Statistics 1
Chapter 5

Probability Theory 1

Chapter Five

A Survey of Probability Concepts

GOALS

When you have completed this chapter, you will be able to:

ONE

Define probability.

TWO

Describe the classical, the empirical, and the subjective approaches to probability.

THREE

Understand the terms: experiment, event, outcome, permutations, and combinations.

FOUR

Define the terms: conditional probability and joint probability.

Chapter Five *continued*

A Survey of Probability Concepts

GOALS

When you have completed this chapter, you will be able to:

FIVE

Calculate probabilities applying the rules of addition and the rules of multiplication.

SIX

Use a tree diagram to organize and compute probabilities.

SEVEN

Calculate a probability using Bayes' theorem.

EIGHT

Determine the number of permutations and the number of combinations.

Definitions

- Probability: A measure of the likelihood that an event in the future will happen; it can only assume a value between 0 and 1, inclusive.
- Experiment: The observation of some activity or the act of taking some measurement.
- Outcome: A particular result of an experiment.
- Event: A collection of one or more outcomes of an experiment.

Approaches to Probability

- Classical probability is based on the assumption that the outcomes of an experiment are equally likely.
- Using this classical viewpoint,

$$\text{Probability of an event} = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

EXAMPLE 1

- Consider the experiment of tossing two coins once.
- The sample space S = {HH, HT, TH, TT}
- Consider the event of *one* head.
- Probability of one head = 2/4 = 1/2.

Mutually Exclusive Events

- Mutually Exclusive Events: The occurrence of any one event means that none of the others can occur at the same time.
- In EXAMPLE 1, the four possible outcomes are mutually exclusive.

Collectively Exhaustive Events

- Collectively exhaustive: At least one of the events must occur when an experiment is conducted.
- In EXAMPLE 1, the four possible outcomes are collectively exhaustive. In other words, the sum of probabilities = 1 (.25 + .25 + .25 + .25).

Relative Frequency Concept

- The probability of an event happening in the long run is determined by observing what fraction of the time like events happened in the past:

$$\text{Probability of event} = \frac{\text{Number of times event occured in the past}}{\text{Total number of observations}}$$

EXAMPLE 2

- Throughout her career Professor Jones has awarded 186 A's out of the 1200 students she has taught. What is the probability that a student in her section this semester will receive an A?
- By applying the relative frequency concept, the probability of an A= 186/1200=.155

Subjective Probability

- Subjective probability: The likelihood (probability) of a particular event happening that is assigned by an individual based on whatever information is available.
- Examples of subjective probability are estimating the probability the Washington Redskins will win the Super Bowl next year and estimating the probability of an earthquake in Los Angeles this year.

Basic Rules of Probability

- If events are mutually exclusive, then the occurrence of any one of the events precludes any of the other events from occurring.
- Rules of addition: If two events A and B are mutually exclusive, the special rule of addition states that the probability of A or B occurring equals the sum of their respective probabilities:
  P(A or B) = P(A) + P(B)

EXAMPLE 3

- New England Commuter Airways recently supplied the following information on their commuter flights from Boston to New York:

Arrival	Frequency
Early	100
On Time	800
Late	75
Canceled	25
Total	1000

EXAMPLE 3 *continued*

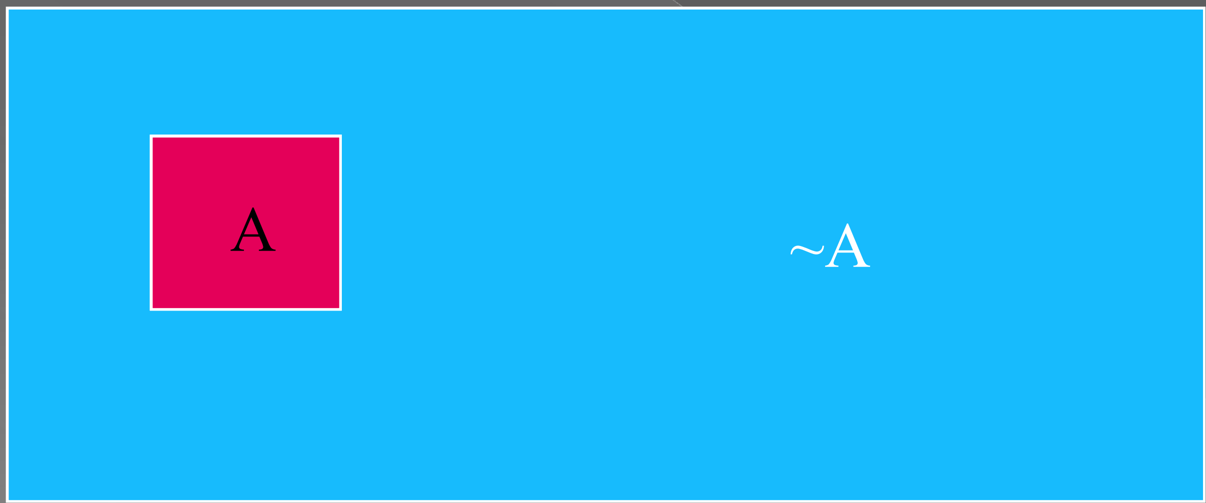
- If A is the event that a flight arrives early, then P(A) = 100/1000 = .1.
- If B is the event that a flight arrives late, then P(B) = 75/1000 = .075.
- The probability that a flight is either early or late is P(A or B) = P(A) + P(B) = .1 + .075 =.175.

The Complement Rule

- The complement rule is used to determine the probability of an event occurring by subtracting the probability of the event *not* occurring from 1. If P(A) is the probability of event A and P(~A) is the complement of A, P(A) + P(~A) = 1 OR P(A) = 1-P(~A).

The Complement Rule *continued*

- A Venn diagram illustrating the complement rule would appear as:

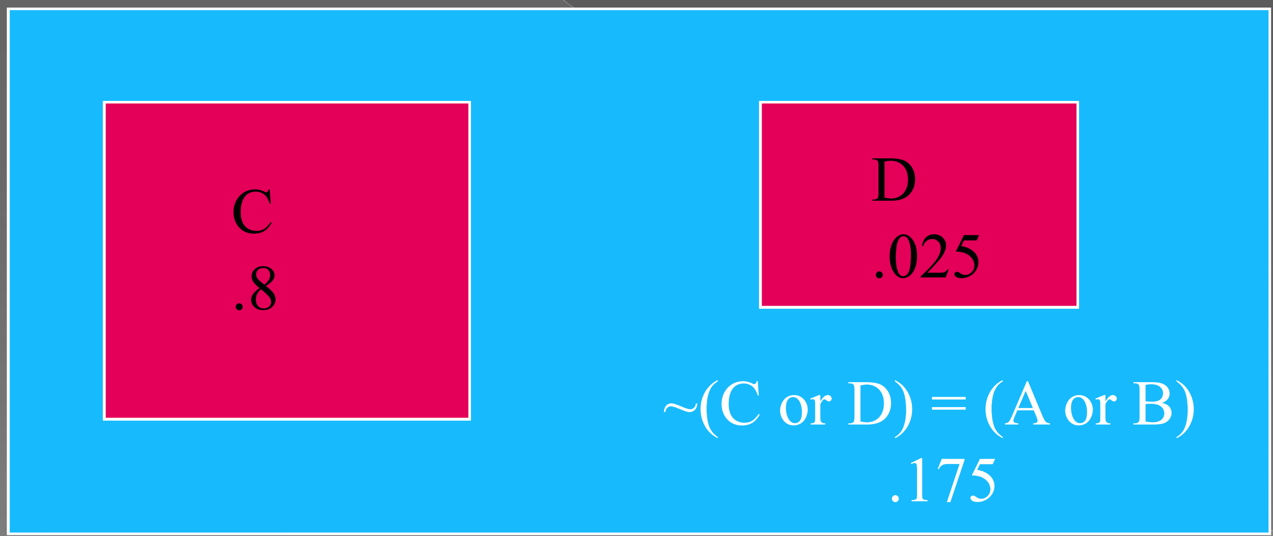


EXAMPLE 4

- Recall EXAMPLE 3.
- If C is the event that a flight arrives on time, then P(C) = 800/1000 = .8.
- If D is the event that a flight is canceled, then P(D) = 25/1000 = .025.
- Use the complement rule to show that the probability of an early (A) or a late (B) flight is .175.

EXAMPLE 4 *continued*

- P(A or B) = 1 - P(C or D) = 1 -[.8 +.025] =.175

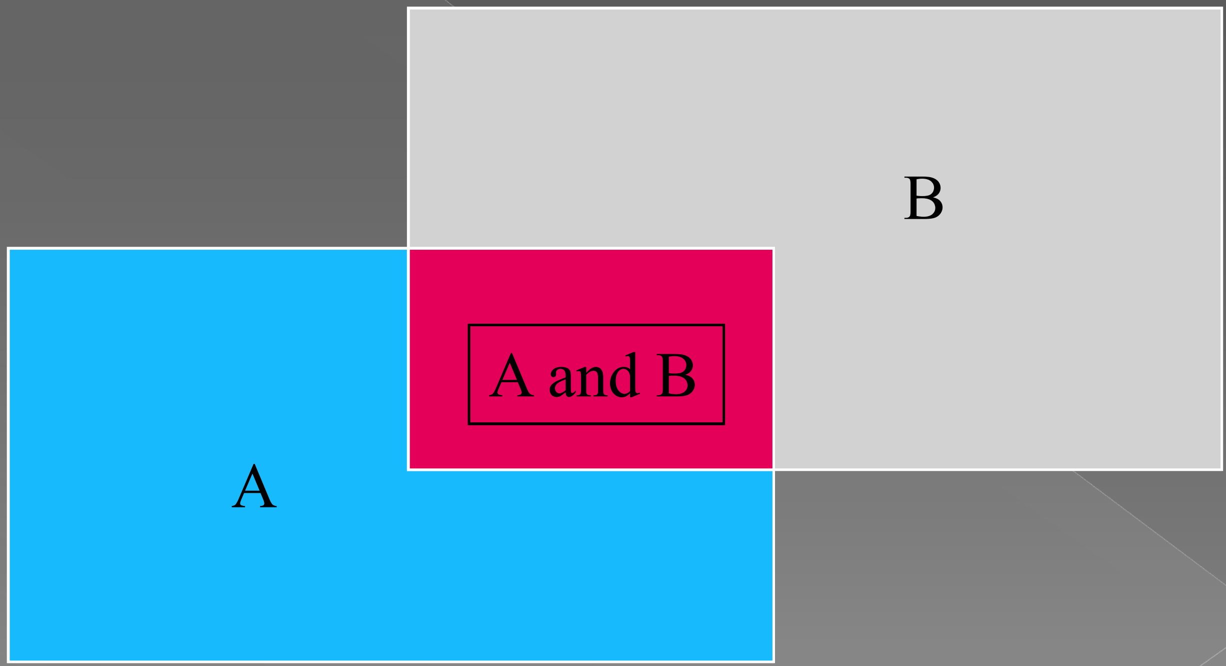


The General Rule of Addition

- If A and B are two events that are not mutually exclusive, then P(A or B) is given by the following formula:
- $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

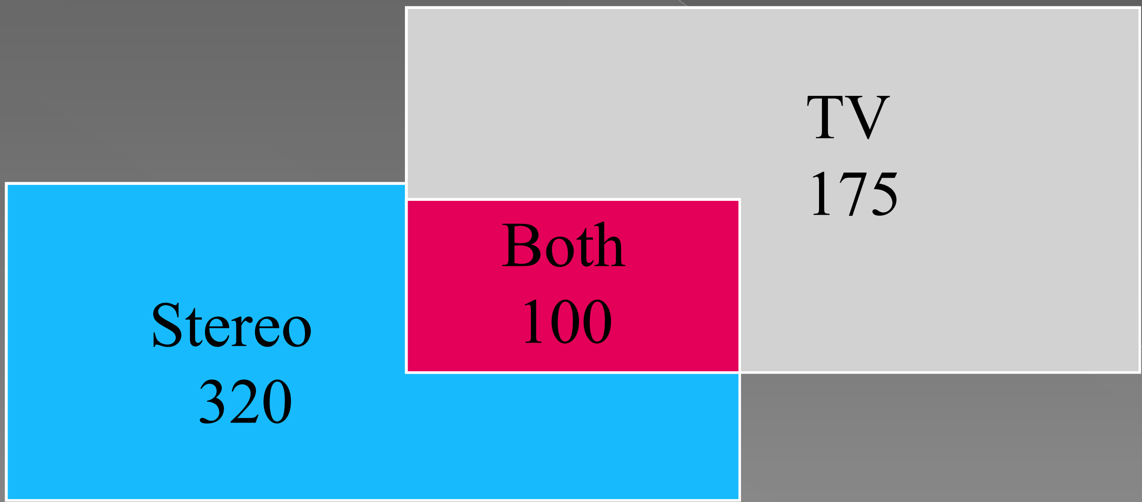
The General Rule of Addition

- The Venn Diagram illustrates this rule:



EXAMPLE 5

- In a sample of 500 students, 320 said they had a stereo, 175 said they had a TV, and 100 said they had both:



EXAMPLE 5 *continued*

- If a student is selected at random, what is the probability that the student has only a stereo, only a TV, and both a stereo and TV?
- P(S) = 320/500 = .64.
- P(T) = 175/500 = .35.
- P(S and T) = 100/500 = .20.

EXAMPLE 5 *continued*

- If a student is selected at random, what is the probability that the student has either a stereo or a TV in his or her room?
- $P(S \text{ or } T) = P(S) + P(T) - P(S \text{ and } T) = .64 + .35 - .20 = .79$.