Models with Ordinary Differential Equations (Energy Balance)

The conservation statement for total energy under steady conditions takes the form

(Rate of energy in) − (Rate of energy out) + (Interphase heat transfer rate, Q,) + (Rate of work on the system) = 0

The conservation statement for total energy under un teady steady conditions takes the form

(Rate of energy in) − (Rate of energy out) + (Interphase heat transfer rate, Q,) + (Rate of work on the system) = (Rate of energy accumulation)

EXAMPLE

Suppose that in the winter day time temperature in a certain office building is maintained at 70 °F and the outside temperature was 45 °F. The temperature of the building after 2h was found to be 65 °F. What was the temperature inside the building after 8h?

The conservation statement for total energy under unteady steady conditions takes the form

$$\dot{m}_{in}H_{in} - \dot{m}_{out}H_{out} + \dot{Q} + \dot{W}_s = m * C_p \frac{dT}{dt}$$

$$m * C_p \frac{dT}{dt} = h * A * (T - T_{outside})$$

$$\frac{dT}{dt} = k * (T - T_o)$$

$$\frac{dT}{T - T_o} = kdt$$

$$ln(T - T_o) - lnC = kt$$

$$ln\frac{(T - T_o)}{C} = kt$$

$$e^{ln\frac{(T - T_o)}{C}} = e^{kt}$$

$$\frac{(T - T_o)}{C} = e^{kt}$$

$$T = T_o + Ce^{kt}$$

Initial condition: t = 0, T = 70 °F

$$70\ °F = 45\ °F + Ce^{0} = 45\ °F + C$$

$$\Rightarrow C = 25\ °F$$

$$T = 45 + 25e^{kt}$$

Boundary condition: t = 120 min, T = 65 °F

$$65\ °F = 45\ °F + 25e^{120k}$$

$$e^{120k} = \frac{20\ °F}{25\ °F} \Rightarrow lne^{120k} = ln\left(\frac{20}{25}\right) = -0.223$$

$$120k = -0.223$$

$$k = -0.00186$$

$$T = 45 + 25e^{-0.00186t}$$

If t = 8 h = 480 min,

$$T = 45 + 25e^{-0.00186.480}$$

T = 55.24 °F