Models with Ordinary Differential Equations (Material Balance)

Total Material Balance Equations (Steady-State Balances)

One of the basic principles of modelling is that of the conservation of mass or matter. For a steady-state flow process, this can be expressed by the statement:

Mass flow rate into the system = Mass flow rate out of the system

Total Material Balance Equations ,Unsteady-State (Dynamic) Balances

Most real situations are, however, such that conditions change with respect to time. Under these circumstances, a steady-state material balance is inappropriate and must be replaced by a dynamic or unsteady-state material balance, expressed as:

Rate of accumulation of mass in the system = Mass flow rate into the system − Mass flow rate out of the system

Here the rate of accumulation term represents the rate of change in the total mass of the system, with respect to time, and at steady state, this is equal to zero.

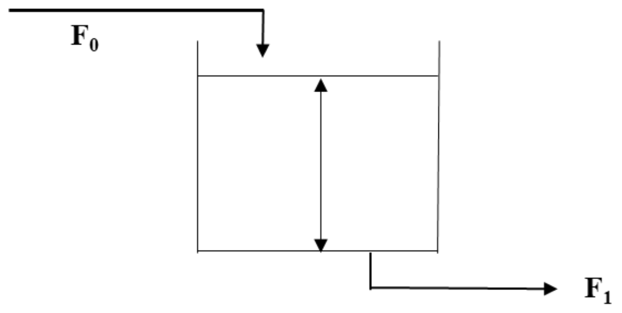
EXAMPLE (Total Material Balance Equations,(Unsteady-State (Dynamic) Balances)

A cylindrical tank with a height of 2 m and a diameter of 1.5 m is initially filled with water until halfway. Water is sent at a speed of 10 m/s with a pipe with diameter of 0.03 m.

The flow rate at the outlet of the tank is directly proportional to the liquid level in the tank and given by $F_1 = 0.01h$. Under these conditions;

a) Does the liquid level in the tank decrease or increase?

b) What is the time required for 90% of the tank to be filled?



Solution

a) Flow rate of feed

$$F_o = v_0 A_o = \frac{\Pi D_o^2}{4} v_o = \frac{\Pi (0.05m)^2}{4}(5m/s) = 9.813x10^{-3} m^3/s$$

Height of the initial water level: h_o=h/2= 1.5/2= 0.75 m

At the beginning, the flow rate of F_1:

$$F_1 = Rh = (0.008m^2/s)(0.75m) = 0.006m^3/s$$

Tank is going to be filled with water since $F_1 < F_o$

b) Total Mass balance

$$F_o\rho_o - F_1\rho_1 = \frac{d(V\rho)}{dt}$$

Density is constant and F_1 =Rh , V=A_th=0.785 h

$$F_o - Rh = A_t \frac{dh}{dt}$$

$$A_t \int_{h_o}^{h} \frac{dh}{F_o - Rh} = \int_0^t dt$$

$$\ln \frac{h - F_o / R}{h_o - F_o / R} = -(R / A_t)t$$

In order to fill 90 % of tank (h=1.5*0.90= 1.35m)

$$\ln \frac{(1.35 - 0.009817)/0.008}{(0.75 - 0.009817)/0.008} = -(0.008)/(0.7854)t$$

There is no solution for h=1.35 m since the maximum h is obtained as

Fo=Rh

*0.009813=0.008*h*

h=1.23 m