PROBABILITY AND RANDOM VARIABLES

Textbook:

1) D. P. Bertsekas, J. N. Tsitsiklis, "Introduction to Probability", 2^{nd} Ed., Athena Science 2008.

Reference Books:

1) A. Papoulis, S. U. Pillai, "Probability, Random Variables and Stochastic Processes", McGraw-Hill 2002.
2) A. Leon-Garcia, "Probability, Statistics, and Random Processes for Electrical Engineering", 3^{rd} Ed. Pearson 2008
3) X. Rong Li, "Probability Random Signals and Statistics", CRC Press 1999
4) S. M. Ross, "A First Course In Probability", Pearson 2009.

Outline

- Probabilistic models,
- Conditional probability,
- Total probability theorem and Bayes' rule
- Independence, Counting
- Discrete random variables
- Probability mass function
- Function of random variables
- Expectation, mean and variance
- Discrete random variables: Joint PMFs of two random variables
- Discrete random variables: Conditioning and Independence
- Midterm

Textbook: D. P. Bertsekas, J. N. Tsitsiklis, "Introduction to Probability", 2nd Ed., Athena Science 2008.

Outline (Continued)

- Continuous random variables
- Probability density function
- Cumulative density function
- Normal random variables
- Some special random variables and their properties
- Continuous random variables: Joint PDFs of two random variables

  Continuous random variables: Conditioning and Continuous Bayes' rule
- Covariance and correlation
- Conditional expectation and variance
- Function of random variables
- Markov and Chebyshev inequalities
- Law of large numbers, Convergence theorems, Central limit theorem

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“Probability is common sense reduced to calculation.”

Laplace

Interpretations of Probability

- **Frequentist**

  repeated trials: flipping the coin

- **Bayesian**

  Quantifying **uncertainty**

  Modeling based on information

- which interpretation do we use?
- advantages of each interpretation

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➢ Motivation

- describing uncertainty
    - noise
- probabilistic modeling
    - thermal noise, awgn
- probabilistic inference

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SET THEORY

$x \in A$: *x* is a member of *A*

$x \notin A$: *x* is not a member of *A*

$\varnothing$: empty (null) set

$A = \{x_1, x_2, ..., x_n\}$ finite number of elements

$A = \{x_1, x_2, ...\}$ countable infinite

$\{x \mid x \text{ satisfies a property}\}$

"$|$" : "such that"

$A \subset B$ subset

Ω universal set

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SET THEORY
Set Operations

- $A^C = \{x : x \notin A\}$ Complement
- $A \cup B = \{x : x \in A \text{ or } x \in B\}$ Union
- $A \cap B = AB = \{x : x \in A \text{ and } x \in B\}$ Intersection

$A \cap B = AB = \varnothing$ Mutually Exclusive or Disjoint

SET THEORY

The Algebra Of Sets

- $\left(A^C\right)^C = A$
- $A \cup B = B \cup A$ and $A \cap B = B \cap A$
- $A \cap (B \cap C) = (A \cap B) \cap C$

  $A \cup (B \cup C) = (A \cup B) \cup C$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

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SET THEORY

The Algebra Of Sets

- $A \cap A^C = \varnothing$
- $A \cup \varnothing = A$
- $A \cup \Omega = \Omega$
- $A \cap \Omega = A$

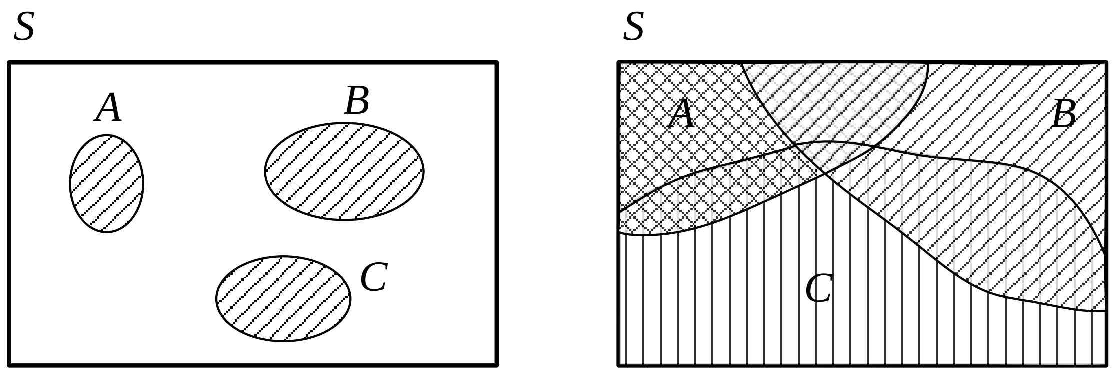
De Morgan's Law

$$\left(\bigcup_n A_n\right)^C = \bigcap_n A_n^C$$

$$\left(\bigcap_n A_n\right)^C = \bigcup_n A_n^C$$

SET THEORY

- Mutually exclusive or disjoint sets
- Collectively exhaustive



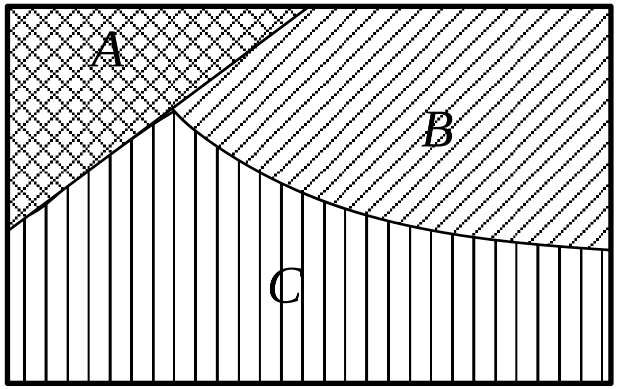
Mutually Exclusive   Collectively Exhaustive

Textbook: D. P. Bertsekas, J. N. Tsitsiklis, “Introduction to Probability”, 2nd Ed., Athena Science 2008.

SET THEORY

A **partition** of the universal set S.

- Mutually exclusive and collectively exhaustive



Mutually Exclusive

and

Collectively Exhaustive

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