

# Statistics 1

## Chapter 8

### The Normal Probability Distribution

# Chapter Eight

## The Normal Probability Distribution

### GOALS

When you have completed this chapter, you will be able to:

#### ONE

List the characteristics of the normal probability distribution.

#### TWO

Define and calculate  $z$  values.

#### THREE

Determine the probability that an observation will lie between two points using the standard normal distribution.

#### FOUR

Determine the probability that an observation will be above or below a given value using the standard normal distribution.

## *Chapter Seven* continued

# A Survey of Probability Concepts

### **GOALS**

When you have completed this chapter, you will be able to:

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#### **FIVE**

Compare two or more observations that are on different probability distributions.

#### **SIX**

Use the normal distribution to approximate the binomial probability distribution.

# Characteristics of a Normal Probability Distribution

- The normal curve is *bell-shaped* and has a single peak at the exact center of the distribution.
- The arithmetic mean, median, and mode of the distribution are equal and located at the peak.
- Half the area under the curve is above the peak, and the other half is below it.

# Characteristics of a Normal Probability Distribution

- The normal probability distribution is symmetrical about its mean.
- The normal probability distribution is **asymptotic** - the curve gets closer and closer to the x-axis but never actually touches it.

# Characteristics of a Normal Distribution

Normal  
curve is  
symmetrical

Theoretically,  
curve  
extends to  
infinity

Mean, median, and  
mode are equal

# The Standard Normal Probability Distribution

- ◉ A normal distribution with a mean of 0 and a standard deviation of 1 is called the standard normal distribution.
- ◉ **Z value:** The distance between a selected value, designated  $X$ , and the population mean  $\mu$ , divided by the population standard deviation,

$$Z = \frac{X - \mu}{\sigma}$$

# EXAMPLE 1

- The monthly incomes of recent MBA graduates in a large corporation are normally distributed with a mean of \$2000 and a standard deviation of \$200. What is the **Z value** for an income of \$2200? An income of \$1700?
- For  $X = \$2200$ ,  $Z = (2200 - 2000) / 200 = 1$ .



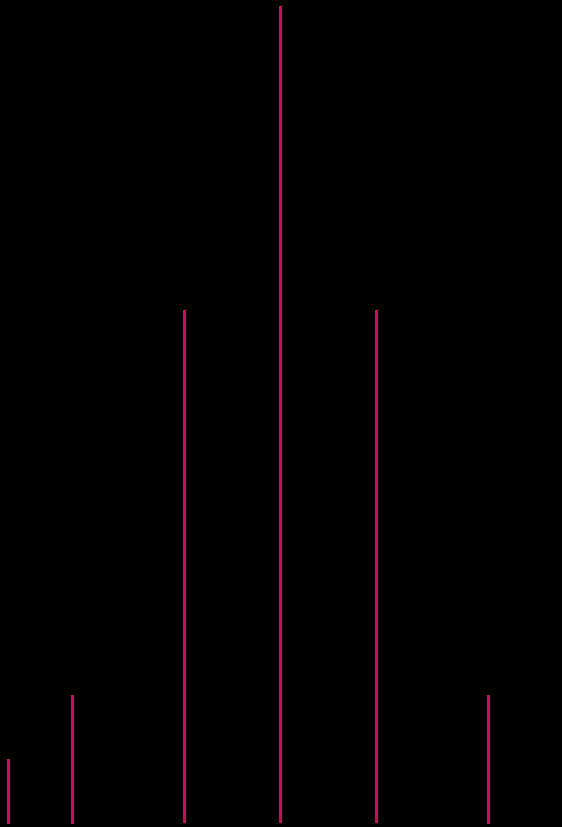
# EXAMPLE 1 *continued*

- For  $X = \$1700$ ,  $Z = (1700 - 2000) / 200 = -1.5$
- A *Z value* of 1 indicates that the value of \$2200 is 1 standard deviation above the mean of \$2000, while a *Z value* of \$1700 is 1.5 standard deviation below the mean of \$2000.

# Areas Under the Normal Curve

- About 68 percent of the area under the normal curve is within one standard deviation of the mean.  
 $\mu \pm 1\sigma$
- About 95 percent is within two standard deviations of the mean.  
 $\mu \pm 2\sigma$
- 99.74 percent is within three standard deviations of the mean.  
 $\mu \pm 3\sigma$

# Areas Under the Normal Curve



Between:  
1.68.26%  
2.95.44%  
3.99.74%

## EXAMPLE 2

- The daily water usage per person in New Providence, New Jersey is normally distributed with a mean of 20 gallons and a standard deviation of 5 gallons.

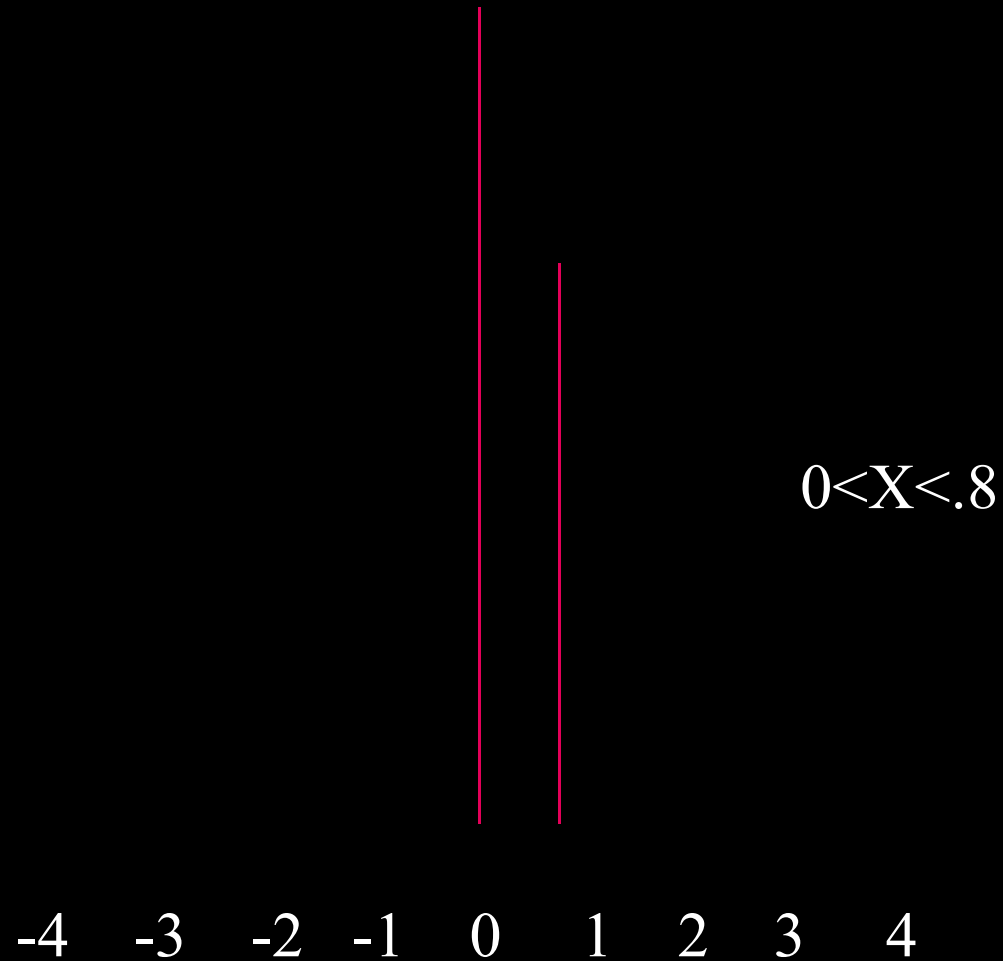
- About 68% of the daily water usage per person in New Providence lies between what two values?  
 $\mu \pm 1\sigma = 20 \pm 1(5)$ .

- That is, about 68% of the daily water usage will lie between 15 and 25 gallons.

## EXAMPLE 3

- What is the probability that a person from New Providence selected at random will use less than 20 gallons per day?
- The associated **Z value** is  $Z = (20 - 20) / 5 = 0$ . Thus,  $P(X < 20) = P(Z < 0) = .5$
- What percent uses between 20 and 24 gallons?
- The **Z value** associated with  $X = 20$  is  $Z = 0$  and with  $X = 24$ ,  $Z = (24 - 20) / 5 = .8$ . Thus,  
 $P(20 < X < 24) = P(0 < Z < .8) = 28.81\%$

## EXAMPLE 3



$$P(0 < Z < .8) = .2881$$

$$0 < X < .8$$

## EXAMPLE 3 *continued*

- ◉ What percent of the population uses between 18 and 26 gallons?
- ◉ The *Z value* associated with  $X=18$  is  $Z=(18-20)/5=-.4$ , and for  $X=26$ ,  $Z=(26-20)/5=1.2$ . Thus  $P(18<X<26)=P(-.4<Z<1.2)=.1554+.3849=.5403$

## EXAMPLE 4

- Professor Mann has determined that the final averages in his statistics course is normally distributed with a mean of 72 and a standard deviation of 5. He decides to assign his grades for his current course such that the top 15% of the students receive an A. What is the lowest average a student can receive to earn an A?
- Let  $X$  be the lowest average. Find  $X$  such that  $P(X > X) = .15$ . The corresponding Z value is 1.04. Thus we have  $(X - 72) / 5 = 1.04$  or  $X = 77.2$



# EXAMPLE 4

0 1 2 3 4



## EXAMPLE 5

- The amount of tip the servers in an exclusive restaurant receive per shift is normally distributed with a mean of \$80 and a standard deviation of \$10. Shelli feels she has provided poor service if her total tip for the shift is less than \$65. What is the probability she has provided poor service?
- Let  $X$  be the amount of tip. The  $Z$  value associated with  $X=65$  is  $Z = (65 - 80) / 10 = -1.5$ . Thus  $P(X < 65) = P(Z < -1.5) = .5 -$

# The Normal Approximation to the Binomial

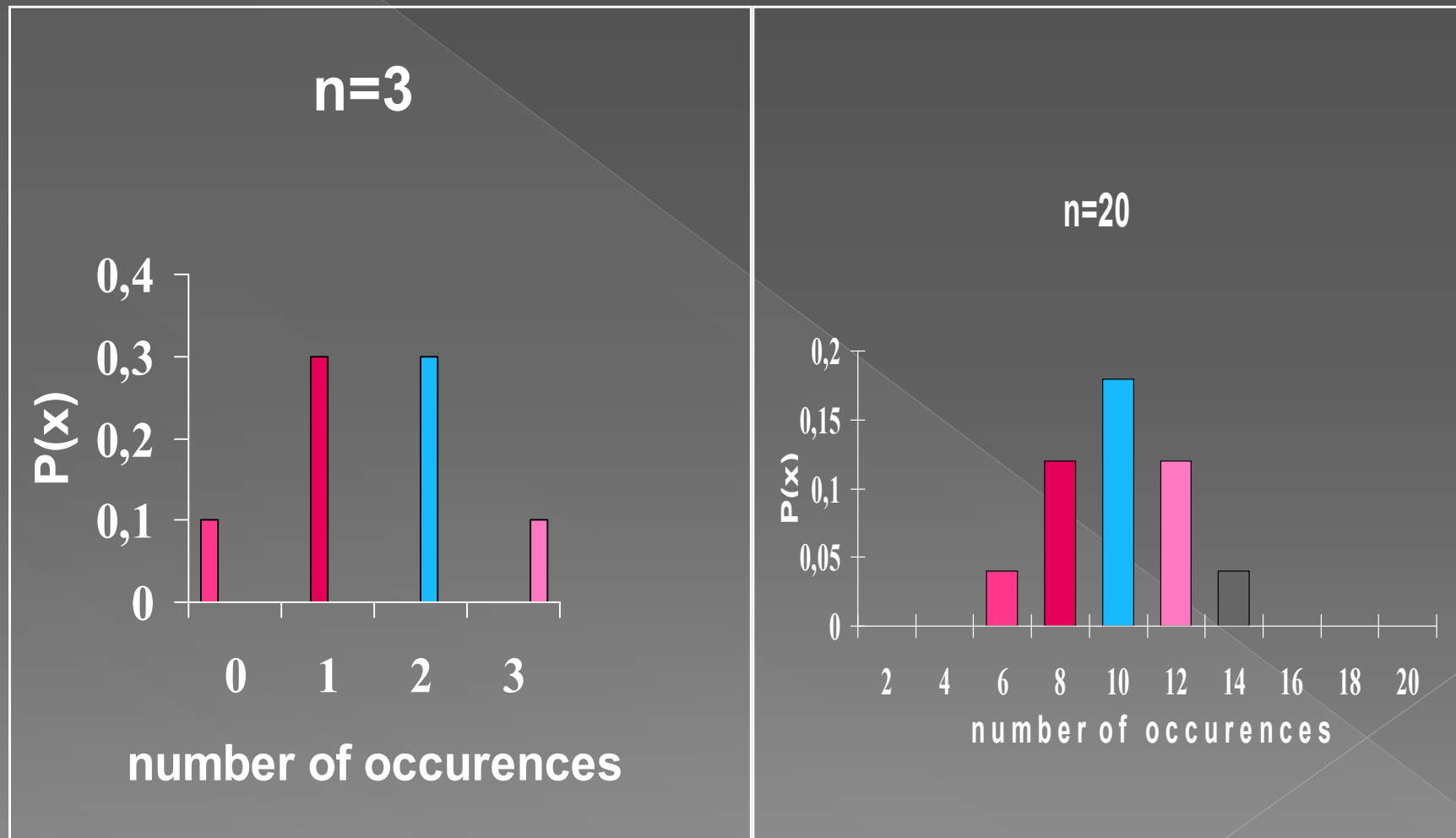
- Using the normal distribution (a continuous distribution) as a substitute for a binomial distribution (a discrete distribution) for large values of  $n$  seems reasonable because as  $n$  increases, a binomial distribution gets closer and closer to a normal distribution.
- The normal probability distribution is generally deemed a good approximation to the binomial probability distribution when  $n$  and  $n\pi$  are both greater than 5.

# The Normal Approximation

*continued*

- ◎ Recall for the binomial experiment:
  - > There are only two mutually exclusive outcomes (success or failure) on each trial.
  - > A binomial distribution results from counting the number of successes.
  - > Each trial is independent.
  - > The probability is fixed from trial to trial, and the number of trials  $n$  is also fixed.

# Binomial Distribution for an $n$ of 3 and 20, where $\pi = .50$



# Continuity Correction Factor

- The value .5 subtracted or added, depending on the problem, to a selected value when a binomial probability distribution (a discrete probability distribution) is being approximated by a continuous probability distribution (the normal distribution).

## EXAMPLE 6

- A recent study by a marketing research firm showed that 15% of American households owned a video camera. A sample of 200 homes is obtained.
- Of the 200 homes sampled how many would you expect to have video cameras?

$$\mu = n\pi = (.15)(200) = 30$$

## EXAMPLE 6

- What is the variance?

$$\sigma^2 = n \pi (1 - \pi) = (30)(1 - .15) = 25.5$$

- What is the standard deviation?

$$\sigma = \sqrt{25.5} = 5.0498$$

- What is the probability that less than 40 homes in the sample have video cameras? We need  $P(X < 40) = P(X < 39)$ .

So using the normal approximation,

$$P(X < 39.5) = P\left[Z < \frac{(39.5 - 30)}{5.0498}\right] = P(Z < 1.8812)$$

$$P(Z < 1.88) = .5 + .4699 = .9699$$



## EXAMPLE 6

$$\begin{aligned} P(Z=1.88) \\ .5+.4699 \\ =.9699 \end{aligned}$$

$Z=1.88$

0 1 2 3 4