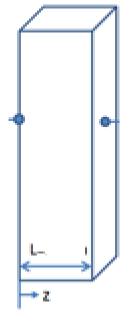
Models with partial differential equations (Energy balances)

Unsteady-state Heat Conduction in One Dimension.

Figure illustrates a section of a flat wall of thickness L whose height and length are both large compared with L. If the temperature distribution is uniform throughout the wall at zero time and the heat is supplied at a fixed rate per unit area to the one surface, it is required to determine the temperature as a function of position and time.

Since the original temperature is uniform, and every part of each wall surface is subjected to the same conditions, no heat will travel parallel to the surface and the temperature will be constant in any plane parallel to the surface. Thus the temperature distribution can be specified in terms of a single coordinate denoted by x in figure and the discussion can be restricted to a section of unit area through the wall. Considering the thermal equilibrium of a slice of the wall between a plane at distance x from the heated surface and a parallel plane at x +dx from the same surface gives the following balance, where T is the temperature and k is the thermal conductivity.



Energy balances:

Rate of heat input - Rate of heat output= Accumulation of heat

$$qA_{x|x} - qA_{x|x+\Delta x} = \frac{mCpT}{\partial t}$$

$$A_{x|x} = \Delta y \Delta z$$

$$V = \Delta y \Delta z \Delta x$$

Taking the limit of equation as x-> 0

$$\frac{qA_{x|x} - qA_{x|x+\Delta x}}{\Delta x \Delta y \Delta z} = \frac{\rho \Delta x \Delta y \Delta z Cp \delta T}{\Delta x \Delta y \Delta z \partial t}$$

$$\lim_{\Delta x \to 0} \frac{q_{x|x} - q_{x|x+\Delta x}}{\Delta x} = \frac{\rho Cp \delta T}{\partial t}$$

$$-\frac{\delta q_{x|}}{\delta x} = \frac{\rho Cp \delta T}{\partial t}$$

$$\frac{k \delta^2 T}{\delta x^2} = \frac{\rho Cp \delta T}{\partial t}$$

$$\frac{\delta^2 T}{\delta x^2} = \frac{1}{\alpha} \frac{\delta T}{\partial t}$$