Statistics 1
Chapter 11
Tests of Hypothesis
Small Samples

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Chapter Eleven

Tests of Hypothesis Small Samples

GOALS

When you have completed this chapter, you will be able to:

ONE

Describe the characteristics of Student's t-distribution.

TWO

Understand the difference between dependent and independent samples.

THREE

Understand the assumptions necessary to conduct a test of hypothesis regarding a population mean, when the number of observations is small.

FOUR

Conduct a test of hypothesis regarding one population mean.

Chapter 11 *continued*

Tests of Hypothesis
Small Samples

GOALS
When you have completed this chapter, you will be able to:

FIVE
Conduct a test of hypothesis regarding the difference in the means of two independent samples.

SIX
Conduct a test of hypothesis regarding the mean difference between paired observations.

Characteristics of Student's *t*-Distribution

- The *t*-distribution has the following properties:
  - › It is continuous, bell-shaped, and symmetrical about zero like the z-distribution.
  - › There is a family of *t*-distributions sharing a mean of zero but having different standard deviations.
  - › The t-distribution is more spread out and flatter at the center than the z-distribution, but approaches the z-distribution as the sample size gets larger.

The degrees of freedom for the t-distribution is $df = n - 1$.

z-distribution

t-distribution
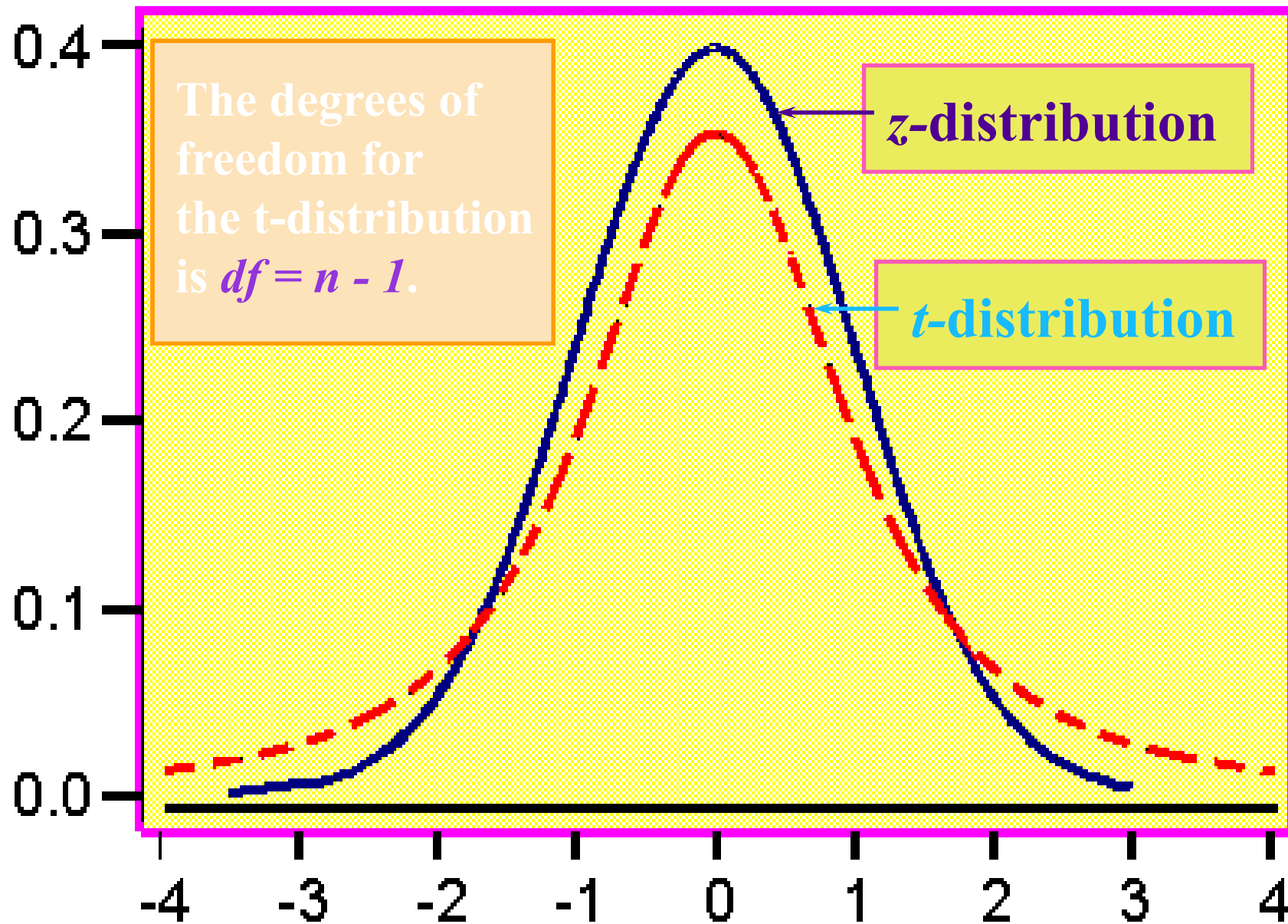
Testing for the Population Mean: Small Sample, Population Standard Deviation Unknown

⊙ The test statistic for the one sample case is given by:

$$t = \frac{\overline{X} - \mu}{s / \sqrt{n}}$$

EXAMPLE 1

- The current rate for producing 5 amp fuses at Neary Electric Co. is 250 per hour. A new machine has been purchased and installed that, according to the supplier, will increase the production rate. A sample of 10 randomly selected hours from last month revealed the mean hourly production on the new machine was 256, with a sample standard deviation of 6 per hour. At the .05 significance level can Neary conclude that the new machine is faster?

EXAMPLE 1 *continued*

⦿ Step 1: $H_0 : \mu \leq 250 \qquad H_1 : \mu > 250$

⦿ Step 2: H_0 is rejected if *t* >1.833, *df=9*

⦿ Step 3: $t = [256 - 250]/[6/\sqrt{10}] = 3.16$

⦿ Step 4: H_0 is rejected. The new machine is faster.

Display of the Rejection Region, Critical Value, and the computed Test Statistic

df=9

.05 region of rejection

Critical Value =1.833

t=3.16

0   1   2   3   4
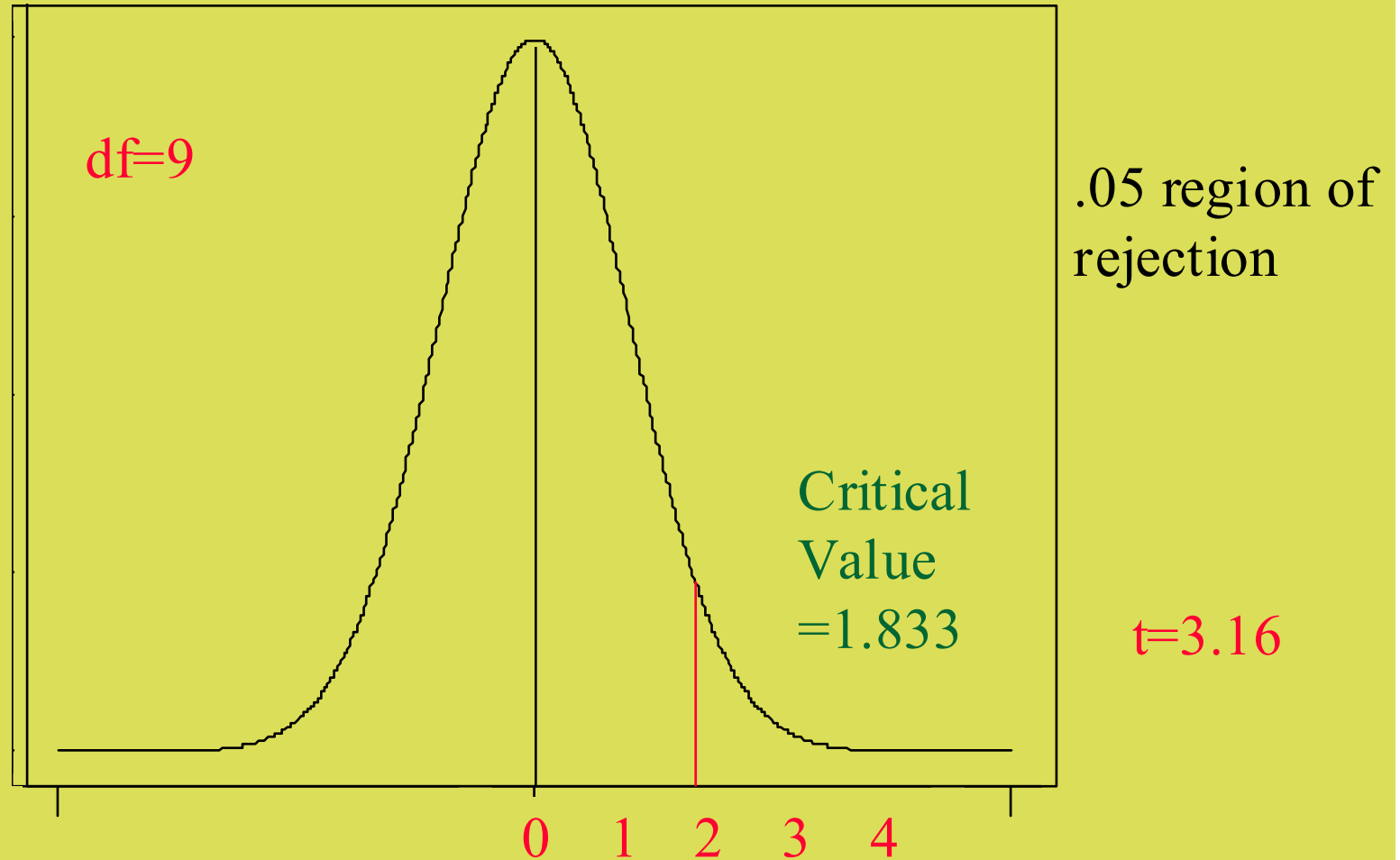
NOTE

- For a two-tail test using the *t*-distribution, you will reject the null hypothesis when the value of the test statistic is greater than $t_{n-1,\alpha/2}$

- $t_{n-1,\alpha/2}$ or if it is less than - $t_{n-1,\alpha/2}$

- For a left-tail test using the *t*-distribution, you will reject the null hypothesis when the value of the test statistic is less than - $t_{n-1,\alpha/2}$

Comparing Two Population Means

- To conduct this test, three assumptions are required:
  - › The populations must be normally or approximately normally distributed.
  - › The populations must be independent.
  - › The population variances must be equal.

Pooled Sample Variance and Test Statistic

⊙ Pooled Sample Variance:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

⊙ Test Statistic:

$$t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{s_p^2 \left(\dfrac{1}{n_1} + \dfrac{1}{n_2} \right)}}$$

EXAMPLE 2

- A recent EPA study compared the highway fuel economy of domestic and imported passenger cars. A sample of 15 domestic cars revealed a mean of 33.7 mpg with a standard deviation of 2.4 mpg. A sample of 12 imported cars revealed a mean of 35.7 mpg with a standard deviation of 3.9. At the .05 significance level can the EPA conclude that the mpg is higher on the imported cars? *(Let subscript 1*

EXAMPLE 2 *continued*

⊙ Step 1: $H_0 : \mu_2 \leq \mu_1$ $\qquad H_1 : \mu_2 > \mu_1$

⊙ Step 2: H_0 is rejected if $t<-1.708$, $df=25$

⊙ Step 3: $t=1.64$ (Verify.)

⊙ Step 4: H_0 is not rejected. There is insufficient sample evidence to claim a higher mpg on the imported cars.

Hypothesis Testing Involving Paired Observations

- **Independent samples** are samples that are not related in any way.

- **Dependent samples** are samples that are paired or related in some fashion.

  › For example, if you wished to buy a car you would look at the *same* car at two (or more) *different* dealerships and compare the prices.

- Use the following test when the samples are dependent:

Hypothesis Testing Involving Paired Observations

$$t = \frac{\overline{d}}{s_d / \sqrt{n}}$$

- where $\overline{d}$ is the average of the differences
- s_d is the standard deviation of the differences
- n is the number of pairs (differences)

EXAMPLE 3

- An independent testing agency is comparing the daily rental cost for renting a compact car from Hertz and Avis. A random sample of eight cities is obtained and the following rental information obtained. At the .05 significance level can the testing agency conclude that there is a difference in the rental charged?

EXAMPLE 3 *continued*

City	Hertz ($)	Avis ($)
Atlanta	42	40
Chicago	56	52
Cleveland	45	43
Denver	48	48
Honolulu	37	32
Kansas City	45	48
Miami	41	39
Seattle	46	50

EXAMPLE 3 *continued*

◉ Step 1: $H_0 : \mu_d = 0 \qquad H_1 : \mu_d \neq 0$

◉ Step 2: H_0 is rejected if t<-2.365 or t>2.365

◉ Step 3: $t = (1.00) / [3.162 / \sqrt{8}] = .89$

◉ Step 4: H_0 is not rejected. There is no difference in the charge.