Numerical Differentiation, Numerical Integration [1-6]

References:

1. Chapra S.C. and Canale R.P. “Numerical Methods for Engineers”, Sixth Edition,McGraw Hill, International Edition 2010.

2. Chapra S.C. and Canale R. P. “Yazılım ve programlama Uygulamalarıyla Mühendisler için Sayısal Yöntemler” 4.Basımdan Çevirenler: Hasan Heperkan ve Uğur Kesgin 2003.

3.Chapra S.C. “Applied Numerical Methods with MATLAB for engineers and Scientists” Third Edition, McGraw Hill, International Edition 2012.

4. Mathews J.H. and Fınk K.D. “Numerical Methods using MATLAB”, Fourth Edition, Pearson P. Hall, International Edition 2004.

5. Fausett L.V. “Applied Numerical Analysis Using MATLAB, Second Edition, PearsonP. Hall, International Edition, 2008.

6. Gilat A. And Subramaniam V. “Numerical Methods, An introduction with Applications Using MATLAB”, Second Edition,John Wiley and Sons. Inc. 2011.

multiple application of Simpson’s 3/8 rule to integrate with 6 segments

$$f(x) = 1 - x - 4x^3 + 2x^5$$

$$A = \int_{-2}^{4} f(x) * dx \qquad a = -2 \quad b = 4 \quad n = 6 \quad h = \frac{b-a}{n} = \frac{4-(-2)}{6} = 1$$

x	$f(x) = 1 - x - 4x^3 + 2x^5$
-2	-29
-1	4
0	1
1	-2
2	31
3	376
4	1789

Simpson's 3/8 rule: (3 segments at a time)

$$I = \frac{3 \times h}{8} \times [f(x_0) + 3 \times f(x_1) + 3 \times f(x_2) + f(x_3)]$$

$$I = \frac{3 \times 1}{8} \times [-29 + 3 \times (4 + 1) - 2] + \frac{3 \times 1}{8} \times [-2 + 3 \times (31 + 376) + 1789] = 1122$$

$$\varepsilon_t = \left| \frac{(1122) - (1122)}{(1122)} \right| \times 100\% = 0\%$$

the first derivative of the unequally spaced data at $x = x_i = 1.6$

x	1	1.5	1.6	2.5	3.5
f(x)	0.6767	0.3734	0.3261	0.08422	0.01596

$$f'(x) = f(x_{i-1})\frac{2x - x_i - x_{i+1}}{(x_{i-1} - x_i)(x_{i-1} - x_{i+1})} + f(x_i)\frac{2x - x_{i-1} - x_{i+1}}{(x_i - x_{i-1})(x_i - x_{i+1})} + f(x_{i+1})\frac{2x - x_{i-1} - x_i}{(x_{i+1} - x_{i-1})(x_{i+1} - x_i)}$$

$$f'(1.6) = 0.3734\frac{2 \times 1.6 - 1.6 - 2.5}{(1.5 - 1.6)(1.5 - 2.5)} + 0.3261\frac{2 \times 1.6 - 1.5 - 2.5}{(1.6 - 1.5)(1.6 - 2.5)} + 0.08422\frac{2 \times 1.6 - 1.5 - 1.6}{(2.5 - 1.5)(2.5 - 1.6)}$$

$$f'(1.6) = -0.4526$$

$$\varepsilon_t = \left|\frac{(-0.4484) - (-0.4526)}{(-0.4484)}\right| \times 100\% = 0.9367\%$$

integral with multiple application Simpson's 3/8 rule

$$d = \int_0^t v(t)dt$$

t	0	2	4	6	8	10	12
v(t)	0	16.4988	28.0425	36.1194	41.7705	45.7245	48.4910

$$I \simeq (b-a)\frac{f(x_0)+3f(x_1)+3f(x_2)+f(x_3)}{8}$$

$$I \simeq (6-0)\frac{0+3*16.4988+3*28.0425+36.1194}{8}$$

$$+(6-0)\frac{36.1194+3*41.7705+3*45.7245+48.4910}{8}$$

$$I \simeq 387.6290\,m$$

$$f'(x=0)=$$

?

$$f'(x=1.25)=$$

x	0	1.25	3.75
F(x)	13.25	12.50	10.75

$$f'(x)=f(x_{i-1})\frac{2x-x_i-x_{i+1}}{(x_{i-1}-x_i)(x_{i-1}-x_{i+1})}+f(x_i)\frac{2x-x_{i-1}-x_i}{(x_i-x_{i-1})(x_i-x_{i+1})}+f(x_{i+1})\frac{2x-x_{i-1}-x_i}{(x_{i+1}-x_{i-1})(x_{i+1}-x_i)}$$

$$f'(x=0)=13.25*\frac{2(0)-1.25-3.75}{(0-1.25)(0-3.75)}+12.50*\frac{2(0)-0-3.75}{(1.25-0)(1.25-3.75)}+10.75*\frac{2(0)-0-1.25}{(3.75-0)(3.75-1.25)}$$

$$f'(x=0) = -14.133 + 15 - 1.433 = -0.566$$

$$f'(x=1.25) = 13.25 * \frac{2(1.25) - 1.25 - 3.75}{(0-1.25)(0-3.75)} + 12.50 * \frac{2(1.25) - 0 - 3.75}{(1.25-0)(1.25-3.75)} + 10.75 * \frac{2(1.25) - 0 - 1.25}{(3.75-0)(3.75-1.25)}$$

$$f'(x=1.25) = -7.066 + 5 + 1.433 = -0.633$$