vec{R}} = 0$$

$$\vec{E}_0 \bullet \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial x}\hat{y} + \frac{\partial}{\partial x}\hat{z}\right)\nabla e^{-jk_x x + k_y y + k_z z} = 0$$

$$\vec{E}_0 \bullet \hat{k}=0$$

i.e., $\vec{E}_0$ is transverse to the direction of the propagation.

The corresponding magnetic field can be computed as follows:

$$\vec{H}(R) = -\frac{1}{j\omega\mu}\nabla \times \vec{E}$$

$$\vec{H}(R) = -\frac{1}{j\omega\mu}\nabla \times \vec{E}_0 e^{-j\vec{k}\cdot\vec{R}}$$

Using the vector identity,

$$\nabla \times (\alpha\vec{E}) = \alpha\nabla \times \vec{E} + \nabla\alpha \times \vec{E}$$

Since $\vec{E}_0$ is constant one can write,

$$\vec{H}(R) = -\frac{1}{j\omega\mu}\nabla \times \vec{E}$$

$$\vec{H}(R) = -\frac{1}{j\omega\mu}\nabla e^{-j\vec{k}\cdot\vec{R}} \times \vec{E}_0$$

$$\vec{H}(R) = -\frac{1}{j\omega\mu}(-j\vec{k} \times \vec{E}_0 e^{-j\vec{k}\cdot\vec{R}})$$

$$\vec{H}(R) = \frac{k}{\omega\mu}(\hat{n} \times \vec{E}_0 e^{-j\vec{k}\cdot\vec{R}})$$

$$\vec{H}(R) = \frac{1}{\eta}(\hat{n} \times \vec{E}_0 e^{-j\vec{k}\cdot\vec{R}})$$

where η is the intrinsic impedance of the medium and $\vec{k} = k\hat{n}$

$\vec{H}(R)$ is perpendicular to both $\vec{k}$ and $\vec{E}(R)$.

Thus the electromagnetic wave represented by $\vec{E}(R)$ and $\vec{H}(R)$ is a TEM wave.