MAGNETOSTATIC FIELDS

Static electric fields are characterized by $\vec{\boldsymbol{E}}$ or $\vec{\boldsymbol{D}}$**.** Static magnetic fields, are characterized by $\vec{\boldsymbol{H}}$ or $\vec{\boldsymbol{B}}$**.** There are similarities and dissimilarities between electric and magnetic fields. As $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{D}}$ are related according to $\vec{\boldsymbol{D}} = \varepsilon\vec{E}$ for linear material space, $\vec{\boldsymbol{H}}$ and $\vec{\boldsymbol{B}}$ are related according to

$$\vec{\boldsymbol{B}} = \mu\vec{H}$$

A definite link between electric and magnetic fields was established by Oersted in 1820. An electrostatic field is produced by static or stationary charges. If the charges are moving with constant velocity, a static magnetic (or magnetostatic) field is produced. A magnetostatic field is produced by a constant current flow (or direct current). This current flow may be due to magnetization currents as in permanent magnets, electron-beam currents as in vacuum tubes, or conduction currents as in current-carrying wires.

The development of the motors, transformers, microphones, compasses, telephone bell ringers, television focusing controls, advertising displays, magnetically levitated high speed vehicles, memory stores, magnetic separators, and so on, involve magnetic phenomena and play an important role in our everyday life.

There are two major laws governing magnetostatic fields:

(1) Biot-Savart's law, and

(2) Ampere's circuit law.

Like Coulomb's law, Biot-Savart's law is the general law of magnetostatics. Just as Gauss's law is a special case of Coulomb's law, Ampere's law is a special case of Biot-Savart's law and is easily applied in problems involving symmetrical current distribution.

MAGNETIC FLUX DENSITY

The magnetic flux density **B** is similar to the electric flux density $\vec{\mathbf{D}}$. As $\mathbf{D} = \varepsilon_0 \mathbf{E}$ in free space, the magnetic flux density **B** is related to the magnetic field intensity **H** according to

$$\mathbf{B} = \mu_0 \mathbf{H} \tag{1.21}$$

where, μ_0 is a constant known as the *permeability of free space.* The constant is in henrys/meter (H/m) and has the value of

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m} \tag{1.22}$$

The precise definition of the magnetic field **B,** in terms of the magnetic force, can be discussed later.

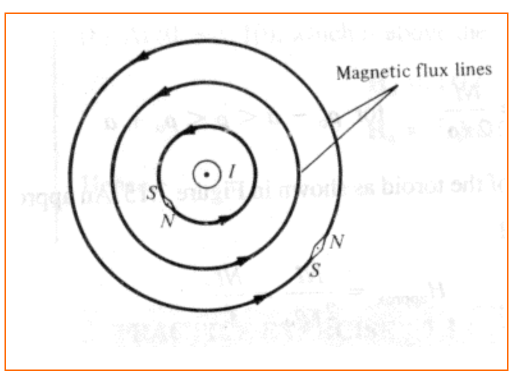

Figure 1.8: Magnetic flux lines due to a straight wire with current coming out of the page

The magnetic flux through a surface S is given by

$$\psi = \oint_S B \cdot dS \tag{1.23}$$

Where the magnetic flux ψ is in webers (Wb) and the magnetic flux density is a webers/square meter (Wb/m^2) or teslas.

An isolated magnetic charge does not exit.

Total flux through a closed surface in a magnetic field must be zero;

that is,

$$\oint B \cdot dS = 0 \tag{1.24}$$

This equation is referred to as the *law of conservation of magnetic flux or Gauss''s law for magnetostatic fields just* as $\oint$ D. dS = Q is Gauss's law for electrostatic fields. Although the magnetostatic field is not conservative, magnetic flux is conserved.

By applying the divergence theorem to eq. (1.24), we obtain

$$\oint_S B \cdot dS = \int_v \nabla \cdot B \; dv = 0$$

Or

$$\nabla \cdot \mathbf{B} = \mathbf{0} \tag{1.25}$$

This equation is the fourth Maxwell's equation to be derived. Equation (1.24) or (1.25) shows that magnetostatic fields have no sources or sinks. Equation (1.25) suggests that magnetic field lines are always continuous.

TABLE 1.2: Maxwell's Equations for Static EM Fields

Differential (or Point) Form	Integral Form	Remarks
$\nabla \cdot D = \rho_v$	$\oint_S D \cdot dS = \int_v \rho_v dv$	Gauss's law
$\nabla \cdot B = 0$	$\oint_S B \cdot dS = 0$	Nonexistence of magnetic monopole
$\nabla \times E = 0$	$\oint_L E \cdot dl = 0$	Conservativeness of electrostatic field
$\nabla \times H = J$	$\oint_L H \cdot dl = \int_s J \cdot dS$	Ampere's law

The **Table 1.2** gives the information related to Maxwell's Equations for Static Electromagnetic Fields.

AMPERE'S CIRCUIT LAW

Ampere's circuit law states that the line integral of the tangential components of **H** around a closed path is the same as the net current I_{enc} enclosed by the path

In other words, the circulation of **H** equals I_{enc} ; that is,

$$\oint H \cdot dl = I_{enc} \quad (1.16)$$

Ampere's law is similar to Gauss's law and it is easily applied to determine **H** when the current distribution is symmetrical. It should be noted that eq. (1.16) always holds whether the current distribution is symmetrical or not but we can only use the equation to determine **H** when symmetrical current distribution exists. Ampere's law is a special case of Biot-Savart's law; the former may be derived from the latter.

By applying Stoke's theorem to the left-hand side of eq. (1.16), we obtain

$$I_{enc} = \oint_L H \cdot dl = \oint_S (\nabla \times H) \cdot dS \tag{1.17}$$

But

$$I_{enc} = \oint_S J \cdot dS \tag{1.18}$$

Comparing the surface integrals in eqs. (7.17) and (7.18) clearly reveals that

$$\nabla \times \mathbf{H} = \mathbf{J} \tag{1.19}$$

This is the third Maxwell's equation to be derived; it is essentially Ampere's law in differential (or point) form whereas eq. (1.16) is the integral form. From eq. (1.19), we should observe that $\nabla \times \mathbf{H} = \mathbf{J} \neq 0$; that is, magnetostatic field is not conservative.

APPLICATIONS OF AMPERE'S LAW

Infinite Line Current

Consider an infinitely long filamentary current I along the z-axis as in Figure 1. 7. To determine **H** at an observation point P, we allow a closed path pass through P. This path on, which Ampere's law is to be applied, is known as an *Amperian path* (analogous to the term Gaussian surface). We choose a concentric circle as the Amperian path in view of eq. (1.14), which shows that **H** is constant provided p is constant. Since this path encloses the whole current I, according to Ampere's law

$$I = \int H_\phi a_\phi \cdot \rho \ d\phi \ a_\phi = H_\phi \int \rho \ d\phi = H_\phi \cdot 2\pi\rho$$

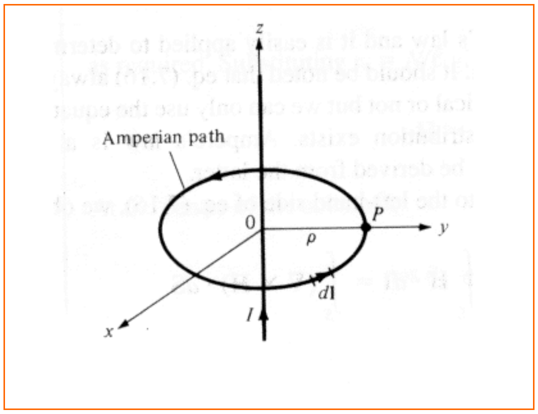

Figure 1.7: Ampere's law applied to an infinite filamentary, line current.

Or

$$H = \frac{1}{2\pi\rho} a_\phi \qquad (1.20)$$

As expected from eq. (1.14).

Fundamental Postulates of Magnetostatics

We know that for a magnetostatic field, **∇ x B = 0** as stated in eq. (1.25). In order to satisfy eqs. (1.25) and (1.27) simultaneously, we can define the *vector magnetic potential* A (in Wb/m) such that

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad (1.31)$$

Just as we defined

$$V = \int \frac{dQ}{4\pi\varepsilon_0 r} \qquad (1.32)$$

We can define

$$A = \int_L \frac{\mu_0 I \; dl}{4\pi R} \qquad \text{for line current} \qquad (1.33)$$

$$A = \int_S \frac{\mu_0 K \; dS}{4\pi R} \qquad \text{for surface current} \qquad (1.34)$$

$$A = \int_v \frac{\mu_0 J \; dv}{4\pi R} \qquad \text{for volume current} \qquad (1.35)$$

Illustration 1: Given the magnetic vector potential $\mathbf{A} = -\rho^2/4\ \mathbf{a_z}$ Wb/m, calculate the total magnetic flux crossing the surface $\phi = \pi/2$, $1 \le \rho \le 2$m, $0 \le z \le 5$m.

Solution:

$$B = \nabla \times A = -\frac{\partial A_z}{\partial \rho} a_\phi = \frac{\rho}{2} a_\phi, \qquad dS = d\rho \;\, dz \;\, a_\phi$$

$$\psi = \int B \cdot dS \;\, \frac{1}{2}\int_{z=0}^{5} \int_{\rho=1}^{2} \rho \;\, d\rho \;\, dz = \frac{1}{4}\rho^2 \Big| (5) = \frac{15}{4}$$

$\psi = 3.75$ Wb

Illustration 2:

Identify the configuration in figure 1.9 that is not a correct representation of I and H.

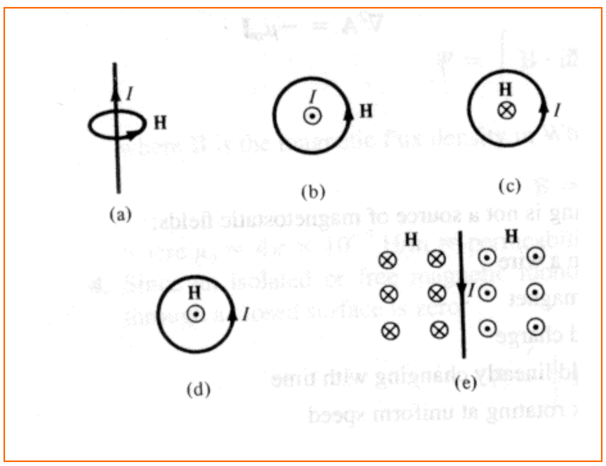

Figure 1.9: Different I and H representations (related to Illustration 2)

Solution:

Figure 1.9 (c) is not a correct representation. The direction of H field should have been outwards for the given I direction.