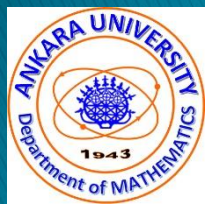


MTH 102 Calculus -II

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Topic-10-Alternating Series
Power Series



An alternating series is a series whose terms are alternately positive and negative. Here are two examples:

$$\sum_{n=1}^{\infty} (-1)^n = -1 + 1 - 1 + \dots$$
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n} = -\frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \dots$$

Theorem (Leibniz's Test)

If the alternating series

$$\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + \dots$$

satisfies

$$(i) 0 < b_{n+1} \leq b_n \text{ for all } n \geq 1,$$

$$(ii) \lim_{n \rightarrow \infty} b_n = 0,$$

then the series is convergent.

Absolute and Conditional Convergence

Definition: A series $\sum a_n$ is **absolutely convergent** if the corresponding series of absolute values, $\sum |a_n|$, is convergent.

A series $\sum a_n$ is called **conditionally convergent** if it is convergent but not absolutely convergent.

Theorem If $\sum |a_n|$ converges, then $\sum a_n$ converges.

Power Series

Definition: A series of the form

$$\sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \dots$$

is called a power series in $(x-a)$ or power series centered at a or a power series about a , where x is a variable and the c_n 's are constants called the coefficients of the series.

In this section our question is for what values of x is the power series convergent?

Definition: Let the power series $\sum c_n(x - a)^n$ be convergent for $|x - a| < R$. The number R is called the radius of convergence of the power series. If the series converges only when $x = a$, then $R = 0$. If the series converges for all x , then $R = \infty$. The interval of convergence of a power series is the interval that consists of all values of x for which the series converges.

	Series	Radius of convergence	Interval of convergence
Geometric series	$\sum_{n=0}^{\infty} x^n$	$R = 1$	$(-1, 1)$
Example 1	$\sum_{n=0}^{\infty} n! x^n$	$R = 0$	$\{0\}$
Example 2	$\sum_{n=1}^{\infty} \frac{(x - 3)^n}{n}$	$R = 1$	$[2, 4)$
Example 3	$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$	$R = \infty$	$(-\infty, \infty)$