Curves and Parametrization

$$\vec{R} = x(t)\hat{x} + y(t)\hat{y} + z(t)\hat{z} \qquad a \le t \le b$$

$$\vec{v}(t) = \frac{d\vec{R}}{dt}$$

Arc length = s

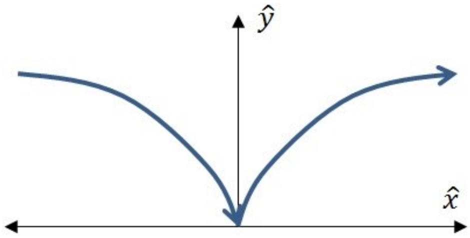
$$s = \int_{t_1}^{t_2} |\vec{v}(t)|\,dt$$

$$= \int_{t_1}^{t_2} \left|\frac{d\vec{R}}{dt}\right| dt \qquad \int_{t_1}^{t_2} v(t)\,dt$$

$$\int_{t_1}^{t_2} v(t)\,dt$$

$$v(t) = |\vec{v}(t)| = \left|\frac{d\vec{R}}{dt}\right|$$

Ex.



$$\vec{R} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$x = t^3$$

$$y = t^2$$

$$z = 0$$

$$\vec{R} = t^3\hat{x} + t^2\hat{y} \qquad -\infty \le t \le +\infty$$

$$\vec{v}(t) = 3t^2\hat{x} + 2t^1\hat{y}$$

$|\vec{v}(t)| =$ 0 @ $t = 0$

$$x = t^3 = 0$$

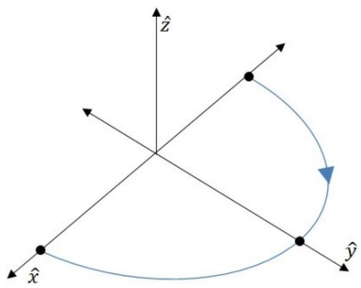
$$y = t^2 = 0$$

$$z = 0$$

Not smooth at $t = 0$

Piecewise smooth for t>0 & t<0

Ex.



$\vec{R}_1 = \sin(t)\,.\hat{x} + \cos(t)\hat{y} + 0\hat{z}$ $\quad t: -\frac{\pi}{2} \to +\frac{\pi}{2}$

$\vec{v}_1 = \frac{d\vec{R}_1}{dt} =$

$v_1 = |\vec{v}_1| = |\frac{d\vec{R}_1}{dt}| =$

t	x=$\sin(t)$	y= $\cos(t)$	z=0
$-\frac{\pi}{2}$			
0			
$\frac{\pi}{2}$			

$x^2 + y^2 = 1$

$\vec{R}_2 = (\mathrm{t}-1)\hat{x} + \sqrt{2\mathrm{t} - t^2}\hat{y} + 0\hat{z}$ $\quad t: 0 \to 2$

$\vec{v}_1 = \frac{d\vec{R}_1}{dt} =$

$v_1 = |\vec{v}_1| = |\frac{d\vec{R}_1}{dt}| =$

t	x=$(\mathrm{t}-1)$	y= $\sqrt{2\mathrm{t}-t^2}$	z=0
0			
1			
2			

$x^2 + y^2 = 1$

$\vec{R}_3 = \mathrm{t}\sqrt{2-t^2}\hat{x} + (1-t^2)\hat{y} + 0\hat{z}$ $\quad t: -1 \to 1$

$\vec{v}_1 = \frac{d\vec{R}_1}{dt} =$

$v_1 = |\vec{v}_1| = |\frac{d\vec{R}_1}{dt}| =$

t	x=$\mathrm{t}\sqrt{2-t^2}$	y= $(1-t^2)$	z=0
-1			
0			
1			

$x^2 + y^2 = 1$

Ex

Parametrize the intersection of two surfaces

$$3x + 2y + z = 4 \qquad \text{a plane}$$

$$x^2 + y^2 = 4 \qquad \text{An elliptic cylinder}$$

Choose $x = 2\cos(t) \qquad t: 0 \to 2\pi$

$$y = sin(t)$$

$\Rightarrow$ $z = 4 - 3x - 2y$

$$z = 4 - 6\cos(t) - 2\sin(t)$$

$$\vec{R} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$2\cos(t).\hat{x} + \sin(t)\hat{y} + (4 - 6\cos(t) - 2\sin(t))\hat{z}$$

$$\overrightarrow{dR} = dt.\{-2\sin(t).\hat{x} + \cos(t)\hat{y} + (6\sin(t) - 2\cos(t))\hat{z}\}$$

Ex

Parametrize the intersection of two surfaces

$$xy + z = 1$$

$$x^2 + y + z = 2$$

Choose $x = t$

$\Rightarrow$ $z = 1 - xy = 1 - ty$

$\Rightarrow$ $t^2 + y + 1 - ty = 2$

$\Rightarrow$ $y = \dfrac{1 - t^2}{1 - t} = 1 + t$

$\Rightarrow$ $z = 1 - xy = 1 - t.(1 + t)$

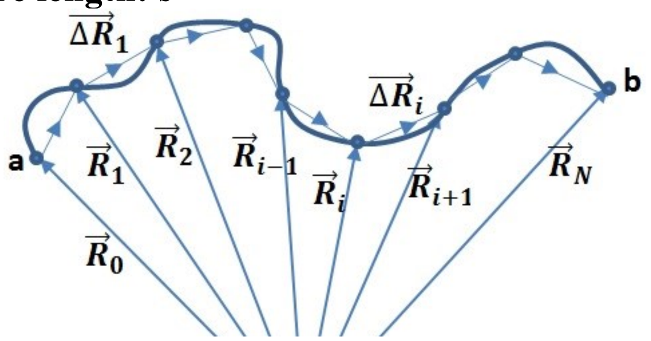
$$z = 1 - t - t^2$$

$$\vec{R} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$t\hat{x} + (1 + t)\hat{y} + (1 - t - t^2)\hat{z}$$

$$\overrightarrow{dR} = dt.\{1.\hat{x} + 1.\hat{y} + (-1 - 2t).\hat{z}\}$$

Arc length: s



$$\boldsymbol{s} = \sum_{i=1}^{N} \left|\frac{\overrightarrow{\Delta R_i}}{\Delta t_i}\right|.\Delta t_i$$

$$\boldsymbol{s} = \int_a^b \left|\frac{\overrightarrow{dR_i}}{dt_i}\right| dt$$

$$s=\int_a^b |\vec{v}|\,dt$$

$$\overrightarrow{\Delta R_i}=\vec{R}_{i+1}-\vec{R}_i$$

$$s=\int_a^b v\,dt$$

Assume $\vec{R}=x\hat{x}+f(x)\hat{y}$

$$\frac{\overrightarrow{dR}}{dx}=1.\hat{x}+f'(x).\hat{x}$$

$$ds=\left|\frac{\overrightarrow{dR}}{dt}\right|dt=\left|\frac{\overrightarrow{dR}dx}{dxdt}\right|dt=\left|\frac{\overrightarrow{dR}}{dx}\right|dx$$

$$\left|\frac{\overrightarrow{dR}}{dx}\right|=|1.\hat{x}+f'(x).\hat{x}|=\sqrt{1+(f'(x))^2}$$

$$ds=\sqrt{1+(f'(x))^2}.dx$$

Ex: Find arclength s for the circular helix: $\vec{R}=acos(t)\hat{x}+asin(t)\hat{y}+bt\hat{z}$ between (a,0,0) and (a,0,2πb).

$x=\ acos(t)$ $\quad$ $y=\ acos(t)$ $\quad$ $z=\ bt$

$$x^2+y^2=a$$

$$\vec{v}=\frac{\overrightarrow{dR}}{dt}=\quad -asin(t)\hat{x}+acos(t)\hat{y}+b\hat{z}$$

$$|\vec{v}|=v=\quad \sqrt{(-asin(t))^2+(acos(t))^2+b^2}$$

$$v=\quad \sqrt{a^2+b^2}$$

$$s=\quad \int_{Pstart}^{Pstop} v\,dt=\int_0^{2\pi}\sqrt{a^2+b^2}\,dt \quad =2\pi\sqrt{a^2+b^2}$$

Type equation here.

$$ds=|\overrightarrow{dR}|=|\overrightarrow{dl}|=dl$$

$$ds=\quad |\overrightarrow{dl}|=|(1.\mathrm{dr})\hat{\mathrm{r}}+(1.\mathrm{d}\varphi)\hat{\varphi}+(1.\mathrm{dz})\hat{\mathrm{z}}|$$

$$ds=\quad |\overrightarrow{dl}|=\left|\left(1.\frac{\mathrm{dr}}{dt}\right)\hat{\mathrm{r}}+\left(\mathrm{r}.\frac{\mathrm{d}\varphi}{dt}\right)\hat{\varphi}+\left(1.\frac{\mathrm{dz}}{dt}\right)\hat{\mathrm{z}}\right|dt$$

$$ds = |\overrightarrow{dl}| = \sqrt{\left(1.\frac{dr}{dt}\right)^2 + \left(r.\frac{d\varphi}{dt}\right)^2 + \left(1.\frac{dz}{dt}\right)^2}\, dt$$

$$ds = |\overrightarrow{dl}| = |(1.dR)\hat{R} + (R.d\theta)\hat{\theta} + (R\sin\theta.d\varphi)\hat{\varphi}|$$

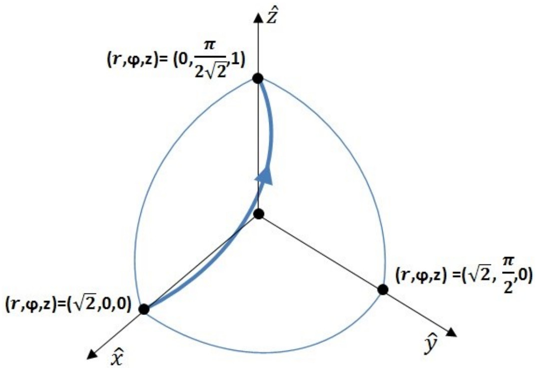
$$ds = |\overrightarrow{dl}| = |\left(1.\frac{dR}{dt}\right)\hat{R} + \left(R.\frac{d\theta}{dt}\right)\hat{\theta} + \left(R\sin\theta.\frac{d\varphi}{dt}\right)\hat{\varphi}|dt$$

$$ds = |\overrightarrow{dl}| = \sqrt{\left(1.\frac{dR}{dt}\right)^2 + \left(R.\frac{d\theta}{dt}\right)^2 + \left(R\sin\theta.\frac{d\varphi}{dt}\right)^2}\, dt$$

Ex: Desribe the curve and find the arc length for $t: 0 \to \frac{\pi}{2}$ where

$r(t) = \sqrt{2}\cos(t)$ $\qquad \varphi = \frac{\pi}{2}$ $\qquad z = \sin(t)$

Solution



See that $\qquad r^2 + 2z^2 = 2 \qquad$ (Ellipsoid in 3D)

$$x^2 + y^2 + 2z^2 = 2$$

$$s = \int_a^b |\vec{v}|\, dt = \int_a^b ds = \int_a^b \sqrt{\left(1.\frac{dr}{dt}\right)^2 + \left(r.\frac{d\varphi}{dt}\right)^2 + \left(1.\frac{dz}{dt}\right)^2}\, dt$$

$$s = \int_0^{\frac{\pi}{2}} \sqrt{(1.2\sin(t))^2 + \left(2\cos(t).\frac{1}{\sqrt{2}}\right)^2 + (1.\cos(t))^2}\, dt$$

$$s = \frac{\pi}{\sqrt{2}} \text{ units}$$

Arc length parametrization

$$ds = v(t)dt \qquad \text{general parametrization}$$

$$ds = v(s)ds \qquad t = s \; (arc\ lenght\ parametrization)$$

$\Rightarrow$ $$v(s) = 1 = constant$$

$\Rightarrow$ *Arc lenght parametrized curves are trajected at constant speed of* $v(s) = 1$

Ex: Parametrize circular helix in arc length parametrization

$$\vec{R} = a\cos(t)\hat{x} + a\sin(t)\hat{y} + bt\hat{z}$$

$$\vec{v} = \frac{d\vec{R}}{dt} = -a\sin(t)\hat{x} + a\cos(t)\hat{y} + b\hat{z}$$

$$|\vec{v}| = v = \sqrt{(-a\sin(t))^2 + (a\cos(t))^2 + b^2}$$

$$v = \sqrt{a^2 + b^2}$$

$$s = \int_{Pstart}^{Pstop} v\,dt = \int_0^t \sqrt{a^2 + b^2}\,dt = t\sqrt{a^2 + b^2}$$

$$\Rightarrow t = \frac{s}{\sqrt{a^2 + b^2}}$$

$$\Rightarrow \vec{R} = a\cos\left(\frac{s}{\sqrt{a^2 + b^2}}\right)\hat{x} + a\sin\left(\frac{s}{\sqrt{a^2 + b^2}}\right)\hat{y} + b\left(\frac{s}{\sqrt{a^2 + b^2}}\right)\hat{z}$$

LINE INTEGRAL

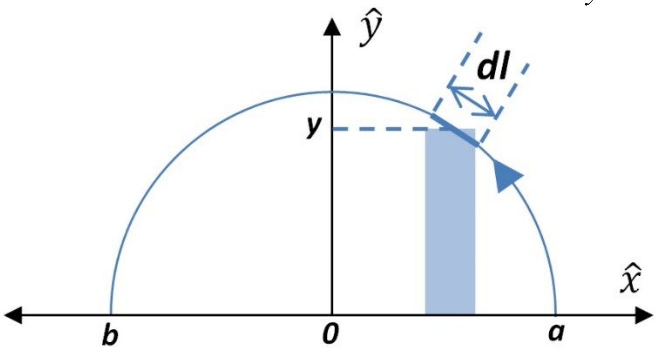
Length of C=

$$\int_C ds = \int_a^b dl = \int_a^b \left|\frac{d\vec{R}}{dt}\right| dt$$

In general,

$$\int_C f(\vec{R}(t))dl = \int_a^b f(\vec{R}(t)) \left|\frac{d\vec{R}}{dt}\right| dt$$

Ex: Calculate the moment of half circle about y=0



$$M = \int_C y.dl$$

$$x = a\cos(t) \quad y = a\sin(t)$$

$$\vec{R} = x\hat{x} + y\hat{y}$$

$$\vec{R} = a\cos(t)\hat{x} + a\sin(t)\hat{y}$$

$$\frac{d\vec{R}}{dt} = -a\sin(t)\hat{x} + a\cos(t)\hat{y} \qquad t: 0 \to \pi$$

$$\left|\frac{d\vec{R}}{dt}\right| = |-a\sin(t)\hat{x} + a\cos(t)\hat{y}|$$

$$\left|\frac{d\vec{R}}{dt}\right| = \sqrt{(-asin(t)\hat{x} + acos(t)\hat{y}) \bullet (-asin(t)\hat{x} + acos(t)\hat{y})}$$

$$\left|\frac{d\vec{R}}{dt}\right| = \sqrt{a^2\big(sin^2(t) + cos^2(t)\big) = a}$$

$$dl = \left|\frac{d\vec{R}}{dt}\right| dt = a.dt$$

$$M = \int_C y.dl = \int_0^\pi asin(t).a.dt = 2a^2$$

Alternative parametrization

$x = \ x = t$ $\qquad y = \ \pm\sqrt{a^2 - x^2}$ $\qquad x: -a \rightarrow a$

$$\vec{R} = x\hat{x} + y\hat{y}$$

$$\vec{R} = x\hat{x} + \sqrt{a^2 - x^2}\hat{y}$$

$$\frac{d\vec{R}}{dx} = 1\hat{x} - \frac{x}{\sqrt{a^2 - x^2}}\hat{y}$$

$t: 0 \rightarrow \pi$

$$\left|\frac{d\vec{R}}{dt}\right| = \left|1\hat{x} - \frac{x}{\sqrt{a^2 - x^2}}\hat{y}\right|$$

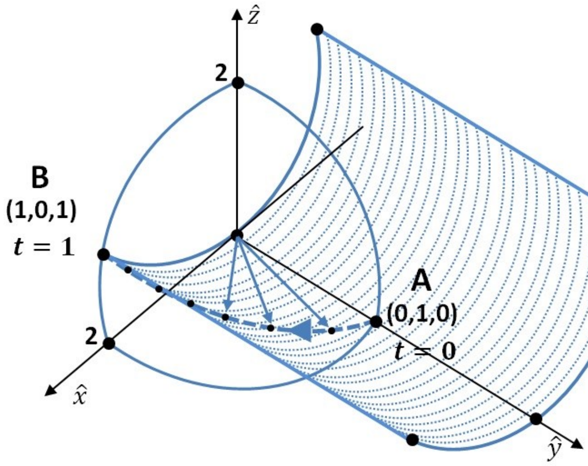
$$\left|\frac{d\vec{R}}{dt}\right| = \sqrt{(1\hat{x} - \frac{x}{\sqrt{a^2 - x^2}}\hat{y}) \bullet (1\hat{x} - \frac{x}{\sqrt{a^2 - x^2}}\hat{y})}$$

$$\left|\frac{d\vec{R}}{dt}\right| = \sqrt{\left(1 + \frac{x^2}{a^2 - x^2}\right)} = \sqrt{\left(\frac{a^2}{a^2 - x^2}\right)}$$

$$dl = \left|\frac{d\vec{R}}{dt}\right| dt = \sqrt{\left(\frac{a^2}{a^2 - x^2}\right)}.dx = \frac{a}{\sqrt{a^2 - x^2}}$$

$$M = \int_C y.dl = \int_{-a}^{a} \sqrt{a^2 - x^2}.\frac{a}{\sqrt{a^2 - x^2}}.dx = 2a^2$$

Ex: Assume a wire with density $\boldsymbol{\rho(x,y,z) = xy}\ (\frac{grams}{m^2})$. Find the mass of the wire which is padded on the contour from $(\mathbf{0,1,0})$ to $(\mathbf{1,0,1})$ if the contour is obtained by the intersection of the surfaces $\boldsymbol{z = 2 - x^2 - 2y^2}$ and $\boldsymbol{z = x^2}$.



$$x = t$$

$$y = \sqrt{1 - t^2}$$

$$z = t^2$$

t	0	$\rightarrow$	1
x	0		1
y	1		0
z	0		1
	A		B

$$\vec{R} = x\hat{x} + y\hat{y} + z\hat{z} = t\hat{x} + \sqrt{1-t^2}\,\hat{y} + t^2\hat{z}$$

$$t: 0 \rightarrow 1$$

$$\frac{d\vec{R}}{dx} = 1\hat{x} - \frac{t}{\sqrt{1-t^2}}\hat{y} + 2t.\hat{z}$$

$$\left|\frac{d\vec{R}}{dt}\right| = \left|\frac{d\vec{R}}{dx} = 1\hat{x} - \frac{t}{\sqrt{1-t^2}}\hat{y} + 2t.\hat{z}\right|$$

$$\left|\frac{d\vec{R}}{dt}\right| = \sqrt{\left(1\hat{x} - \frac{t}{\sqrt{1-t^2}}\hat{y} + 2t.\hat{z}\right) \bullet \left(1\hat{x} - \frac{t}{\sqrt{1-t^2}}\hat{y} + 2t.\hat{z}\right)}$$

$$\left|\frac{d\vec{R}}{dt}\right| = \sqrt{1 + \frac{t^2}{1-t^2} + 4t^2} = \sqrt{\frac{1 + 4t^2 - 4t^4}{1-t^2}}$$

$$dl = \left|\frac{d\vec{R}}{dt}\right| dt = \sqrt{\frac{1 + 4t^2 - 4t^4}{1-t^2}}\,.dt$$

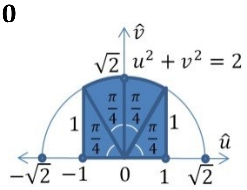
$$Mass\ m = \int_C \rho(x,y,z)\,.dl = \int_C xy\,.dl = \int_{t=0}^{1} t.\sqrt{1-t^2}\ .dl$$

$$m = \int_{t=0}^{1} t.\sqrt{1-t^2}.\sqrt{\frac{1 + 4t^2 - 4t^4}{1-t^2}}dt = \int_{t=0}^{1} \sqrt{1 + 4t^2 - 4t^4}.t.dt$$

$$t^2 = k$$

$$tdt = \frac{dk}{2} \quad \Rightarrow \quad m = \int_{k=0}^{1} \sqrt{1 + 4k - 4k^2}\frac{dk}{2} = \int_{k=0}^{1} \sqrt{2 - (2k-1)^2}\frac{dk}{2}$$

$$m = \int_{v=-1}^{1} \sqrt{2 - u^2}\frac{du}{4} = \frac{1}{4}.2.\left(\frac{\pi}{4} + \frac{1}{2}\right)$$



Stoke's Theorem

$$\int_S \nabla \times \vec{A} \cdot \overrightarrow{dS} = \oint_C \vec{A} \cdot \overrightarrow{dl}$$

Null Identities

1) $\nabla \times (\nabla V) = 0$

The curl of a gradient of a scalar field is identically zero.

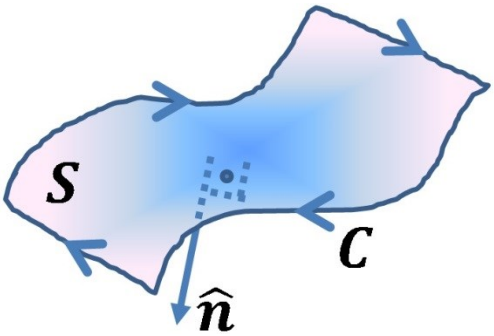
Proof: Consider an arbitrary surface S bounded by a closed contour C.

$$\int_S \nabla \times (\nabla V) \cdot \overrightarrow{dS} = \oint_C \nabla V \cdot \overrightarrow{dl}$$ (By Stoke's Theorem)

$$\nabla V \cdot \hat{l}$$

is the directional derivative i.e., the derivative of V in the $\hat{l}$ direction

$$= \oint_C \nabla V \cdot \hat{l} dl = \oint_C \frac{dV}{dl} dl$$



$$\oint_C dV = V \Big|_{P_1}^{P_1 = P_2} = V(P_2) - V(P_1)$$

$$= 0$$

⇒

$$\oint_C \nabla V \cdot \overrightarrow{dl} = \int_S \nabla \times (\nabla V) \cdot \overrightarrow{dS} = 0 \qquad \forall S$$

⇒

$$\nabla \times (\nabla V) = 0$$

If a vector is curl-free (i.e., if the curl a vector field is 0), then it can be expressed as a gradient of a vector field.

Such curl-free vector fields are called "irrotational" or "conservative".

Ex: In electrostatics,

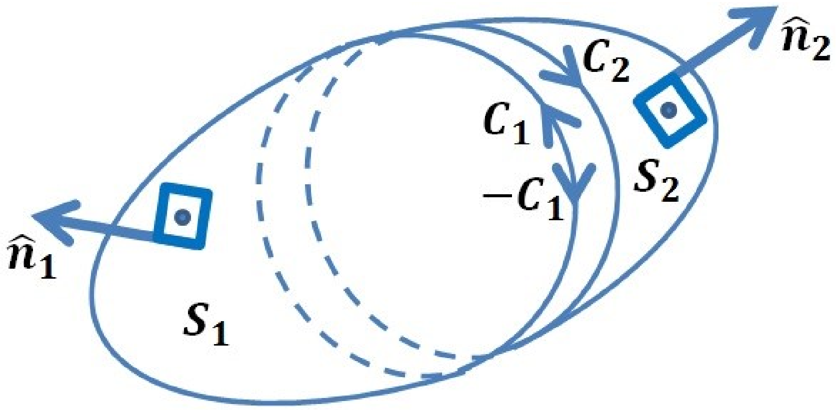
$$\vec{E} = \nabla V \quad \Leftrightarrow \quad \nabla \times \vec{E} = 0$$

$\vec{E}$: Electric field intensity vector | **$\vec{E}$: Conservative, Irrotational**

$$\mathbf{2)}\ \nabla \bullet (\nabla \times \vec{A}) = 0$$

Proof:

Split the closed surface S into two open surfaces S_1 and S_2 with common rim C:



By Divergence Theorem:

$$S = S_1 \cup S_2$$

$$\overrightarrow{dS}_1 = \hat{n}_1 dS \qquad \overrightarrow{dS}_2 = \hat{n}_2 dS$$

$$-\oint_{C_1} \vec{A} \bullet \overrightarrow{dl}_1 = \oint_{-C_1} \vec{A} \bullet \overrightarrow{dl}_1 = \oint_{C_2} \vec{A} \bullet \overrightarrow{dl}_2$$

$$\int_V \nabla \bullet (\nabla \times \vec{A}) dV = \oint_S \nabla \times \vec{A} \bullet \overrightarrow{dS}$$

$$= \int_{S_1} \nabla \times \vec{A} \bullet \overrightarrow{dS} + \int_{S_2} \nabla \times \vec{A} \bullet \overrightarrow{dS}$$

Using Stoke's Theorem

$$\int_V \nabla \bullet (\nabla \times \vec{A}) dV = \oint_{C_1} \vec{A} \bullet \overrightarrow{dl}_1 + \oint_{C_2} \vec{A} \bullet \overrightarrow{dl}_2$$

$$\int_V \nabla \bullet (\nabla \times \vec{A}) dV = -\oint_{C_2} \vec{A} \bullet \overrightarrow{dl}_2 + \oint_{C_2} \vec{A} \bullet \overrightarrow{dl}_2$$

$$\int_V \nabla \bullet (\nabla \times \vec{A}) dV = 0 \qquad \forall\, V$$

$$\nabla \bullet (\nabla \times \vec{A}) = 0$$

If a vector field is divergenceless, then it is called as Solenodial Field and It can be expressed as the curl of an another vector:

$\vec{B}$**:** Magnetic Flux Density

$\vec{A}$**:** Vector Potential

$$\nabla \cdot \vec{B} = 0 \Leftrightarrow \vec{B} = \nabla \times \vec{A}$$

Field Classification and Helmholtz Theorem

1) Static electric field in a charge free region

Solenoidal	Irrotational
$\nabla \cdot \vec{F} = 0$	$\nabla \times \vec{F} = 0$

2) Steady magnetic field in a current carrying conductor

Solenoidal	Not Irrotational
$\nabla \cdot \vec{F} = 0$	$\nabla \times \vec{F} \neq 0$

3) Static electric field in a charged region

Not Solenoidal	Irrotational
$\nabla \cdot \vec{F} \neq 0$	$\nabla \times \vec{F} = 0$

4) Time-varying electric field in a charged region

Not Solenoidal	Not Irrotational
$\nabla \cdot \vec{F} \neq 0$	$\nabla \times \vec{F} \neq 0$

In general, a vector field $\vec{F}$ can be decomposed into its solenoidal ($\vec{F}_s$) and irrotational ($\vec{F}_i$) parts as:

$$\vec{F} = \vec{F}_s + \vec{F}_i$$

$$\nabla \cdot \vec{F}_s = 0 \qquad \nabla \times \vec{F}_i = 0$$

$$\nabla \times \vec{F}_s \neq 0 \qquad \nabla \cdot \vec{F}_i \neq 0$$

Thus,

$$\begin{aligned} \nabla \cdot \vec{F} &= \nabla \cdot (\vec{F}_s + \vec{F}_i) \\ &= \nabla \cdot \vec{F}_s + \nabla \cdot \vec{F}_i \\ &= \nabla \cdot \vec{F}_i \end{aligned}$$

$$\begin{aligned} \nabla \times \vec{F} &= \nabla \times (\vec{F}_s + \vec{F}_i) \\ &= \nabla \times \vec{F}_s + \nabla \times \vec{F}_i \\ &= \nabla \times \vec{F}_s \end{aligned}$$

According to Helmholtz Theorem, a vector field is determined if both its divergence and its curl are specified everywhere in the space of interest:

$\nabla \bullet \vec{F}$ → Measure of strength of flow sources (scalar sources)

$\nabla \times \vec{F}$ → Measure of strength of vortex sources (vector sources)

⇒

$$\vec{F} = \vec{F}_s + \vec{F}_i$$

$$\vec{F} = \nabla \times \vec{A} + (-\nabla V)$$

where

V: Scalar Potential

$\vec{A}$: Scalar Potential

$$\nabla \bullet \vec{F} = \nabla \bullet (\vec{F}_s + \vec{F}_i)$$

$$\nabla \bullet \vec{F} = \nabla \bullet (-\nabla V + \nabla \times \vec{A})$$

$$\nabla \bullet \vec{F} = -\nabla \bullet (\nabla V) + \nabla \bullet (\nabla \times \vec{A}))$$

$$\nabla \bullet \vec{F} = -\nabla \bullet (\nabla V) = -\nabla \bullet \nabla(V)$$

$$\nabla \bullet \vec{F} = -\nabla^2 V \neq 0$$

$$\nabla \times \vec{F} = \nabla \times (\vec{F}_s + \vec{F}_i)$$

$$\nabla \times \vec{F} = \nabla \times (-\nabla V + \nabla \times \vec{A})$$

$$\nabla \times \vec{F} = -\nabla \times (\nabla V) + \nabla \times (\nabla \times \vec{A}))$$

$$\nabla \times \vec{F} = \nabla \times (\nabla \times \vec{A}))$$

$$\nabla \times \vec{F} = \nabla\left(\nabla \bullet \vec{A}\right) - \nabla^2 \vec{A} \neq 0$$

Knowing $\nabla \bullet \vec{F}$

One can solve for V using $\nabla \bullet \vec{F} = -\nabla^2 V$

Knowing $\nabla \times \vec{F}$

One can solve for $\vec{A}$ using $\nabla \times \vec{F} = \nabla\left(\nabla \bullet \vec{A}\right) - \nabla^2 \vec{A}$

Having obtained V and $\vec{A}$ one can write $\vec{F}$ as

$$\vec{F} = \nabla \times \vec{A} + (-\nabla V)$$

Static Electric Field

Electric field intensity is defined as as the force per unit charge that a very small stationary test charge experiences when it is placed in a region where an electric field exists.

$$\vec{E} = \lim_{q \to 0} \frac{\vec{F}}{q}$$

$$\vec{F} = q\vec{E}$$

$[\vec{F}] =$	$Newton$	
$[\vec{E}] =$	$\frac{Newton}{Coulomb} = \frac{Volt}{m}$	Electric Field Intensity

Fundamental Postulates of Electrostatics in free space (Differential Form)

$\nabla \cdot \vec{E} =$	$\frac{\rho_v}{\epsilon_0}$	Divergence of Electric Field
$\nabla \times \vec{E} =$	0	Curl of Electric Field

(Remember Helmholtz Theorem)

$[\rho_v] =$	$\frac{Coulomb}{m^3}$	Free Volume Charge Density
$\epsilon_0 =$	$\frac{1}{36\pi}.10^{-9}$	
$[\epsilon_0] =$	$\frac{Farad}{m}$	Dielectric constant for free space

Notes:

- Static electric fields are always irrotational but they are solenoidal at points where $\rho_v = 0$,
- All other relations in electrostatics can be obtained using these fundamental postulates,
- These postulates are independent of the chosen coordinate system.