Ordinary Differential Equations, Euler's Method [1-6]

References:

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3. Chapra S.C. "Applied Numerical Methods with MATLAB for engineers and Scientists" Third Edition, McGraw Hill, International Edition 2012.
4. Mathews J.H. and Fink K.D. "Numerical Methods using MATLAB", Fourth Edition, Pearson P. Hall, International Edition 2004.
5. Fausett L.V. "Applied Numerical Analysis Using MATLAB, Second Edition, PearsonP. Hall, International Edition, 2008.
6. Gilat A. And Subramaniam V. "Numerical Methods, An introduction with Applications Using MATLAB", Second Edition,John Wiley and Sons. Inc. 2011.

Euler's Method to solve the following ordinary differential equation

$$\frac{dy}{dx} = -1.2y + 7e^{-0.3x}$$

h=0.5 from x=0 to x=1 with the initial condition y=3 at x=0

Euler's method formula

$$y_{i+1} = y_i + f(x_i, y_i)h$$

$$x_{i+1} = x_i + h$$

The first point of the solution is (0,3), which is the point where the initial condition is given. For the first point i=1. The values of x and y are $x_1=0$ and $y_1=3$.

$$x_{i+1} = x_i + h = x_i + 0.5$$

$$y_{i+1} = y_i + f(x_i, y_i)h = y_i + (-1.2y_i + 7e^{-0.3x_i})0.5$$

First step : (i=1)

$$x_2 = x_1 + 0.5 = 0 + 0.5 = 0.5$$

$$y_2 = y_1 + (-1.2y_1 + 7e^{-0.3x_1})0.5 = 3 + (-1.2 * 3 + 7e^{-0.3*0})0.5 = 4.7$$

```
>> X1=0;Y1=3;h=0.5
h =
    0.5000
>> Y2=Y1+(-1.2*Y1+7*(exp(-0.3*X1)))*h
Y2 =
    4.7000
```

Second step : (i=2)

$$x_3 = x_2 + 0.5 = 0.5 + 0.5 = 1.0$$

$$y_3 = y_2 + (-1.2y_2 + 7e^{-0.3x_2})0.5 = 4.7 + (-1.2 * 4.7 + 7e^{-0.3*0.5})0.5 = 4.893$$

```
>> X1=0.5;Y1=4.7;h=0.5
h =
    0.5000
>> Y2=Y1+(-1.2*Y1+7*(exp(-0.3*X1)))*h
Y2 =
    4.8925
```

i	1	2	3
x_i	0.0	0.5	1.0
y_i	3.0	4.70	4.893

Step=h=0.05

```
>> X1=0;Y1=3;h=0.05
h =
    0.0500
>> Y2=Y1+(-1.2*Y1+7*(exp(-0.3*X1)))*h
Y2 =
    3.1700
```

```
>> X1=0.05;Y1=3.17;h=0.05
h =
    0.0500
>> Y2=Y1+(-1.2*Y1+7*(exp(-0.3*X1)))*h
Y2 =
    3.3246
```

```
>> X1=0.1;Y1=3.3246;h=0.05
h =
    0.0500
>> Y2=Y1+(-1.2*Y1+7*(exp(-0.3*X1)))*h
Y2 =
    3.4648
```

```
>> X1=0.15;Y1=3.4648;h=0.05
h =
    0.0500
>> Y2=Y1+(-1.2*Y1+7*(exp(-0.3*X1)))*h
Y2 =
    3.5915
```

```
>> X1=0.2;Y1=3.5915;h=0.05
h =
    0.0500
>> Y2=Y1+(-1.2*Y1+7*(exp(-0.3*X1)))*h
Y2 =
    3.7056
```

```
>> X1=0.25;Y1=3.7056;h=0.05
h =
    0.0500
>> Y2=Y1+(-1.2*Y1+7*(exp(-0.3*X1)))*h
Y2 =
    3.8080
```

```
>> X1=0.3;Y1=3.8080;h=0.05
h =
    0.0500
>> Y2=Y1+(-1.2*Y1+7*(exp(-0.3*X1)))*h
Y2 =
    3.8994
```

```
>> X1=0.35;Y1=3.8994;h=0.05
h =
    0.0500
>> Y2=Y1+(-1.2*Y1+7*(exp(-0.3*X1)))*h
Y2 =
    3.9805
```

```
>> X1=0.4;Y1=3.9805;h=0.05
h =
    0.0500
>> Y2=Y1+(-1.2*Y1+7*(exp(-0.3*X1)))*h
Y2 =
    4.0521
```

```
>> X1=0.45;Y1=4.0521;h=0.05
h =
    0.0500
>> Y2=Y1+(-1.2*Y1+7*(exp(-0.3*X1)))*h
Y2 =
    4.1148

>> X1=0.5;Y1=4.1148;h=0.05
h =
    0.0500
>> Y2=Y1+(-1.2*Y1+7*(exp(-0.3*X1)))*h
Y2 =
    4.1692
```

dy.m

```
function dydx = dy( x,y)
dydx=-1.2*y+7*exp(-0.3*x);
end
```

Euler111.m

```
%xinitiai independent value
x=0;
%yinitiai idependent value
y=3;
%h stepsize of indepentent value
h=0.5;
%xf final value of independent variable
xf=1;
while (1)
    if x+h>xf
        h=xf-x;
    end
    dydx=dy(x,y);
    y=y+dydx*h;
    x=x+h;
    if x>= xf, break, end
end
yy=y;
save euleryyresponse.mat yy
```

Command Window

```
>> Euler111
>>

yy=

4,89247791748770
```