İnifinitesimal alan büyüklüğü

$ds_r = dl_\phi . dl_z = (h_\phi d\phi)(h_z dz)$

$ds_\phi = dl_r . dl_z = (h_r dr)(h_z dz)$

$ds_z = dl_r . dl_\phi = (h_r dr)(h_\phi d\phi)$

İnifinitesimal alan vektörü

$\overrightarrow{ds_r} = ds_r \hat{r} = dl_\phi . dl_z \hat{r}$

$= (h_\phi d\phi)(h_z dz)\, \hat{r}$

$=(r.d\phi)(1.dz)\hat{r}$

$\overrightarrow{ds_\varphi} = ds_\phi \widehat{\varphi} = dl_r . dl_z \widehat{\varphi}$

$= (h_r dr)(h_z dz)\, \widehat{\varphi}$

$=(1.dr)(1.dz)\widehat{\varphi}$

$\overrightarrow{ds_z} = ds_z \hat{z} = dl_r . dl_\phi \hat{z}$

$= (h_r dr)(h_\phi d\phi)\, \hat{z}$

$=(1.dr)(r.d\phi)\hat{z}$

İnifinitesimal hacim büyüklüğü

$dv = dl_r dl_\phi dl_z$

$= (h_r dr)(h_\phi d\phi)(h_z dz)$

$dv = dl_r dl_\phi dl_z$

$= (1.dr)(r.d\phi)(1.dz)$

$=r.dr.d\phi.dz$

Slindirik Koordinat Sistemi

Eksen	Metrik katsayı	İnfinitesimal uzunluk
$\hat{r}$	$h_r = 1$	$dl_r = h_r dr = 1.dr$
$\hat{\varphi}$	$h_\phi = r$	$dl_\phi = h_\phi d\phi = r.d\phi$
$\hat{z}$	$h_z = 1$	$dl_z = h_z dz = 1.dz$

İnifinitesimal yer değiştirme vektörü

$$\vec{dl} = \vec{dl}_r + \vec{dl}_\varphi + \vec{dl}_z$$

$$\vec{dl} = (h_r dr)\hat{r} + (h_\phi d\varphi)\hat{\varphi} + (h_z dz)\hat{z}$$

$$\vec{dl} = (1.dr)\hat{r} + (1.d\varphi)\hat{\varphi} + (1.dz)\hat{z}$$

Küresel Koordinat Sistemi

Eksen	Metrik katsayı	İnfinitesimal uzunluk
$\hat{R}$	$h_R = 1$	$dl_R = h_R dR = 1.Dr$
$\hat{\theta}$	$h_\theta = R$	$dl_\theta = h_\theta d\theta = R.d\theta$
$\hat{\varphi}$	$h_\phi = r = R\sin(\theta)$	$dl_\phi = h_\phi d\phi = r.d\phi$

İnifinitesimal yer değiştirme vektörü

$$\vec{dl} = \vec{dl}_R + \vec{dl}_\theta + \vec{dl}_\varphi$$

$$\vec{dl} = (h_R dR)\hat{R} + (h_\theta d\theta)\hat{\theta} + (h_\phi d\varphi)\hat{\varphi}$$

$$\vec{dl} = (1.dR)\hat{R} + (R.d\theta)\hat{\theta} + (R\sin(\theta).d\varphi)\hat{\varphi}$$

Küresel Koordinat Sistemi

İnifinitesimal alan büyüklüğü

$ds_R = dl_\theta \, dl_\phi = (h_\theta d\theta).(h_\phi d\phi)$

$dl_\theta = dl_R . \, dl_\phi = (h_R dR).(h_\phi d\phi)$

$ds_\phi = dl_R . \, dl_\theta = (h_R dR).(h_\theta d\theta)$

İnifinitesimal alan vektörü

$\overrightarrow{ds}_R = ds_R \hat{R} = dl_\theta . dl_\phi \hat{R} = (h_\theta d\theta).(h_\phi d\phi) \hat{R} = (R.d\theta).(R\sin(\theta).d\phi)\hat{R}$

$\overrightarrow{ds}_\theta = ds_\theta \hat{\theta} = dl_R . dl_\phi \hat{\theta} = (h_R dR).(h_\phi d\phi) \hat{\theta} = (1.dR).(R\sin(\theta).d\phi)\hat{\theta}$

$\overrightarrow{ds}_\varphi = ds_\phi \hat{\varphi} = dl_R . dl_\theta \hat{\varphi} = (h_R dR).(h_\theta d\theta) \hat{\varphi} = (1.dR).(R.d\theta)\hat{\varphi}$

İnfinitesimal hacim büyüklüğü

$dv = dl_R \, dl_\theta \, dl_\phi$

$= (h_R dR) \, (h_\theta d\theta) \, (h_\phi d\phi)$

$= (1.dR) \, (R.d\theta)(R\sin(\theta).d\phi)$

$= R^2 . \sin(\theta).dR. \, d\theta \, .d\phi$

- $\hat{\varphi}$, x ve y bağlıdır
- $\hat{\varphi}$, z'ye bağlı değildir
- $\hat{r}$, x ve y bağlıdır
- $\hat{r}$, z'ye bağlı değildir

x=rcos(ϕ),
y=rsin(ϕ),

$r=\sqrt{x^2+y^2}$

$\tan(\varphi)=y/x$

(i) ve (ii) denklemlerinden,

$$\hat{\varphi}=-sin(\varphi)\hat{x}+cos(\varphi)\hat{y}=\frac{(-sin(\varphi)\hat{x}+cos(\varphi)\hat{y})}{1}$$

$$=\frac{r(-sin(\varphi)\hat{x}+cos(\varphi)\hat{y})}{r}$$

$$=\frac{-rsin(\varphi)\hat{x}+rcos(\varphi)\hat{y}}{r}$$

$$=\frac{(-y\hat{x}+x\hat{y})}{\sqrt{x^2+y^2}}$$

$$=-\frac{y}{\sqrt{x^2+y^2}}\hat{x}+\frac{x}{\sqrt{x^2+y^2}}\hat{y}$$

$$=-\frac{y}{r}\hat{x}+\frac{x}{r}\hat{y}$$

$$\hat{r}=cos(\varphi)\hat{x}+sin(\varphi)\hat{y}=\frac{(cos(\varphi)\hat{x}+sin(\varphi)\hat{y})}{1}$$

$$=\frac{r(cos(\varphi)\hat{x}+sin(\varphi)\hat{y})}{r}$$

$$=\frac{rcos(\varphi)\hat{x}+rsin(\varphi)\hat{y}}{r}$$

$$=\frac{(x\hat{x}+y\hat{y})}{\sqrt{x^2+y^2}}$$

$$=\frac{x}{\sqrt{x^2+y^2}}\hat{x}+\frac{y}{\sqrt{x^2+y^2}}\hat{y}$$

$$=\frac{x}{r}\hat{x}+\frac{y}{r}\hat{y}$$

- $\hat{R}$ ve $\hat{\theta}$ (x,y,z)'ye bağlıdır:

$$x=R\sin(\theta)\cos(\varphi)$$

$$y=R\sin(\theta)\sin(\varphi)$$

$$z=R\cos(\theta)$$

$$\sin(\theta)=\frac{r}{R}$$

$$\cos(\theta)=\frac{z}{R}$$

$$sin^2(\theta)+cos^2(\theta)=1=\left(\frac{r}{R}\right)^2+\left(\frac{z}{R}\right)^2 \Rightarrow$$

$$r^2+z^2=R^2 \Rightarrow$$

$$R=\sqrt{r^2+z^2}$$

$$=\sqrt{x^2+y^2+z^2}$$

$$= \sqrt{(\mathrm{Rsin}(\theta)\cos(\varphi))^{2} + \left(\mathrm{Rsin}(\theta)\sin(\varphi)\right)^{2} + (\mathrm{Rcos}(\theta))^{2}}$$

$$= \sqrt{(\mathrm{Rsin}(\theta)\cos(\varphi))^{2} + \left(\mathrm{Rsin}(\theta)\sin(\varphi)\right)^{2} + \left(\mathrm{Rcos}(\theta)\right)^{2}}$$

$$= \sqrt{R^2(\sin^2(\theta)\cos^2(\varphi) + \sin^2(\theta)\sin^2(\varphi) + \cos^2(\theta)}$$

$$= \sqrt{R^2(\sin^2(\theta)\,(\cos^2(\varphi) + \sin^2(\varphi)) + \cos^2(\theta)}$$

$$= \sqrt{R^2(\sin^2(\theta) + \cos^2(\theta))}$$

$$= R$$

$$\hat{R} = \left(\hat{R} \cdot \hat{x}\right)\hat{x} + \left(\hat{R} \cdot \hat{y}\right)\hat{y} + \left(\hat{R} \cdot \hat{z}\right)\hat{z}$$

$$= \mathrm{s}in(\theta)\cos(\varphi)\hat{x} + sin(\theta)\sin(\varphi)\hat{y} + cos(\theta)\hat{z}$$

$$\hat{\theta} = \left(\hat{\theta} \cdot \hat{x}\right)\hat{x} + \left(\hat{\theta} \cdot \hat{y}\right)\hat{y} + \left(\hat{\theta} \cdot \hat{z}\right)\hat{z}$$

$$= cos(\theta)\cos(\varphi)\hat{x} + cos(\theta)\sin(\varphi)\hat{y} + cos(90 + \theta)\hat{z}$$

$$\vec{R} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$R = \sqrt{\vec{R}\cdot\vec{R}}$$

$$= \sqrt{(x\hat{x} + y\hat{y} + z\hat{z})\cdot(x\hat{x} + y\hat{y} + z\hat{z})}$$

$$= \sqrt{x^2 + y^2 + z^2}$$

$$\vec{R} = R\widehat{\mathrm{R}}$$

$$\widehat{\mathrm{R}} = \frac{\vec{R}}{R} \qquad (0)$$

$$= \frac{x\hat{x} + y\hat{y} + z\hat{z}}{R}$$

$$x = R.sin\theta.cos\varphi$$

$$y = R.sin\theta.sin\varphi$$

$$z = R.cos\theta$$

$$\hat{x} \text{ x } \widehat{\mathrm{R}} = \hat{x} \times \frac{x\hat{x} + y\hat{y} + z\hat{z}}{R}$$

$$= -\frac{z}{R}\hat{y} + \frac{y}{R}\hat{y}$$

$$= -cos\theta\hat{y} + sin\theta sin\varphi\hat{z} \qquad (1)$$

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$

$$A_x = \vec{A} \bullet \hat{x}$$

$$A_y = \vec{A} \bullet \hat{y}$$

$$A_z = \vec{A} \bullet \hat{z}$$

$$\vec{A} = (\vec{A} \bullet \hat{x})\hat{x} + (\vec{A} \bullet \hat{y})\hat{y} + (\vec{A} \bullet \hat{z})\hat{z}$$

$\vec{A} = \hat{\theta}$ in cartesian unit vectors

$$\vec{A} = \hat{\theta} = (\hat{\theta} \bullet \hat{x})\hat{x} + (\hat{\theta} \bullet \hat{y})\hat{y} + (\hat{\theta} \bullet \hat{z})\hat{z}$$

$$\hat{\theta} = (cos\theta.cos\varphi)\hat{x} + (cos\theta.sin\varphi)\hat{y} + (\cos(\frac{\pi}{2} + \theta)\hat{z}$$

$$\hat{\theta} = (cos\theta.cos\varphi)\hat{x} + (cos\theta.sin\varphi)\hat{y} + (-sin\theta)\hat{z} \quad (2)$$

$\vec{A} = \hat{x}$ in spherical unit vectors

$$\vec{A} = \hat{x} = (\hat{x} \bullet \hat{R})\hat{R} + (\hat{x} \bullet \hat{\theta})\hat{\theta} + (\hat{x} \bullet \hat{\varphi})\hat{\varphi}$$

$$= (sin\theta.cos\varphi)\hat{R} + (cos\theta.cos\varphi)\hat{\theta} + \cos(\frac{\pi}{2} + \varphi)\hat{\varphi}$$

$$= (sin\theta.cos\varphi)\hat{R} + (cos\theta.cos\varphi)\hat{\theta} + (-\sin\varphi)\hat{\varphi} \quad (A)$$

$$\hat{x} \text{ x } \hat{R} = -sin\theta.\hat{\theta} - \cos\theta.\cos\varphi)\hat{\varphi} \quad (3)$$

$$\vec{A} = \hat{\varphi} = (\hat{\varphi} \bullet \hat{x})\hat{x} + (\hat{\varphi} \bullet \hat{y})\hat{y} + (\hat{\varphi} \bullet \hat{z})\hat{z}$$

$$cos\left(\frac{\pi}{2} + \varphi\right).\hat{x} + cos\varphi\hat{y} + \cos(\frac{\pi}{2})\hat{z}$$

$$-sin\varphi\hat{x} + cos\varphi\hat{y} + 0\hat{z} \quad (4)$$

Insert (2) & (4) -> (3)=(1)

$$\hat{x} \text{ x } \hat{R} = -cos\theta.sin\varphi.cos\varphi.\hat{x} - cos\theta.sin^2\varphi.\hat{y} + sin\theta.sin\varphi.\hat{z} + cos\theta.sin\varphi.cos\varphi.\hat{x} - cos\theta.cos^2\varphi.\hat{y}$$

$$= -cos\theta.\hat{y} + sin\theta.sin\varphi.\hat{z} \quad (1) \dagger$$

$$\hat{x} \text{ x } \hat{R} = -cos\theta.\hat{y} + sin\theta.sin\varphi.\hat{z}$$

$$= -sin\varphi.\hat{\theta} - \cos\theta.\cos\varphi.\hat{\varphi}$$

See that

$$\hat{x} \perp (\hat{x} \times \hat{R})$$

$$\hat{x} \bullet (\hat{x} \times \hat{R}) = 0$$

$$\hat{R} \perp (\hat{x} \times \hat{R})$$

$$\hat{R} \bullet (\hat{x} \times \hat{R}) = 0$$

$$\hat{y} \times \hat{R} = \hat{y} \times \frac{x\hat{x} + y\hat{y} + z\hat{z}}{R}$$

$$= \frac{z}{R}\hat{x} - \frac{x}{R}\hat{z}$$

$$= cos\theta\hat{x} - sin\theta.cos\varphi.\hat{z} \qquad (5)$$

$\vec{A} = \hat{y}$ in spherical unit vectors

$$\vec{A} = \hat{y} = (\hat{y} \bullet \hat{R})\hat{R} + (\hat{y} \bullet \hat{\theta})\hat{\theta} + (\hat{y} \bullet \hat{\varphi})\hat{\varphi}$$

$$= (sin\theta.sin\varphi)\hat{R} + (cos\theta.sin\varphi)\hat{\theta} + \cos(\varphi)\hat{\varphi} \qquad (B)$$

$$\hat{y} \times \hat{R} = cos\varphi.\hat{\theta} - \cos\theta.\cos\varphi)\hat{\varphi} \qquad (6)$$

$$\vec{A} = \hat{\varphi} = -sin\varphi\hat{x} + cos\varphi\hat{y} + 0\hat{z} \qquad (4)$$

Insert (2) & (4) -> (6)=(5)

$$\hat{y} \times \hat{R} = cos\theta.cos^2\varphi.\hat{x} + cos\theta.sin\varphi.cos\varphi.\hat{y} - sin\theta.cos\varphi.\hat{z} + cos\theta.sin^2\varphi.\hat{x} - cos\theta.sin\varphi.cos\varphi.\hat{y}$$

$$= cos\theta.\hat{x} - sin\theta.sin\varphi.\hat{z} \qquad (6)\ \dagger$$

$$\hat{y} \times \hat{R} = cos\theta.\hat{x} - sin\theta.sin\varphi.\hat{z}$$

$$= cos\varphi.\hat{\theta} - \cos\theta.\cos\varphi.\hat{\varphi}$$

See that

$$\hat{y} \perp (\hat{y} \times \hat{R})$$

$$\hat{y} \bullet (\hat{y} \times \hat{R}) = 0$$

$$\hat{R} \perp (\hat{y} \times \hat{R})$$

$$\hat{R} \bullet (\hat{y} \times \hat{R}) = 0$$

$$\hat{R} = \frac{x\hat{x} + y\hat{y} + z\hat{z}}{R}$$

$$\hat{z} \times \hat{R} = \hat{z} \times \frac{x\hat{x} + y\hat{y} + z\hat{z}}{R}$$

$$= -\frac{y}{R}\hat{x} + \frac{x}{R}\hat{y}$$

$$\vec{r} = x\hat{x} + y\hat{y}$$

$$r = \sqrt{x^2 + y^2}$$

$$\hat{r} = \frac{\vec{r}}{r}$$

$$\hat{r} = \frac{x\hat{x} + y\hat{y}}{\sqrt{x^2 + y^2}}$$

$$\vec{r} = r\hat{r}$$

$$\vec{r} = \sqrt{x^2 + y^2}.\frac{x\hat{x} + y\hat{y}}{\sqrt{x^2 + y^2}}$$

$$\vec{R} \quad x\hat{x} + y\hat{y} + z\hat{z}$$

$$R \quad \sqrt{x^2 + y^2 + z^2}$$

$$\sqrt{r^2 + z^2} \quad \text{(F)}$$

$$\hat{R} = \frac{\vec{r} + z\hat{z}}{R}$$

$$\hat{R} = \frac{r\hat{r} + z\hat{z}}{R}$$

$$\hat{R} = \frac{r\hat{r} + z\hat{z}}{\sqrt{r^2 + z^2}} \quad \text{(E)}$$

$$\hat{z} \times \hat{R} = \hat{z} \times \frac{r\hat{r} + z\hat{z}}{\sqrt{r^2 + z^2}} \quad (5)$$

$$= \frac{r}{\sqrt{r^2 + z^2}}\hat{\varphi}$$

$$= \sin\varphi.\hat{\varphi} \quad \text{(E)}$$

$\vec{A} = \hat{z}$ in spherical unit vectors

$\vec{A} = \hat{z} \quad = \quad (\hat{z} \bullet \hat{R})\hat{R} + (\hat{z} \bullet \hat{\theta})\hat{\theta} + (\hat{z} \bullet \hat{\varphi})\hat{\varphi}$

$= \quad cos\theta.\hat{R} - sin\theta.\hat{\theta} + \cos(\frac{\pi}{2})\hat{\varphi} \qquad$ (C)

Use (2)

$\hat{x} \times \hat{\theta} \quad = \quad sin\theta.\hat{y} + cos\theta.sin\varphi.\hat{z} \qquad$ (7)

using (A):

$\hat{x} \times \hat{\theta} \quad = \quad sin\varphi.\hat{R} + sin\theta.cos\varphi.\hat{\varphi} \qquad$ (8)

(0) & (4) -> (8)=(7)

using (2) → (9)

$\hat{y} \times \hat{\theta} \quad = \quad -sin\theta.\hat{x} - cos\theta.cos\varphi.\hat{z} \qquad$ (9)

using (B) → (10)

$\hat{y} \times \hat{\theta} \quad = \quad -cos\theta.\hat{R} + sin\theta.sin\varphi.\hat{\varphi} \qquad$ (10)

(0) & (4) -> (10)=(9)

(A) & (C) -> (9)=(10)

using (2) → (11)

$\hat{z} \times \hat{\theta} \quad = \quad -cos\theta.cos\varphi.\hat{x} + cos\theta.cos\varphi.\hat{y} \qquad$ (11)

using (2) → (12)

$\hat{z} \times \hat{\theta} \quad = \quad cos\theta.\hat{\varphi} \qquad$ (12)

(A) & (B) -> (11)=(12)

Insert (4) -> (2)=(11)

Using RHR & (4)

$\hat{x} \times \hat{\varphi} \quad = \quad cos\varphi.\hat{z}$

using (C):

$\hat{x} \times \hat{\varphi} \quad = \quad cos\theta cos\varphi.\hat{R} - sin\theta cos\varphi.\hat{\theta}$

Using RHR & (4)

$\hat{y} \times \hat{\varphi} \quad = \quad sin\varphi.\hat{z}$

using (C):

$\hat{y} \times \hat{\varphi} \quad = \quad cos\theta sin\varphi.\hat{R} - sin\theta sin\varphi.\hat{\theta}$

$\hat{z} \times \hat{\varphi} \quad = \quad \hat{r}$

Cylindrical <-> spherical unit vectors

$$\hat{r} \times \hat{R} = -\frac{z}{\sqrt{r^2 + z^2}}.\hat{\varphi}$$ Using (E)

$$= -cos\theta.\hat{\varphi}$$

$$\hat{\varphi} \times \hat{R} = \hat{\theta}$$ Use RHR

$$\hat{z} \times \hat{R} = \frac{r}{\sqrt{r^2 + z^2}}.\hat{\varphi}$$ Using (E)

$$= sin\theta.\hat{\varphi}$$

---

using RHR:

$$\hat{r} \text{ x } \hat{\theta} = sin\theta.\hat{\varphi}$$ (13a)

$$\hat{r} = \frac{x\hat{x} + y\hat{y}}{\sqrt{x^2 + y^2}}$$

$$= \frac{x}{\sqrt{x^2 + y^2}}\hat{x} + \frac{y}{\sqrt{x^2 + y^2}}\hat{y}$$

$$\hat{r} = cos(\varphi)\hat{x} + sin(\varphi)\hat{y}$$ (G)

$$= \frac{x}{r}\hat{x} + \frac{y}{r}\hat{y}$$

Use (A) & (B) ->(G) =>

$$\hat{r} = sin\theta.cos^2\varphi.\hat{R} + cos\theta.cos^2\varphi.\hat{\theta} - cos\theta.\sin\varphi.\hat{\varphi} + sin\theta.sin^2\varphi.\hat{R} + cos\theta.sin^2\varphi.\hat{\theta} - sin\theta.cos\varphi.\hat{\varphi}$$

$$\hat{r} = sin\theta.\hat{R} + cos\theta.\hat{\theta}$$

$$\sqrt{\hat{r} \bullet \hat{r}} = \sqrt{(sin\theta.\hat{R} + cos\theta.\hat{\theta}) \bullet (sin\theta.\hat{R} + cos\theta.\hat{\theta})}$$

$$= \sqrt{sin^2\theta.\hat{R} \bullet \hat{R} + cos^2\theta.\hat{\theta} \bullet \hat{\theta} + 2.sin\theta.\hat{R} \bullet \hat{\theta}}$$

$$= 1$$

$$\hat{r} \text{ x } \hat{\theta} = sin\theta.\hat{\varphi}$$ (13b)

$$\hat{\varphi} \text{ x } \hat{\theta} = -\hat{R}$$ Use RHR

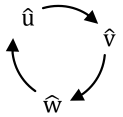
$$\hat{z} \text{ x } \hat{\theta} = cos\theta.\hat{\varphi}$$ <->(12)

$$\cos(\theta) = \frac{z}{\sqrt{r^2 + z^2}} = \frac{z}{R}$$

$$\hat{r} \times \hat{\varphi} = \hat{z}$$ Use RHR

$$\hat{\varphi} \times \hat{\varphi} = 0$$ Use RHR

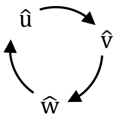
$$\hat{z} \times \hat{\hat{\varphi}} = \hat{r}$$ Use RHR



Gradyan (Skaler alanın Gradyan'ı):

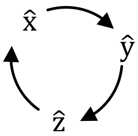
$\hat{n}$ yönü, (x,y,z) noktasında pozisyona bağlı bir skaler alan olan $\alpha(x,y,z)$'nın değişimin en fazla olduğu yönü ifade etmek üzere, $\nabla\alpha(x,y,z) = \hat{n}\frac{\partial\alpha(x,y,z)}{\partial n}$ olarak hesaplanan vektörel alan, skaler alan α'nın gradyanıdır. $(\hat{u},\hat{v},\hat{w})$ genel olarak dik koordinat sistemi olmak üzere, $(\hat{u},\hat{v},\hat{w})$ dairesel rotasyonuna göre, gradyan alttaki ifadelerle hesaplanır.

Genel olarak dik koordinat sistemi



$$\nabla\alpha(u,v,w) = \hat{n}\frac{\partial\alpha(u,v,w)}{\partial n} = \frac{1}{h_u}\frac{\partial\alpha}{\partial u}\hat{u} + \frac{1}{h_v}\frac{\partial\alpha}{\partial v}\hat{v} + \frac{1}{h_w}\frac{\partial\alpha}{\partial w}\hat{w}$$

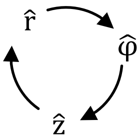
Kartezyen koordinat sistemi



$$\nabla\alpha(x,y,z) = \hat{n}\frac{\partial\alpha(x,y,z)}{\partial n} = \frac{1}{h_x}\frac{\partial\alpha}{\partial x}\hat{x} + \frac{1}{h_y}\frac{\partial\alpha}{\partial y}\hat{y} + \frac{1}{h_z}\frac{\partial\alpha}{\partial z}\hat{z}$$

$$\nabla\alpha(x,y,z) = \hat{n}\frac{\partial\alpha(x,y,z)}{\partial n} = \frac{1}{1}\frac{\partial\alpha}{\partial x}\hat{x} + \frac{1}{1}\frac{\partial\alpha}{\partial y}\hat{y} + \frac{1}{1}\frac{\partial\alpha}{\partial z}\hat{z}$$

Silindirik koordinat sistemi

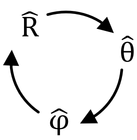


$$\nabla\alpha(r,\varphi,z) = \hat{n}\frac{\partial\alpha(r,\varphi,z)}{\partial n} = \frac{1}{h_r}\frac{\partial\alpha}{\partial r}\hat{r} + \frac{1}{h_\varphi}\frac{\partial\alpha}{\partial\varphi}\hat{\varphi} + \frac{1}{h_z}\frac{\partial\alpha}{\partial z}\hat{z}$$

$$\nabla\alpha(r,\varphi,z) = \hat{n}\frac{\partial\alpha(r,\varphi,z)}{\partial n} = \frac{1}{1}\frac{\partial\alpha}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\alpha}{\partial r}\hat{\varphi} + \frac{1}{1}\frac{\partial\alpha}{\partial z}\hat{z}$$

$$\nabla\alpha(R,\theta,\varphi) = \hat{n}\frac{\partial\alpha(R,\theta,\varphi)}{\partial n} = \frac{1}{h_R}\frac{\partial\alpha}{\partial R}\hat{R} + \frac{1}{h_\theta}\frac{\partial\alpha}{\partial\theta}\hat{\theta} + \frac{1}{h_\varphi}\frac{\partial\alpha}{\partial\varphi}\hat{\varphi}$$

Küresel koordinat sistemi



$$\nabla\alpha(R,\theta,\varphi) = \hat{n}\frac{\partial\alpha(R,\theta,\varphi)}{\partial n}$$

$$= \frac{1}{1}\frac{\partial\alpha}{\partial R}\hat{R} + \frac{1}{R}\frac{\partial\alpha}{\partial\theta}\hat{\theta} + \frac{1}{Rsin(\theta)}\frac{\partial\alpha}{\partial\varphi}\hat{\varphi}$$