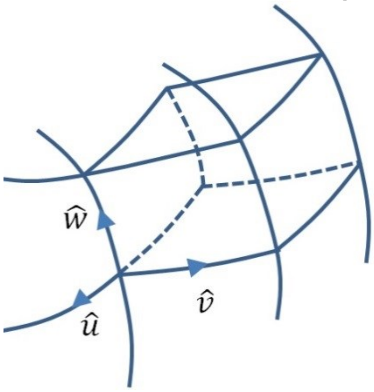
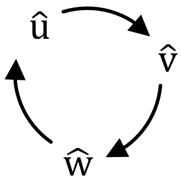
Divergence in Curvilinear Orthogonal Coordinate Systems





$$\hat{u} \times \hat{v} = \hat{w}$$

$$\hat{v} \times \hat{w} = \hat{u}$$

$$\hat{w} \times \hat{u} = \hat{v}$$

$$\vec{F} = F_u(u,v,w)\hat{u} + F_v(u,v,w)\hat{v} + F_w(u,v,w)\hat{w} = \vec{F}(u,v,w)$$

$$\overrightarrow{dS_1} = dS_1\hat{u} = h_v.h_w.dv.dw.\hat{u}$$

$$\overrightarrow{dS_2} = dS_2(-\hat{u}) = -h_v.h_w.dv.dw.\hat{u}$$

On S_1

$$\vec{F}\left(u+\frac{\Delta u}{2},v,w\right) \bullet \overrightarrow{dS_1} =$$

$$F_u\left(u+\frac{\Delta u}{2},v,w\right)\hat{u} \bullet dS_1\hat{u} + F_v\left(u+\frac{\Delta u}{2},v,w\right)\hat{v} \bullet dS_1\hat{u} + F_w\left(u+\frac{\Delta u}{2},v,w\right)\hat{w} \bullet dS_1\hat{u}$$

$$= F_u\left(u+\frac{\Delta u}{2},v,w\right) h_v\left(u+\frac{\Delta u}{2},v,w\right).h_w\left(u+\frac{\Delta u}{2},v,w\right).dv.dw.\hat{u} \bullet \hat{u}$$

$$= F_u\left(u+\frac{\Delta u}{2},v,w\right) h_v\left(u+\frac{\Delta u}{2},v,w\right).h_w\left(u+\frac{\Delta u}{2},v,w\right).dv.dw.1$$

On S_2

$$\vec{F}\left(u-\frac{\Delta u}{2},v,w\right) \bullet \overrightarrow{dS_1} =$$

$$F_u\left(u-\frac{\Delta u}{2},v,w\right)\hat{u} \bullet dS_1(-\hat{u}) + F_v\left(u-\frac{\Delta u}{2},v,w\right)\hat{v} \bullet dS_1(-\hat{u}) + F_w\left(u-\frac{\Delta u}{2},v,w\right)\hat{w} \bullet dS_1(-\hat{u})$$

$$= F_u\left(u-\frac{\Delta u}{2},v,w\right).h_v\left(u-\frac{\Delta u}{2},v,w\right).h_w\left(u-\frac{\Delta u}{2},v,w\right).dv.dw.\hat{u} \bullet (-\hat{u})$$

$$= F_u\left(u-\frac{\Delta u}{2},v,w\right).h_v\left(u-\frac{\Delta u}{2},v,w\right).h_w\left(u-\frac{\Delta u}{2},v,w\right).dv.dw.(-1)$$

$$\vec{F} \bullet \overrightarrow{dS_1}\Big|_{S_1} + \vec{F} \bullet \overrightarrow{dS_2}\Big|_{S_2} = \vec{F}\left(u+\frac{\Delta u}{2},v,w\right) \bullet \overrightarrow{dS_1} + \vec{F}\left(u-\frac{\Delta u}{2},v,w\right) \bullet \overrightarrow{dS_2}$$

$$= F_u\left(u+\frac{\Delta u}{2},v,w\right) h_v\left(u+\frac{\Delta u}{2},v,w\right).h_w\left(u+\frac{\Delta u}{2},v,w\right).dv.dw - F_u\left(u-\frac{\Delta u}{2},v,w\right).h_v\left(u-\frac{\Delta u}{2},v,w\right).h_w\left(u-\frac{\Delta u}{2},v,w\right).dv.dw$$

$$= \frac{\partial}{\partial u}(F_u.h_v.h_w).du.dv.dw$$

$$\overrightarrow{dS_3} = dS_3\hat{v} = h_u.h_w.du.dw.\hat{v}$$
$$\overrightarrow{dS_4} = dS_4(-\hat{v}) = -h_u.h_w.du.dw.\hat{v}$$

On S_3

$$\vec{F}\left(u,v+\frac{\Delta v}{2},w\right)\bullet\ \overrightarrow{dS_3} =$$

$$F_u\left(u,v+\frac{\Delta v}{2},w\right)\hat{u}\bullet dS_3\hat{v}+F_v\left(u,v+\frac{\Delta v}{2},w\right)\hat{v}\bullet dS_3\hat{v}+F_w\left(u,v+\frac{\Delta v}{2},w\right)\hat{w}\bullet dS_3\hat{v}$$

$$= F_v\left(u,v+\frac{\Delta v}{2},w\right)h_u\left(u,v+\frac{\Delta v}{2},w\right).h_w\left(u,v+\frac{\Delta v}{2},w\right).du.dw.\hat{v}\bullet\hat{v}$$
$$= F_v\left(u,v+\frac{\Delta v}{2},w\right)h_u\left(u,v+\frac{\Delta v}{2},w\right).h_w\left(u,v+\frac{\Delta v}{2},w\right).du.dw.1$$

On S_4

$$\vec{F}\left(u,v-\frac{\Delta v}{2},w\right)\bullet\ \overrightarrow{dS_4} =$$

$$F_u\left(u,v-\frac{\Delta v}{2},w\right)\hat{u}\bullet dS_3(-\hat{v})+F_v\left(u,v-\frac{\Delta v}{2},w\right)\hat{v}\bullet dS_3(-\hat{v})+F_w\left(u,v-\frac{\Delta v}{2},w\right)\hat{w}\bullet dS_3(-\hat{v})$$
$$= F_v\left(u,v-\frac{\Delta v}{2},w\right)h_u\left(u,v-\frac{\Delta v}{2},w\right).h_w\left(u,v-\frac{\Delta v}{2},w\right).du.dw.\hat{v}\bullet(-\hat{v})$$
$$= F_v\left(u,v-\frac{\Delta v}{2},w\right)h_u\left(u,v-\frac{\Delta v}{2},w\right).h_w\left(u,v-\frac{\Delta v}{2},w\right).du.dw.(-1)$$

$$\vec{F}\bullet\ \overrightarrow{dS_3}\ \Big|_{S_3}\ +\vec{F}\bullet\ \overrightarrow{dS_4}\Big|_{S_4} = \vec{F}\left(u,v+\frac{\Delta v}{2},w\right)\bullet\ \overrightarrow{dS_3}+\vec{F}\left(u,v-\frac{\Delta v}{2},w\right)\bullet\ \overrightarrow{dS_4}$$
$$= F_v\left(u,v-\frac{\Delta v}{2},w\right)h_u\left(u,v-\frac{\Delta v}{2},w\right).h_w\left(u,v-\frac{\Delta v}{2},w\right).du.dw$$
$$-F_v\left(u,v-\frac{\Delta v}{2},w\right).h_u\left(u,v-\frac{\Delta v}{2},w\right).h_w\left(u,v-\frac{\Delta v}{2},w\right).du.dw$$
$$= \frac{\partial}{\partial v}(F_v.h_u.h_w).du.dv.dw$$

$$\overrightarrow{dS_5} = dS_5\hat{w} = h_u.h_v.du.dv.\hat{w}$$
$$\overrightarrow{dS_6} = dS_6(-\hat{w}) = -h_u.h_v.du.dv.\hat{w}$$

On S_5

$$\vec{F}\left(u,v,w+\frac{\Delta w}{2}\right)\bullet\ \overrightarrow{dS_5} =$$

$$F_u\left(u,v,w+\frac{\Delta w}{2}\right)\hat{u}\bullet dS_5\hat{w}+F_v\left(u,v,w+\frac{\Delta w}{2}\right)\hat{v}\bullet dS_5\hat{w}+F_w\left(u,v,w+\frac{\Delta w}{2}\right)\hat{w}\bullet dS_5\hat{w}$$

$$= F_w\left(u,v,w+\frac{\Delta w}{2}\right)h_u\left(u,v,w+\frac{\Delta w}{2}\right).h_v\left(u,v,w+\frac{\Delta w}{2}\right).du.dv.\hat{w}\bullet\hat{w}$$
$$= F_w\left(u,v,w+\frac{\Delta w}{2}\right)h_u\left(u,v,w+\frac{\Delta w}{2}\right).h_v\left(u,v,w+\frac{\Delta w}{2}\right).du.dv.1$$

On S_6 $$\vec{F}\left(u,v,w-\frac{\Delta w}{2}\right)\bullet\ \overrightarrow{dS_6} =$$

$$F_u\left(u,v,w-\frac{\Delta w}{2}\right)\hat{u}\bullet dS_6(-\widehat{w}) + F_v\left(u,v,w-\frac{\Delta w}{2}\right)\hat{v}\bullet dS_6(-\widehat{w}) + F_w\left(u,v,w-\frac{\Delta w}{2}\right)\widehat{w}\bullet dS_6(-\widehat{w})$$

$$= F_w\left(u,v,w-\frac{\Delta w}{2}\right)h_u\left(u,v,w-\frac{\Delta w}{2}\right).h_v\left(u,v,w-\frac{\Delta w}{2}\right).du.dv.\widehat{w}\bullet(-\widehat{w})$$

$$= F_w\left(u,v,w-\frac{\Delta w}{2}\right)h_u\left(u,v,w-\frac{\Delta w}{2}\right).h_v\left(u,v,w-\frac{\Delta w}{2}\right).du.dv.(-1)$$

$$\vec{F}\bullet\ \overrightarrow{dS_5}\ \Big|_{S_5} + \vec{F}\bullet\ \overrightarrow{dS_6}\Big|_{S_6} = \vec{F}\left(u,v,w+\frac{\Delta w}{2}\right)\bullet\ \overrightarrow{dS_5} + \vec{F}\left(u,v,w-\frac{\Delta w}{2}\right)\bullet\ \overrightarrow{dS_6}$$

$$= F_w\left(u,v,w+\frac{\Delta w}{2}\right)h_u\left(u,v,w+\frac{\Delta w}{2}\right).h_v\left(u,v,w+\frac{\Delta w}{2}\right).du.dv$$
$$-F_w\left(u,v,w-\frac{\Delta w}{2}\right).h_u\left(u,v,w-\frac{\Delta w}{2}\right).h_v\left(u,v,w-\frac{\Delta w}{2}\right).du.dv$$

$$= \frac{\partial}{\partial w}(F_w.h_u.h_v).du.dv.dw$$

$$\nabla\bullet\vec{F} = \frac{\oiint \vec{F}\bullet\overrightarrow{dS}}{dV}$$

$$= \frac{\frac{\partial}{\partial u}(F_u.h_v.h_w).du.dv.dw + \frac{\partial}{\partial v}(F_v.h_u.h_w).du.dv.dw + \frac{\partial}{\partial w}(F_w.h_u.h_v).du.dv.dw}{(h_u.h_v.h_w).du.dv.dw}$$

$$= \frac{1}{(h_u.h_v.h_w)}\left\{\frac{\partial}{\partial u}(F_u.h_v.h_w) + \frac{\partial}{\partial v}(F_v.h_u.h_w) + \frac{\partial}{\partial w}(F_w.h_u.h_v)\right\}$$

Diverjans (**Vektörel** alanın Diverjans'ı):

$$\vec{F}(u,v,w) = F_u(u,v,w)\hat{u} + F_v(u,v,w)\hat{v} + F_w(u,v,w)\widehat{w}$$

Genel olarak dik koordinat sistemi

$$\nabla\cdot\vec{F}(u,v,w) = \frac{1}{h_u h_v h_w}\left[\frac{\partial(h_v h_w F_u(u,v,w))}{\partial u} + \frac{\partial(h_u h_w F_v(u,v,w))}{\partial v} + \frac{\partial(h_u h_v F_w(u,v,w))}{\partial w}\right]$$

Kartezyen koordinat sistemi, Diverjans

$$\vec{F}(x,y,z) = F_x(x,y,z)\hat{x} + F_y(x,y,z)\hat{y} + F_z(x,y,z)\hat{z}$$

hesabı

$$\nabla \cdot \vec{F}(x,y,z) = \frac{1}{h_x h_y h_z}\left[\frac{\partial(h_y h_z F_x(x,y,z))}{\partial x} + \frac{\partial(h_x h_z F_y(x,y,z))}{\partial y} + \frac{\partial(h_x h_y F_z(x,y,z))}{\partial z}\right]$$

$$\nabla \cdot \vec{F}(x,y,z) = \frac{1}{1.1.1}\left[\frac{\partial(1.1.F_x(x,y,z))}{\partial x} + \frac{\partial(1.1.F_y(x,y,z))}{\partial y} + \frac{\partial(1.1.F_z(x,y,z))}{\partial z}\right]$$

$$\vec{F}(r,\varphi,z) = F_r(r,\varphi,z)\hat{r} + F_\varphi(r,\varphi,z)\hat{\varphi} + F_z(r,\varphi,z)\hat{z}$$

Silindirik koordinat sistemi, Diverjans hesabı

$$\nabla \cdot \vec{F}(r,\varphi,z) = \frac{1}{h_r h_\varphi h_z}\left[\frac{\partial(h_\varphi h_z F_r(r,\varphi,z))}{\partial r} + \frac{\partial(h_r h_z F_\varphi(r,\varphi,z))}{\partial \varphi} + \frac{\partial(h_r h_\varphi F_z(r,\varphi,z))}{\partial z}\right]$$

$$\nabla \cdot \vec{F}(r,\varphi,z) = \frac{1}{1.1.1}\left[\frac{\partial(r.1.F_r(r,\varphi,z))}{\partial r} + \frac{\partial(1.1.F_\varphi(r,\varphi,z))}{\partial \varphi} + \frac{\partial(1.r.F_z(r,\varphi,z))}{\partial z}\right]$$

Küresel koordinat sistemi, Diverjans hesabı

$$\vec{F}(R,\theta,\varphi) = F_R(R,\theta,\varphi)\hat{r} + F_\theta(R,\theta,\varphi)\hat{\theta} + F_\varphi(R,\theta,\varphi)\hat{\varphi}$$

$$\nabla \cdot \vec{F}(R,\theta,\varphi) = \frac{1}{h_R h_\theta h_\varphi}\left[\frac{\partial(h_\theta h_\varphi F_R(R,\theta,\varphi))}{\partial R} + \frac{\partial(h_R h_\varphi F_\theta(R,\theta,\varphi))}{\partial \theta} + \frac{\partial(h_R h_\theta F_\varphi(R,\theta,\varphi))}{\partial \varphi}\right]$$

$$\nabla \cdot \vec{F}(R,\theta,\varphi) = \frac{1}{1.Rsin(\theta).R}\left[\frac{\partial(R.Rsin(\theta).F_R(R,\theta,\varphi))}{\partial R} + \frac{\partial(1.Rsin(\theta).F_\theta(R,\theta,\varphi))}{\partial\theta} + \frac{\partial(1.R.F_\varphi(R,\theta,\varphi))}{\partial\varphi}\right]$$

Bukle (**Vektörel** alanın Buklesi):

Curl of a vector field

Assume $g(u,v,w) = u$

$$\nabla g = \nabla u = \frac{1}{h_u}\frac{\partial g}{\partial u}\hat{u} + \frac{1}{h_v}\frac{\partial g}{\partial v}\hat{v} + \frac{1}{h_w}\frac{\partial g}{\partial w}\hat{w}$$

$$= \frac{1}{h_u}\frac{\partial u}{\partial u}\hat{u} = \frac{1}{h_u}\frac{du}{du}\hat{u} = \frac{1}{h_u}.1.\hat{u}$$

$$\nabla u = \frac{1}{h_u}.\hat{u}$$

$$\nabla v = \frac{1}{h_v}.\hat{v} \qquad \nabla w = \frac{1}{h_w}.\hat{w}$$

$$\hat{u} = h_u.\nabla u \qquad \hat{v} = h_v.\nabla v \qquad \hat{w} = h_w.\nabla w$$

Assume

$$\vec{F} = F_u(u,v,w).\hat{u} + F_v(u,v,w).\hat{v} + F_w(u,v,w).\hat{w}$$

$$= F_u(u,v,w).h_u.\nabla u + F_v(u,v,w).h_v.\nabla v + F_w(u,v,w).h_w.\nabla w$$

$$\nabla g = \nabla u = \frac{1}{h_u}\frac{\partial g}{\partial u}\hat{u} + \frac{1}{h_v}\frac{\partial g}{\partial v}\hat{v} + \frac{1}{h_w}\frac{\partial g}{\partial w}\hat{w}$$

$$= \frac{1}{h_u}\frac{\partial u}{\partial u}\hat{u} = \frac{1}{h_u}\frac{du}{du}\hat{u} = \frac{1}{h_u}.1.\hat{u}$$

$$= \frac{1}{h_u}.\hat{u}$$

$$\nabla \times \vec{F} = \nabla \times \left(F_u(u,v,w).h_u.\nabla u + F_v(u,v,w).h_v.\nabla v + F_w(u,v,w).h_w.\nabla w\right)$$

$$\nabla \times \vec{F} = \nabla \times \left(F_u(u,v,w).h_u.\nabla u\right) + \nabla \times \left(F_v(u,v,w).h_v.\nabla v\right) + \nabla \times \left(F_w(u,v,w).h_w.\nabla w\right)$$

$$\nabla \times \vec{F} = \vec{a} + \vec{b} + \vec{c}$$

$$\nabla \times (F_u(u,v,w).h_u.\nabla u)$$

$$\vec{a} = \nabla \times (F_u(u,v,w).h_u(u,v,w).\nabla u)$$

$$\nabla \times (F_u.h_u.\nabla u)$$

$$\vec{b} = \nabla \times (F_v.h_v.\nabla v)$$

$$\vec{c} = \nabla \times (F_w.h_w.\nabla w)$$

$$\nabla \times z\vec{A} = \nabla z \times \vec{A} + z.\nabla \times \vec{A}$$

$\nabla \times z\vec{A} = \nabla z \times \vec{A}$, if $\vec{A}$ is a constant vector & z(u,v,w)

$$F_u.h_u = z$$

$$\nabla u = \vec{A}$$

$$\nabla \times (F_u.h_u.\nabla u) = \vec{a}$$

$$= \nabla(F_u.h_u) \times \nabla u$$

$$\nabla(F_u.h_u) \times \nabla u = \left(\frac{1}{h_u}\frac{\partial(F_u.h_u)}{\partial u}\hat{u} + \frac{1}{h_v}\frac{\partial(F_u.h_u)}{\partial v}\hat{v} + \frac{1}{h_w}\frac{\partial(F_u.h_u)}{\partial w}\hat{w}\right) \times \nabla u$$

$$= \left(\frac{1}{h_u}\frac{\partial(F_u.h_u)}{\partial u}\hat{u} + \frac{1}{h_v}\frac{\partial(F_u.h_u)}{\partial v}\hat{v} + \frac{1}{h_w}\frac{\partial(F_u.h_u)}{\partial w}\hat{w}\right) \times \frac{1}{h_u}.\hat{u}$$

$$= \frac{1}{h_u h_w}\frac{\partial(F_u.h_u)}{\partial w}\hat{v} + \frac{1}{h_u h_v}\frac{\partial(F_u.h_u)}{\partial v}(-\hat{w})$$

$$= \frac{1}{h_u h_v h_w}\left[h_v\frac{\partial(F_u.h_u)}{\partial w}\hat{v} - h_w\frac{\partial(F_u.h_u)}{\partial v}(\hat{w})\right]$$

$$= \vec{a}$$

$$\nabla(F_v.h_v) \times \nabla v = \left(\frac{1}{h_u}\frac{\partial(F_v.h_v)}{\partial u}\hat{u} + \frac{1}{h_v}\frac{\partial(F_v.h_v)}{\partial v}\hat{v} + \frac{1}{h_w}\frac{\partial(F_v.h_v)}{\partial w}\hat{w}\right) \times \nabla v$$

$$= \left(\frac{1}{h_u}\frac{\partial(F_v.h_v)}{\partial u}\hat{u} + \frac{1}{h_v}\frac{\partial(F_v.h_v)}{\partial v}\hat{v} + \frac{1}{h_w}\frac{\partial(F_v.h_v)}{\partial w}\hat{w}\right) \times \frac{1}{h_v}.\hat{v}$$

$$= -\left[\frac{1}{h_v h_w}\frac{\partial(F_v.h_v)}{\partial w}\hat{u} - \frac{1}{h_u h_v}\frac{\partial(F_v.h_v)}{\partial u}\hat{w}\right]$$

$$= -\frac{1}{h_u h_v h_w}\left[\frac{\partial(F_v.h_v)}{\partial w}\hat{u} - \frac{\partial(F_v.h_v)}{\partial u}\hat{w}\right]$$

$$= \vec{b}$$

$$\nabla(F_w.h_w) \times \nabla w = \left(\frac{1}{h_u}\frac{\partial(F_w.h_w)}{\partial u}\hat{u} + \frac{1}{h_v}\frac{\partial(F_w.h_w)}{\partial v}\hat{v} + \frac{1}{h_w}\frac{\partial(F_w.h_w)}{\partial w}\hat{w}\right) \times \nabla w$$

$$= \left(\frac{1}{h_u}\frac{\partial(F_w.h_w)}{\partial u}\hat{u} + \frac{1}{h_v}\frac{\partial(F_w.h_w)}{\partial v}\hat{v} + \frac{1}{h_w}\frac{\partial(F_w.h_w)}{\partial w}\hat{w}\right) \times \frac{1}{h_w}.\hat{w}$$

$$= \left[\frac{1}{h_v h_w}\frac{\partial(F_w.h_w)}{\partial w}\hat{u} - \frac{1}{h_u h_w}\frac{\partial(F_w.h_w)}{\partial u}\hat{v}\right]$$

$$= -\frac{1}{h_u h_v h_w}\left[h_u\frac{\partial(F_w.h_w)}{\partial w}\hat{u} - h_v\frac{\partial(F_w.h_w)}{\partial u}(\hat{v})\right]$$

$$= \vec{c}$$

$$\nabla \times \vec{F} = \vec{a} + \vec{b} + \vec{c}$$

$$= \frac{1}{h_u h_v h_w}\left[h_v \frac{\partial(F_u.h_u)}{\partial w}\hat{v} - h_w \frac{\partial(F_u.h_u)}{\partial v}(\hat{w})\right] + \frac{1}{h_u h_v h_w}\left[h_v \frac{\partial(F_u.h_u)}{\partial w}\hat{v} - h_w \frac{\partial(F_u.h_u)}{\partial v}(\hat{w})\right] + -\frac{1}{h_u h_v h_w}\left[h_u \frac{\partial(F_w.h_w)}{\partial w}\hat{u} - h_v \frac{\partial(F_w.h_w)}{\partial u}(\hat{v})\right]$$

$$\nabla \times \vec{F} = \frac{1}{h_u h_v h_w}\begin{vmatrix} h_u\hat{u} & h_v\hat{v} & h_w\hat{w} \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_u.F_u & h_v.F_v & h_w.F_w \end{vmatrix}$$

$$\vec{F}(u,v,w) = F_u(u,v,w)\hat{u} + F_v(u,v,w)\hat{v} + F_w(u,v,w)\hat{w}$$

Genel olarak dik koordinat Sistemi, Bukle hesabı

$$\nabla \times \vec{F}(u,v,w) = \frac{1}{h_u h_v h_w}\begin{vmatrix} h_u\hat{u} & h_v\hat{v} & h_w\hat{w} \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_uF_u & h_vF_v & h_wF_w \end{vmatrix}$$

$$\vec{F}(x,y,z) = F_x(x,y,z)\hat{x} + F_y(x,y,z)\hat{y} + F_z(x,y,z)\hat{z}$$

Kartezyen koordinat sistemi, Bukle hesabı

$$\nabla \times \vec{F}(x,y,z) = \frac{1}{h_x h_y h_z}\begin{vmatrix} h_x\hat{x} & h_y\hat{y} & h_z\hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ h_xF_u & h_yF_y & h_zF_z \end{vmatrix}$$

$$\nabla \times \vec{F}(x,y,z) = \frac{1}{1.1.1}\begin{vmatrix} 1\hat{x} & 1\hat{y} & 1\hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 1F_u & 1F_y & 1F_z \end{vmatrix}$$

Silindirik koordinat

$$\vec{F}(r,\varphi,z) = F_r(r,\varphi,z)\hat{r} + F_\varphi(r,\varphi,z)\hat{\varphi} + F_z(r,\varphi,z)\hat{z}$$

sistemi,

Bukle hesabı

$$\nabla \times \vec{F}(r,\varphi,z) = \frac{1}{h_r h_\varphi h_z}\begin{vmatrix} h_r\hat{x} & h_\varphi\hat{y} & h_z\hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ h_r F_u & h_\varphi F_y & h_z F_z \end{vmatrix}$$

$$\nabla \times \vec{F}(r,\varphi,z) = \frac{1}{1.r.1}\begin{vmatrix} 1\hat{r} & r\hat{\varphi} & 1\hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ 1F_r & rF_\varphi & 1F_z \end{vmatrix}$$

$$\vec{F}(R,\theta,\varphi) = F_R(R,\theta,\varphi)\hat{r} + F_\theta(R,\theta,\varphi)\hat{\theta} + F_\varphi(R,\theta,\varphi)\hat{\varphi}$$

Küresel

koordinat

sistemi,

Bukle hesabı

$$\nabla \times \vec{F}(R,\theta,\varphi) = \frac{1}{h_R h_\theta h_\varphi}\begin{vmatrix} h_R\hat{x} & h_\theta\hat{y} & h_\varphi\hat{z} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ h_R F_R & h_\theta F_\theta & h_\varphi F_\varphi \end{vmatrix}$$

$$\nabla \times \vec{F}(r,\varphi,z) = \frac{1}{1.Rsin(\theta).R}\begin{vmatrix} 1\hat{R} & R\hat{\theta} & Rsin(\theta)\hat{\varphi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ 1F_R & RF_\theta & Rsin(\theta)F_\varphi \end{vmatrix}$$

$\hat{x}$ x $\hat{y}$ = 1.1.sin(90) $\hat{z}$ = $\hat{z}$	$\hat{r}$ x $\hat{\varphi}$ = 1.1.sin(90) $\hat{z}$ = $\hat{z}$	$\hat{R}$ x $\hat{\varphi}$ = 1.1.sin(90) $\hat{z}$ = $\hat{z}$
$\hat{y}$ x $\hat{z}$ = 1.1.sin(90) $\hat{x}$ = $\hat{x}$	$\hat{\varphi}$ x $\hat{z}$ = 1.1.sin(90) $\hat{r}$ = $\hat{r}$	$\hat{\varphi}$ x $\hat{z}$ = 1.1.sin(90) $\hat{R}$ = $\hat{R}$
$\hat{z}$ x $\hat{x}$ =1.1.sin(90) $\hat{y}$ = $\hat{y}$	$\hat{z}$ x $\hat{r}$ =1.1.sin(90) $\hat{\varphi}$ = $\hat{\varphi}$	$\hat{z}$ x $\hat{R}$ =1.1.sin(90) $\hat{\varphi}$ = $\hat{\varphi}$
$\hat{y}$ x $\hat{x}$ = 1.1.sin(90)(-$\hat{z}$) = -$\hat{z}$	$\hat{\varphi}$ x $\hat{r}$ = 1.1.sin(90)(-$\hat{z}$) = -$\hat{z}$	$\hat{\varphi}$ x $\hat{R}$ = 1.1.sin(90)(-$\hat{z}$) = -$\hat{z}$
$\hat{z}$ x $\hat{y}$ = 1.1.sin(90)(-$\hat{x}$) = -$\hat{x}$	$\hat{z}$ x $\hat{\varphi}$ = 1.1.sin(90)(-$\hat{r}$) = -$\hat{r}$	$\hat{z}$ x $\hat{\varphi}$ = 1.1.sin(90)(-$\hat{R}$) = -$\hat{R}$
$\hat{x}$ x $\hat{z}$ = 1.1.sin(90)(-$\hat{y}$) = -$\hat{y}$	$\hat{r}$ x $\hat{z}$ = 1.1.sin(90)(-$\hat{\varphi}$) = -$\hat{\varphi}$	$\hat{R}$ x $\hat{z}$ = 1.1.sin(90)(-$\hat{\varphi}$) = -$\hat{\varphi}$

Conservative Fields

a) $\vec{F}$ is a conservative field

b) $$\oint_C \vec{F} \bullet \overrightarrow{dl} = 0$$

c) P_0 and P_1 are two different points in three dimensional space. Any line integral starting from P_0 ending at P_1 gives the same result independent of the path traversed

(a) implies (b)

Proof :

Let $\vec{F}$ be a conservative field. This means $\vec{F} = \nabla\emptyset$ where $\emptyset(x, y, z)$ is a function of (x,y,z).

$$\vec{F} \bullet \overrightarrow{dl} = \nabla\emptyset \bullet \overrightarrow{dl}$$

$$= (\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial x}\hat{y} + \frac{\partial}{\partial x}\hat{z})\emptyset \bullet \overrightarrow{dl}$$

$$= (\frac{\partial\emptyset}{\partial x}\hat{x} + \frac{\partial\emptyset}{\partial x}\hat{y} + \frac{\partial\emptyset}{\partial x}\hat{z}) \bullet (\mathrm{d}x\hat{x} + dy\hat{y} + dz\hat{z})$$

$$= \frac{\partial\emptyset}{\partial x}dx + \frac{\partial\emptyset}{\partial x}dy + \frac{\partial\emptyset}{\partial x}dz$$

$$= \mathrm{d}\emptyset$$

=>

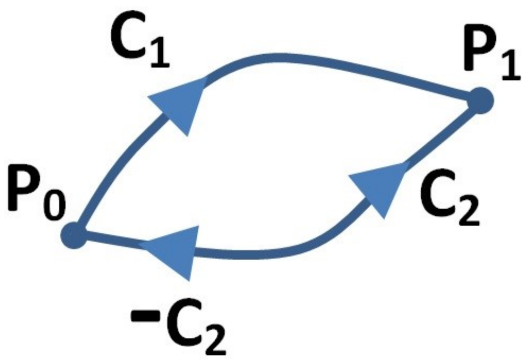
$$\oint_C \vec{F} \bullet \overrightarrow{dl} = \oint_{\emptyset(A)}^{\emptyset(A)} d\emptyset$$

$$= \emptyset\Big|_{\emptyset(A)}^{\emptyset(A)}$$

$$= \emptyset(A) - \emptyset(A)$$

$$= \qquad 0$$

(b) implies (c)



Proof :

$$\int_{C_2} \vec{F} \bullet \overrightarrow{dl} = \int_{P_0}^{P_1} \vec{F} \bullet \overrightarrow{dl} = -\int_{P_1}^{P_0} \vec{F} \bullet \overrightarrow{dl} = -\int_{-C_2} \vec{F} \bullet \overrightarrow{dl}$$

$$\int_{-C_2} \vec{F} \bullet \overrightarrow{dl} = -\int_{C_2} \vec{F} \bullet \overrightarrow{dl}$$

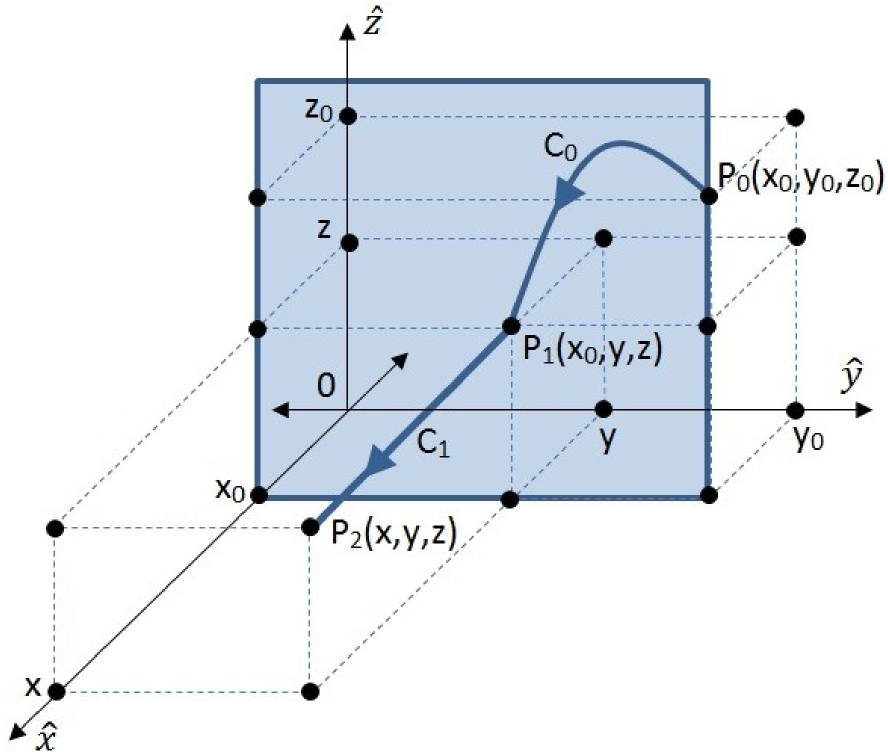
$$0 = \oint_C \vec{F} \bullet \overrightarrow{dl} = \int_{C_1} \vec{F} \bullet \overrightarrow{dl} + \int_{-C_2} \vec{F} \bullet \overrightarrow{dl}$$

$$0 = \oint_C \vec{F} \bullet \overrightarrow{dl} = \int_{C_1} \vec{F} \bullet \overrightarrow{dl} + (-\int_{C_2} \vec{F} \bullet \overrightarrow{dl})$$

$$0 = \int_{C_1} \vec{F} \bullet \overrightarrow{dl} + (-\int_{C_2} \vec{F} \bullet \overrightarrow{dl})$$

$$\int_{C_1} \vec{F} \bullet \overrightarrow{dl} = \int_{C_2} \vec{F} \bullet \overrightarrow{dl}$$

(c) implies (a)



$$\emptyset(x,y,z) = \int_{P_0}^{P_1} \vec{F} \bullet \vec{dl} + \int_{P_1}^{P_2} \vec{F} \bullet \vec{dl}$$

$$= \int_{(x_0,y_0,z_0)}^{(x_0,y,z)} \vec{F} \bullet \vec{dl} + \int_{(x_0,y,z)}^{(x,y,z)} \vec{F} \bullet \vec{dl}$$

$$\frac{\partial\emptyset}{\partial x} = \frac{\partial}{\partial x}\left\{ \int_{(x_0,y_0,z_0)}^{(x_0,y,z)} \vec{F} \bullet \vec{dl} + \int_{(x_0,y,z)}^{(x,y,z)} \vec{F} \bullet \vec{dl} \right\}$$

$$= \frac{\partial}{\partial x} \int_{(x_0,y,z)}^{(x,y,z)} \vec{F} \bullet \vec{dl}$$

$$= F_x(x,y,z)\frac{dx}{dx} + F_x(x,y,z)\frac{dx_0}{dx}$$

$$\frac{\partial\emptyset}{\partial x} = F_x(x,y,z)$$

$$\frac{\partial\emptyset}{\partial y} = F_y(x,y,z) \qquad \frac{\partial\emptyset}{\partial z} = F_z(x,y,z) \qquad \nabla\emptyset = \frac{\partial F_x}{\partial x}\hat{x} + \frac{\partial F_y}{\partial x}\hat{y} + \frac{\partial F_z}{\partial x}\hat{z} = \vec{F}$$