MANAGERIAL ECONOMICS CHAPTER 4

Estimation of Demand

Ankara University, Faculty of Political Science, Department of Economics, Onur Özsoy

Estimation of Demand

Chapter 4

- ***Objective***: Learn how to estimate a demand function using regression analysis, and interpret the results
- A chief uncertainty for managers -- what will happen to their product.
  - forecasting, prediction & estimation
  - need for data: Frank Knight: *"If you think you can't measure something, measure it anyway."*

Sources of information on demand

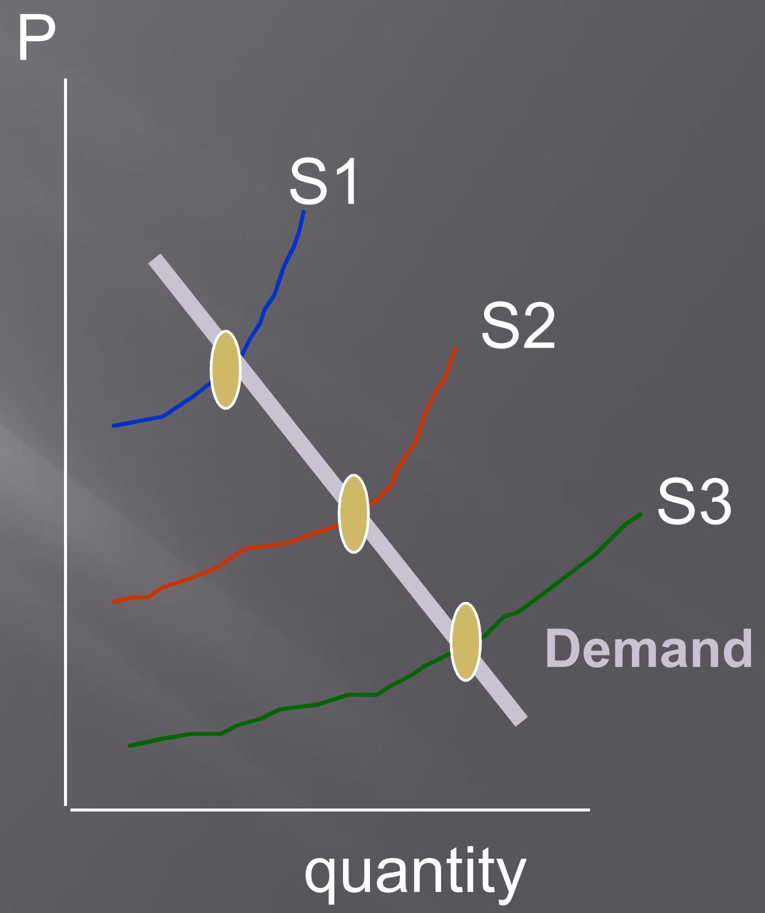
- **Consumer Surveys**
  - ask a sample of consumers their attitudes
- **Consumer Clinics**
  - experimental groups try to emulate a market (Hawthorne effect)
- **Market Experiments**
  - get demand information by trying different prices
- **Historical Data**
  - what happened in the past is guide to the future

Plot Historical Data

- Look at the relationship of price and quantity over time
- Plot it
  - Is it a demand curve or a supply curve?
  - Problem -- not held other things equal

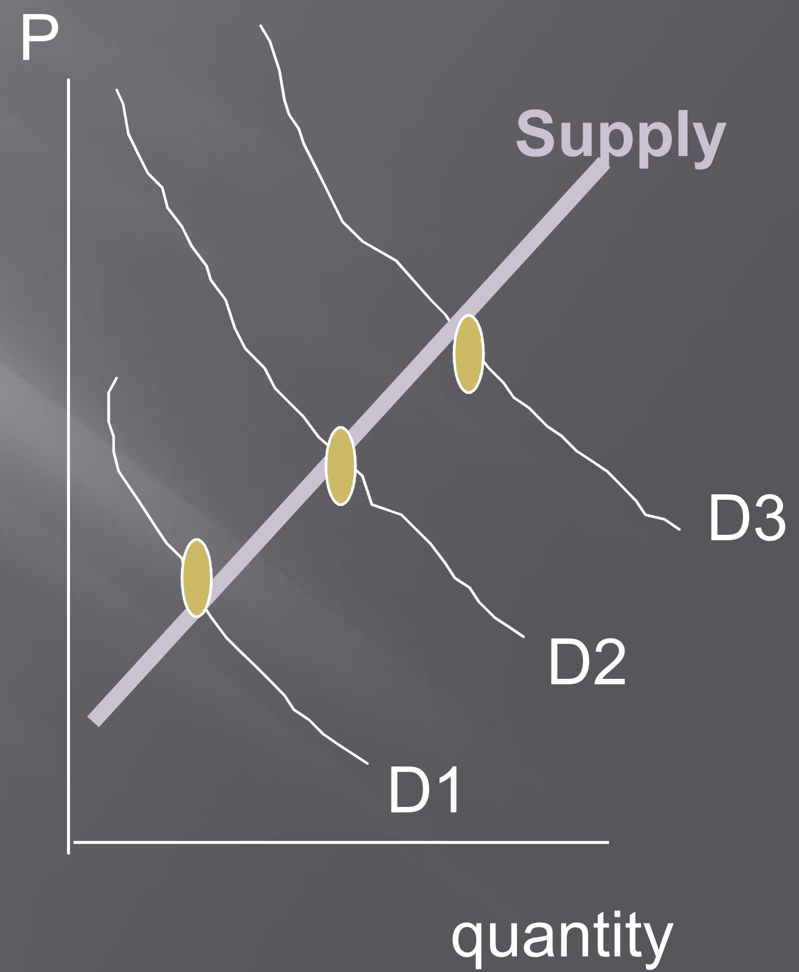
Identification Problem

- $Q = a + b\,P$ can appear upward or downward sloping.
- Suppose Supply varies and Demand is **FIXED**.
- All points lie on the Demand curve



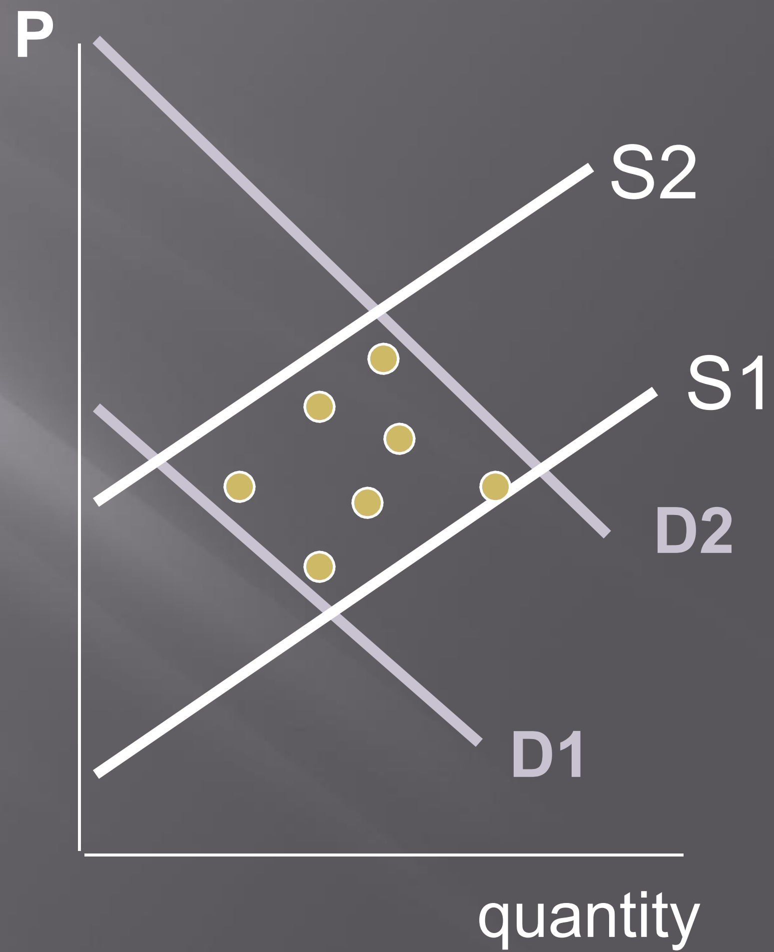
Suppose SUPPLY is Fixed

- Let DEMAND shift and supply **FIXED**.
- All Points are on the SUPPLY curve.
- We say that the SUPPLY curve is identified.



When both Supply and Demand Vary

- Often both supply and demand vary.
- Equilibrium points are in shaded region.
- A regression of **Q = a + b P** will be neither a demand nor a supply curve.



Statistical Estimation of the a Demand Function

- **Steps to take:**
  - Specify the variables -- formulate the demand model, select a **Functional Form**
    - linear $Q = a + b \cdot P + c \cdot I$
    - double log $\ln Q = a + b \cdot \ln P + c \cdot \ln I$
    - quadratic $Q = a + b \cdot P + c \cdot I + d \cdot P^2$
  - Estimate the parameters --
    - determine which are statistically significant
    - try other variables & other functional forms
  - Develop forecasts from the model

Specifying the Variables

- **Dependent Variable** -- quantity in units, quantity in dollar value (as in sales revenues)
- **Independent Variables** -- variables thought to influence the quantity demanded
  - Instrumental Variables -- proxy variables for the item wanted which tends to have a relatively high correlation with the desired variable: ***e.g.,*** **Tastes** **Time Trend**

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Functional Forms

- **Linear** $Q = a + b \cdot P + c \cdot I$
  - The effect of each variable is constant
  - The effect of each variable is independent of other variables
  - Price elasticity is: $E_P = b \cdot P/Q$
  - Income elasticity is: $E_I = c \cdot I/Q$

Functional Forms

- **Multiplicative** $Q = A \bullet P^{b} \bullet I^{c}$
  - The effect of each variable depends on all the other variables and is not constant
  - It is log linear

    $$\text{Ln } Q = a + b \bullet \text{Ln } P + c \bullet \text{Ln } I$$

  - the price elasticity is **b**
  - the income elasticity is **c**

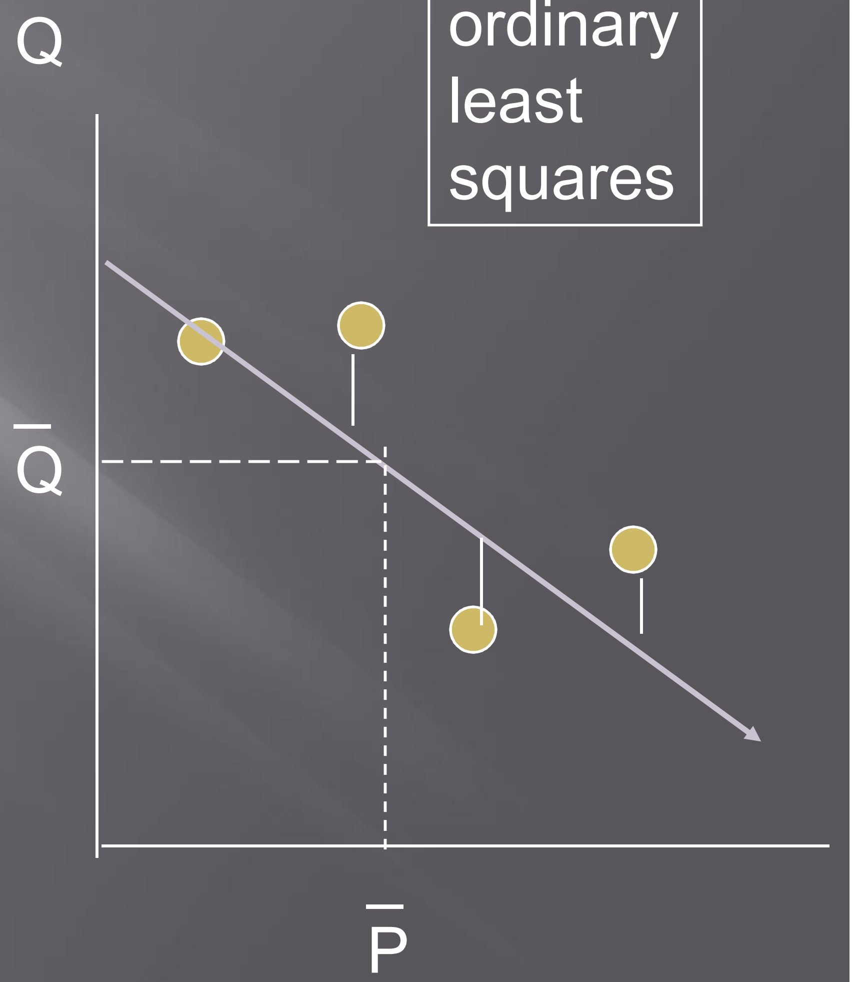
Simple Linear Regression

- $Q_t = a + b\,P_t + \varepsilon_t$
- time subscripts & error term
- Find "best fitting" line

  $\varepsilon_t = Q_t - a - b\,P_t$

  $\varepsilon_t^{\,2} = [Q_t - a - b\,P_t]^2.$

- $\min \Sigma\, \varepsilon_t^{\,2} = \Sigma\, [Q_t - a - b\,P_t]^2.$
- Solution: b = Cov(Q,P)/Var(P) and a = mean(Q) - b • mean(P)



OLS -- ordinary least squares

Ordinary Least Squares:

Assumptions & Solution Methods

- Error term has a mean of zero and a finite variance
- Dependent variable is random
- The independent variables are indeed independent

- Spreadsheets - such as
  - Excel, Lotus 1-2-3, Quatro Pro, or Joe Spreadsheet
- Statistical calculators
- Statistical programs such as
  - Minitab
  - SAS
  - SPSS
  - ForeProfit
  - Mystat

Demand Estimation Case (p. 173)

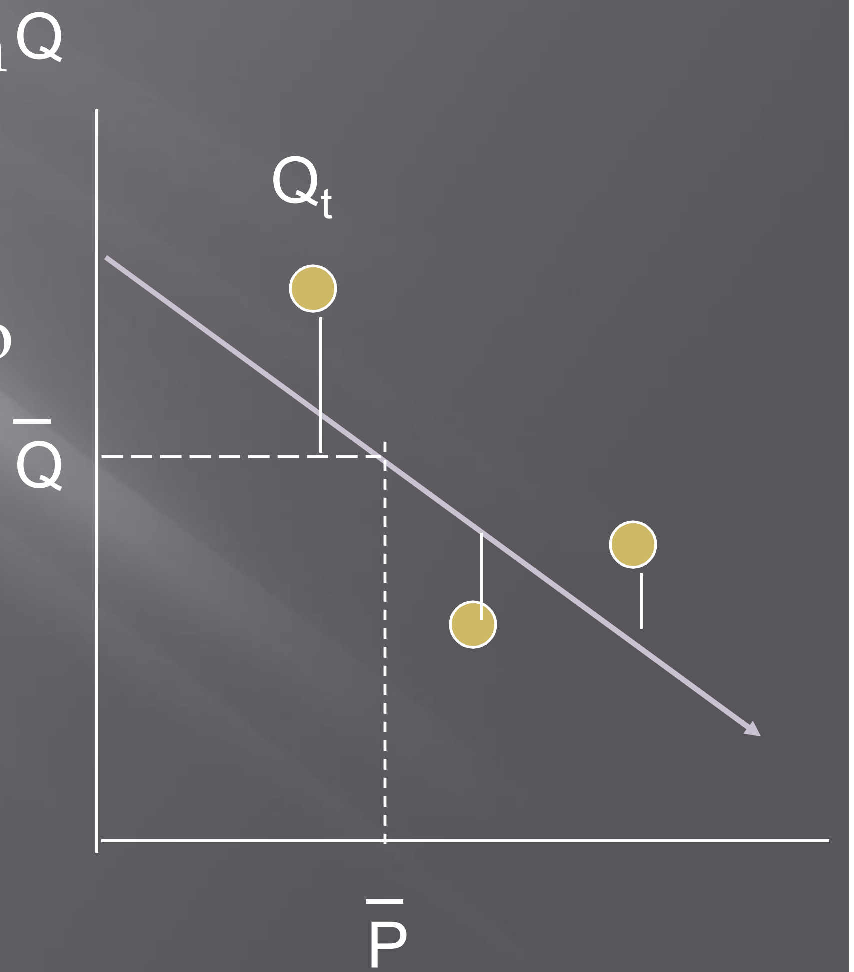
Riders = 785 -2.14 • Price +.110 • Pop +.0015 • Income + .995 • Parking

Predictor	Coef	Stdev	t-ratio	p
Constant	784.7	396.3	1.98	.083
Price	-2.14	.4890	-4.38	.002
Pop	.1096	.2114	.520	.618
Income	.0015	.03534	.040	.966
Parking	.9947	.5715	1.74	.120

R-sq = 90.8% R-sq(adj) = 86.2%

Coefficients of Determination: R^2

- R-square -- % of variation in dependent variable that is explained
- Ratio of $\Sigma [\hat{Q}_t - \bar{Q}_t]^2$ to $\Sigma [Q_t - \bar{Q}_t]^2$.
- As more variables are included, R-square rises
- Adjusted R-square, however, can decline



T-tests

- Different samples would yield different coefficients
- Test the hypothesis that coefficient equals zero
  - H_o: b = 0
  - H_a: $b \neq 0$

- RULE: If absolute value of the estimated t > Critical-t, then REJECT Ho.
  - It's significant.
- estimated $t = (b - 0) / \sigma_b$
- critical t
  - Large Samples, critical $t \cong 2$
    - $N \geq 30$
  - Small Samples, critical t is on Student's t-table
    - D.F. = # observations, minus number of independent variables, minus one.
    - $N < 30$

Double Log or Log Linear

- With the double log form, the coefficients are elasticities
- $Q = A \cdot P^{b} \cdot I^{c} \cdot P_s^{d}$
  - multiplicative functional form
- So: $\text{Ln } Q = a + b \cdot \text{Ln } P + c \cdot \text{Ln } I + d \cdot \text{Ln } P_s$
- Transform all variables into natural logs
- Called the double log, since logs are on the left and the right hand sides. Ln and Log are used interchangeably. We use only natural logs.

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Econometric Problems

- **Simultaneity Problem** -- Indentification Problem:
  - some independent variables may be endogenous
- **Multicollinearity**
  - independent variables may be highly related
- **Serial Correlation** -- Autocorrelation
  - error terms may have a pattern
- **Heteroscedasticity**
  - error terms may have non-constant variance

Identification Problem

- Problem:
  - Coefficients are biased
- Symptom:
  - Independent variables are known to be part of a system of equations
- Solution:
  - Use as many independent variables as possible

Multicollinearity

- Sometimes *independent* variables aren't **independent.**

- EXAMPLE: Q =Eggs

Q = a + b Pd + c Pg

where Pd is for a dozen

and Pg is for a gross.

PROBLEM

- Coefficients are UNBIASED, but t-values are small.

- **Symptoms of Multicollinearity --** high R-sqr, but low t-values.

$$Q = 22 - 7.8\, P_d - .9\, P_g$$

(1.2) (1.45)

R-square = .87

t-values in parentheses

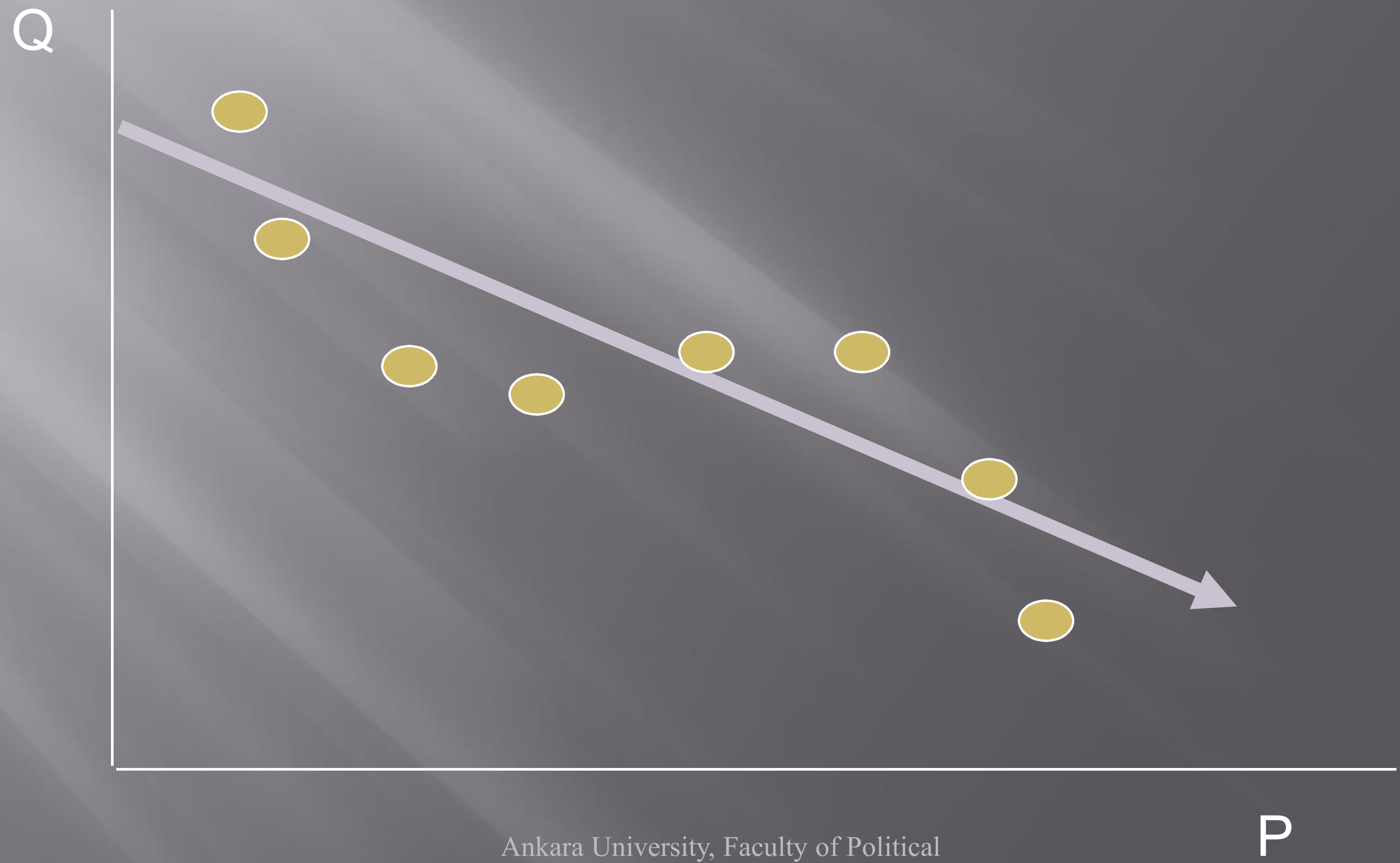
- Solutions:
  - Drop a variable.
  - Do nothing if forecasting

Serial Correlation

- **Problem:**
  - Coefficients are unbiased
  - but t-values are unreliable
- **Symptoms:**
  - look at a scatter of the error terms to see if there is a pattern, or
  - see if ***Durbin Watson*** statistic is far from 2.
- **Solution:**
  - Find more data
  - Take first differences of data: $\Delta Q = a + b \bullet \Delta P$

Scatter of Error Terms

Serial Correlation

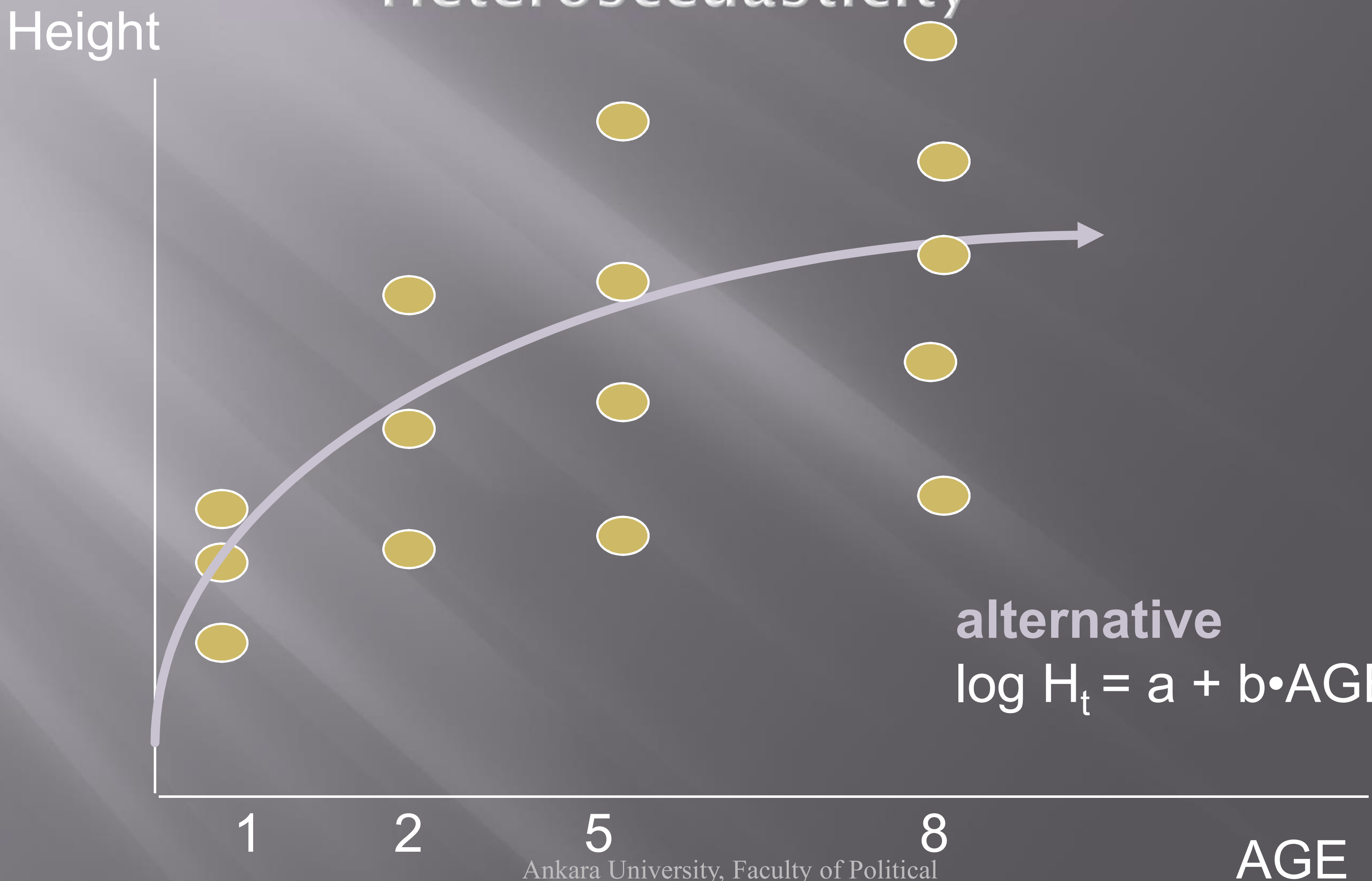


Heteroscedasticity

- **Problem:**
  - Coefficients are unbiased
  - t-values are unreliable
- **Symptoms:**
  - different variances for different sub-samples
  - scatter of error terms shows increasing or decreasing dispersion
- **Solution:**
  - Transform data, *e.g.*, logs
  - Take averages of each subsample: weighted least squares

Scatter of Error Terms

Heteroscedasticity



alternative

$\log H_t = a + b \cdot AGE$

Nonlinear Forms
Appendix 4A

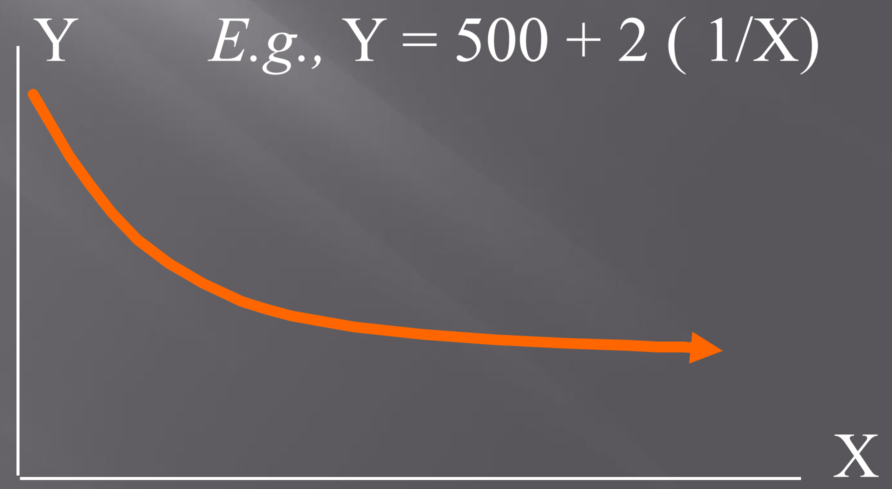
- ***Semi-logarithmic transformations***. Sometimes taking the logarithm of the dependent variable or an independent variable improves the R^2. Examples are:
- **log Y = α + ß·X**.

  - Here, Y grows exponentially at rate ß in X; that is, ß percent growth per period.
- **Y = α + ß·log X**. Here, Y doubles each time X increases by the square of X.

Reciprocal Transformations

- The relationship between variables may be inverse. Sometimes taking the reciprocal of a variable improves the fit of the regression as in the example:
- $Y = \alpha + \beta \cdot (1/X)$
- shapes can be:
  - declining slowly
    - if beta positive
  - rising slowly
    - if beta negative

E.g., $Y = 500 + 2\,(1/X)$



Polynomial Transformations

- Quadratic, cubic, and higher degree polynomial relationships are common in business and economics.
  - Profit and revenue are cubic functions of output.
  - Average cost is a quadratic function, as it is U-shaped
  - Total cost is a cubic function, as it is S-shaped
- $TC = \alpha \cdot Q + \beta \cdot Q^2 + \gamma \cdot Q^3$ is a cubic total cost function.
- If higher order polynomials improve the R-square, then the added complexity may be worth it.