Interpolation, Interpolating Polynomials, Spline interpolation [1-6]

References:

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2nd order Lagrange interpolation

v (m^3/kg)	0.10377	0.108	0.11144	0.1254
s(kJ/kgK)	6.4147	?	6.5453	6.7664

$$x_0 = 0.10377 \quad f(x_0) = 6.4147$$
$$x_1 = 0.11144 \quad f(x_1) = 6.5453$$
$$x_2 = 0.1254 \quad f(x_2) = 6.7664$$

$$f_n(x) = \sum_{i=0}^{n} L_i(x) f_n(x)$$

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$

$$f_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$$

$$f_2(x)=\frac{(0.108-0.11144)(0.108-0.1254)}{(0.10377-0.11144)(0.10377-0.1254)}6.4147+\frac{(0.108-0.10377)(0.108-0.1254)}{(0.11144-0.10377)(0.11144-0.1254)}6.5453$$

$$+\frac{(0.108-0.10377)(0.108-0.11144)}{(0.1254-0.10377)(0.1254-0.11144)}6.7664$$

$$f_2(0.108)=6.4874$$

The Lagrange Cubic Interpolating Polynomial

$$P_3(x)=y_0\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}+y_1\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$
$$+y_2\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}+y_3\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$P_x(x) = 1.000000\frac{(x-0.4)(x-0.8)(x-1.2)}{(0.0-0.4)(0.0-0.8)(0.0-1.2)} + 0.921061\frac{(x-0.0)(x-0.8)(x-1.2)}{(0.4-0.0)(0.4-0.8)(0.4-1.2)}$$
$$+ 0.696707\frac{(x-0.0)(x-0.4)(x-1.2)}{(0.8-0.0)(0.8-0.4)(0.8-1.2)}$$
$$+ 0.362358\frac{(x-0.0)(x-0.4)(x-0.8)}{(1.2-0.0)(1.2-0.4)(1.2-0.8)}$$

$$P_3(x) = -2.604167(x-0.4)(x-0.8)(x-1.2) + 7.195789(x-0.0)(x-0.8)(x-1.2)$$
$$- 5.443021(x-0.0)(x-0.4)(x-1.2) + 0.943641(x-0.0)(x-0.4)(x-0.8)$$

$$P_3(0.6) = -0.062500008 + 0518096808 + 0.3918975 - 0.022647384$$
$$P_3(0.6) = 0.824847 \cong 0.825$$

cubic splines

estimate the value at x=5.5. f(x=5.5)=?

x	3.0	4.5	5.0	7.0
f(x)	2.5	1.0	1.1	2.5

$$f_i(x) = \frac{f_i''(x_{i-1})}{6(x_i - x_{i-1})}(x_i - x)^3 + \frac{f_i''(x_i)}{6(x_i - x_{i-1})}(x - x_{i-1})^3 + \left[\frac{f(x_{i-1})}{(x_i - x_{i-1})} - \frac{f''(x_{i-1})(x_i - x_{i-1})}{6}\right](x_i - x) +$$
$$\left[\frac{f(x_i)}{(x_i - x_{i-1})} - \frac{f''(x_i)(x_i - x_{i-1})}{6}\right](x - x_{i-1})$$

$$(x_i - x_{i-1})f''(x_{i-1}) + 2(x_{i+1} - x_{i-1})f''(x_i) + (x_{i+1} - x_i)f''(x_{i+1}) = \frac{6}{x_{i+1} - x_i}\left[f(x_{i+1}) - f(x_i)\right] + \frac{6}{x_i - x_{i-1}}\left[f(x_{i-1}) - f(x_i)\right]$$

$$(x_i - x_{i-1})f''(x_{i-1}) + 2(x_{i+1} - x_{i-1})f''(x_i) + (x_{i+1} - x_i)f''(x_{i+1}) = \frac{6}{x_{i+1} - x_i}\left[f(x_{i+1}) - f(x_i)\right] + \frac{6}{x_i - x_{i-1}}\left[f(x_{i-1}) - f(x_i)\right]$$

i=1

$$(x_1 - x_0)f''(x_0) + 2(x_2 - x_0)f''(x_1) + (x_2 - x_1)f''(x_2) = \frac{6}{x_2 - x_1}\left[f(x_2) - f(x_1)\right] + \frac{6}{x_1 - x_0}\left[f(x_0) - f(x_1)\right]$$

$$(4.5 - 3)f''(3) + 2(5 - 3)f''(4.5) + (5 - 4.5)f''(5) = \frac{6}{(5 - 4.5)}\left[1.1 - 1\right] + \frac{6}{(4.5 - 3)}\left[2.5 - 1\right]$$

The second derivatives at the end knots are zero

$$f''(3) = 0$$

$$4f''(4.5) + 0.5f''(5) = 7.2 \qquad (1)$$

Same equation can be applied to the second interior point

i=2

$$(x_2 - x_1) f''(x_1) + 2(x_3 - x_1) f''(x_2) + (x_3 - x_2) f''(x_3) = \frac{6}{x_3 - x_2}\left[f(x_3) - f(x_2)\right] + \frac{6}{x_2 - x_1}\left[f(x_1) - f(x_2)\right]$$

$$(5 - 4.5) f''(4.5) + 2(7 - 4.5) f''(5) + (7 - 5) f''(7) = \frac{6}{7 - 5}\left[f(7) - f(5)\right] + \frac{6}{5 - 4.5}\left[f(4.5) - f(5)\right]$$

$$(5 - 4.5) f''(4.5) + 2(7 - 4.5) f''(5) + (7 - 5) f''(7) = \frac{6}{7 - 5}\left[2.5 - 1.1\right] + \frac{6}{5 - 4.5}\left[1 - 1.1\right]$$

$$0.5 f''(4.5) + 5 f''(5) = 3 \qquad (2)$$

$$4 f''(4.5) + 0.5 f''(5) = 7.2 \qquad (1)$$

$$(-8)\{0.5 f''(4.5) + 5 f''(5) = 3\} \qquad (2)$$

$$4f''(4.5) + 0.5f''(5) = 7.2 \qquad (1)$$

$$-4f''(4.5) - 40f''(5) = -24 \qquad (2)$$

and are added $\quad -39.5f''(5) = -16.8 \qquad f''(5) = 0.42531$

If we put $f''(5) = 0.42531$ in equation (1)

$$4f''(4.5) + 0.5(0.4253\ 1) = 7.2 \qquad f''(4.5) = 1.74683$$

$$f_i(x) = \frac{f_i''(\mathrm{x}_{i-1})}{6(x_i - x_{i-1})}(x_i - x)^3 + \frac{f_i''(\mathrm{x}_i)}{6(x_i - x_{i-1})}(x - x_{i-1})^3 + \left[\frac{f(x_{i-1})}{(x_i - x_{i-1})} - \frac{f''(\mathrm{x}_{i-1})(x_i - x_{i-1})}{6}\right](x_i - x) + \left[\frac{f(x_i)}{(x_i - x_{i-1})} - \frac{f''(\mathrm{x}_i)(x_i - x_{i-1})}{6}\right](x - x_{i-1})$$

i=1

$$f_1(x) = \frac{f_1''(x_0)}{6(x_1 - x_0)}(x_1 - x)^3 + \frac{f_1''(x_1)}{6(x_1 - x_0)}(x - x_0)^3 + \left[\frac{f(x_0)}{(x_1 - x_0)} - \frac{f''(x_0)(x_1 - x_0)}{6}\right](x_1 - x) + \left[\frac{f(x_1)}{(x_1 - x_0)} - \frac{f''(x_1)(x_1 - x_0)}{6}\right](x - x_0)$$

$$f_1(x) = \frac{f_1''(3)}{6(4.5 - 3)}(4.5 - x)^3 + \frac{f_1''(4.5)}{6(4.5 - 3)}(x - 3)^3 + \left[\frac{f(3)}{(4.5 - 3)} - \frac{f''(3)(4.5 - 3)}{6}\right](4.5 - x) + \left[\frac{f(4.5)}{(4.5 - 3)} - \frac{f''(4.5)(4.5 - 3)}{6}\right](x - 3)$$

The second derivatives at the end knots are zero

$$f''(3) = f_1''(3) = 0$$

$$f_1(x) = \frac{1.74683}{9}(x - 3)^3 + \left[\frac{2.5}{1.5}(4.5 - x)\right] + \left[\frac{1}{1.5} - 1.74683 * 0.25\right](x - 3)$$

Cubic spline for first interval

$$f_1(x) = 0.194092\,(x-3)^3 + \left[1.66667\,(4.5-x)\right] + 0.229959\,(x-3)$$

i=2

$$f_2(x) = \frac{f_2''(x_1)}{6(x_2-x_1)}(x_2-x)^3 + \frac{f_2''(x_2)}{6(x_2-x_1)}(x-x_1)^3 + \left[\frac{f(x_1)}{(x_2-x_1)} - \frac{f''(x_1)(x_2-x_1)}{6}\right](x_2-x) + \left[\frac{f(x_2)}{(x_2-x_1)} - \frac{f''(x_2)(x_2-x_1)}{6}\right](x-x_1)$$

$$f_2(x) = \frac{f_2''(4.5)}{6(5-4.5)}(5-x)^3 + \frac{f_2''(5)}{6(5-4.5)}(x-4.5)^3 + \left[\frac{f(4.5)}{(5-4.5)} - \frac{f''(4.5)(5-4.5)}{6}\right](5-x) + \left[\frac{f(5)}{(5-4.5)} - \frac{f''(5)(5-4.5)}{6}\right](x-4.5)$$

$$f_2(x) = \frac{1.74683}{3}(5-x)^3 + \frac{0.42531}{3}(x-4.5)^3 + \left[\frac{1}{0.5} - 1.74683\ x 0.08333\right](5-x) + \left[\frac{1.1}{0.5} - 0.42531\ x 0.08333\right](x-4.5)$$

Cubic spline for second interval

$$f_2(x) = 0.582276\,(5-x)^3 + 0.14177\,(x-4.5)^3 + 1.854436\,(5-x) + 2.164558\,(x-4.5)$$

i=3

$$f_3(x) = \frac{f_3''(X_2)}{6(x_3-x_2)}(x_3-x)^3 + \frac{f_3''(X_3)}{6(x_3-x_2)}(x-x_2)^3 + \left[\frac{f(x_2)}{(x_3-x_2)} - \frac{f''(X_2)(x_3-x_2)}{6}\right](x_3-x) + \left[\frac{f(x_3)}{(x_3-x_2)} - \frac{f''(X_3)(x_3-x_2)}{6}\right](x-x_2)$$

$$f_3(x) = \frac{f_3''(5)}{6(7-5)}(7-x)^3 + \frac{f_3''(7)}{6(7-5)}(x-5)^3 + \left[\frac{f(5)}{(7-5)} - \frac{f''(5)(7-5)}{6}\right](7-x) + \left[\frac{f(7)}{(7-5)} - \frac{f''(7)(7-5)}{6}\right](x-5)$$

The second derivatives at the end knots are zero

$$f''(7) = f_3''(7) = 0$$

$$f_3(x) = \frac{0.42531}{12}(7-x)^3 + \left[\frac{1.1}{2} - 0.42531\, x 0.33333\right](7-x) + \left[\frac{2.5}{2}\right](x-5)$$

Cubic spline for third interval

$$f_3(x) = 0.035442\,(7-x)^3 + 0.408231\,(7-x) + 1.25(x-5)$$

x=5.5 falls within the third interval

$$f_3(5.5) = 0.035442\,(7-5.5)^3 + 0.408231\,(7-5.5) + 1.25(5.5-5) = 1.356963$$

Inverse Quadratic Interpolation method:

First iteration:

$$y = f(x) = e^{-x} - x = 0$$

$$x_{i-2} = 0.1 \qquad y_{i-2} = f(0.1) = e^{-0.1} - 0.1 = 0.8048$$

$$x_{i-1} = 0.5 \qquad y_{i-1} = f(0.5) = e^{-0.5} - 0.5 = 0.1065$$

$$x_i = 1.0 \qquad y_i = f(1.0) = e^{-1.0} - 1.0 = -0.6321$$

$$x_{i+1} = \frac{y_{i-1}y_i}{(y_{i-2} - y_{i-1})(y_{i-2} - y_i)} x_{i-2} + \frac{y_{i-2}y_i}{(y_{i-1} - y_{i-2})(y_{i-1} - y_i)} x_{i-1} + \frac{y_{i-2}y_{i-1}}{(y_i - y_{i-2})(y_i - y_{i-1})} x_i$$

$$x_{i+1} = \frac{0.1065\,(-0.6321)}{(0.8048 - 0.1065)(0.8048 - -0.6321)} 0.1 + \frac{0.8048\,(-0.6321)}{(0.1065 - 0.8048)(0.1065 - -0.6321)} 0.5$$

$$+ \frac{0.8048\,(0.1065)}{(-0.6321 - 0.8048)(-0.6321 - 0.1065)} 1.0$$

$$x_{i+1} = -0.0067 + 0.4931 + 0.0807 = 0.5671$$

$$y_{i+1} = f(0.5671) = e^{-0.5671} - 0.5671 = 0.000068 \approx 0$$

$$\varepsilon_t = \left| \frac{x_{true} - x_r^{new}}{x_{true}} \right| 100\% = \varepsilon_t = \left| \frac{0.56714 - 0.5671}{0.56714} \right| 100\% = 0.007\ \%$$