Statistics 1
Chapter 6

Probability Theory 2

Joint Probability

- Joint Probability is a probability that measures the likelihood that two or more events will happen concurrently. An example would be the event that a student has both a stereo and TV in his or her dorm room.

Special Rule of Multiplication

- The special rule of multiplication requires that two events A and B are *independent*.
- Two events A and B are *independent* if the occurrence of one has no effect on the probability of the occurrence of the other.
- The special rule is written:
  P(A and B) = P(A)*P(B).

EXAMPLE 6

- Chris owns two stocks which are independent of each other. The probability that stock A increases in value next year is .5. The probability that stock B will increase in value next year is .7.
- What is the probability that both stocks increase in value next year?
- P(A and B) = (.5)(.7) = .35.

EXAMPLE 6 *continued*

- What is the probability that at least one of these stocks increase in value during the next year (this implies that either one can increase or both)?
- Thus, P(at least one) = (.5)(.3) + (.5)(.7) + (.7)(.5) = .85.

Conditional Probability

- Conditional probability is the probability of a particular event occurring, given that another event has occurred.
- *Note:* The probability of the event A given that the event B has occurred is denoted by $P(A|B)$.

General Multiplication Rule

- The general rule of multiplication is used to find the joint probability that two events will occur, as it states: *for two events A and B, the joint probability that both events will happen is found by multiplying the probability that event A will happen by the conditional probability of B given that A has occurred.*

General Multiplication Rule

- The joint probability, P(A and B) is given by the following formula:
  $P(A \text{ and } B) = P(A)*P(B|A)$
  OR
  $P(A \text{ and } B) = P(B)*P(A|B)$

EXAMPLE 7

- The Dean of the School of Business at Miami collected the following information about undergraduate students in her college:

MAJOR	Male	Female	Total
Accounting	170	110	280
Finance	120	100	220
Marketing	160	70	230
Management	150	120	270
Total	600	400	1000

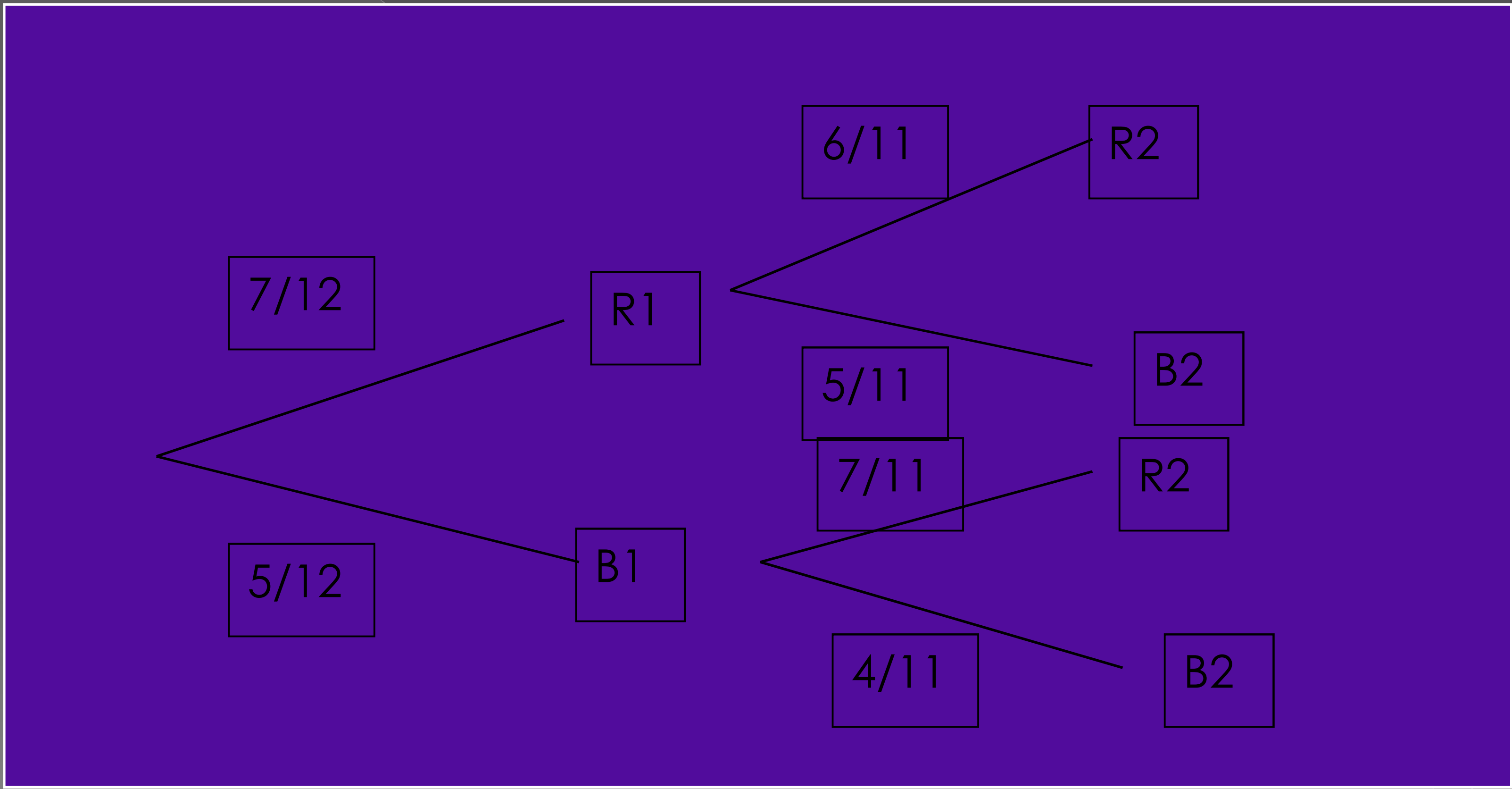
EXAMPLE 7 *continued*

- If a student is selected at random, what is the probability that the student is a female accounting major? *P(A and F) = 110/1000.*
- Given that the student is a female, what is the probability that she is an accounting major? *P(A|F) = [P(A and F)]/[P(F)] = [110/1000]/[400/1000] = .275.*

Tree Diagrams

- A tree diagram is very useful for portraying conditional and joint probabilities and is particularly useful for analyzing business decisions involving several stages.
- EXAMPLE 8: In a bag containing 7 red chips and 5 blue chips you select 2 chips one after the other without replacement. Construct a tree diagram for this information.

EXAMPLE 8 *continued*



Bayes' Theorem

- Bayes' Theorem is given by the formula:

$$P(A_1|B) = \frac{P(A_1) * P(B|A_1)}{P(A_1) * P(B|A_1) + P(A_2) * P(B|A_2)}$$

EXAMPLE 9

- Duff Beer Company has received several complaints that their bottles are under-filled. A complaint was received today but the production manager is unable to identify which of the two Springfield plants (A or B) filled this bottle. What is the probability that the under filled bottle came from plant A?

EXAMPLE 9 *continued*

	% of Total Production	% of under-filled bottles
A	55	3
B	45	4

- P(A | U) = [(.55)(.03)]/[(.55)(.03)+(.45)(.04)] =.4783.

Some Principles of Counting

- The Multiplication Formula: If there are *m* ways of doing one thing and *n* ways of doing another thing, there are *m* x *n* ways of doing both.
- Example 10: Dr. Delong has 10 shirts and 8 ties. How many shirt/tie outfits does he have? *(10)(8) = 80.*

Some Principles of Counting

- Permutation: Any arrangement of *r* objects selected from *n* possible objects.

$$ {}_{n}\mathrm{P}_{\mathrm{r}} = \frac{n!}{(n-r)!} $$

- *Note:* The order of arrangement is important in permutations.

Some Principles of Counting

- Combination: The number of ways to choose *r* objects from a group of *n* objects without regard to order.

$$ {}_{n}C_{r} = \frac{n!}{r!(n-r)!} $$

EXAMPLE 11

- Coach Thompson must pick five players among the twelve on the team to comprise the starting lineup. How many different groups are possible?
  $_{12}C_5 = (12!)/[5!(12-5)!] = 792$
- Suppose Coach Thompson must rank them: $_{12}P_5 = (12!)/(12-5)! = 95{,}040$.