Statistics 2
Chapter 2

Review 2 The Normal Probability Distribution

Chapter 2

The Normal Probability Distribution

GOALS

When you have completed this chapter, you will be able to:

ONE

List the characteristics of the normal probability distribution.

TWO

Define and calculate z values.

THREE

Determine the probability that an observation will lie between two points using the standard normal distribution.

FOUR

Determine the probability that an observation will be above or below a given value using the standard normal distribution.

Chapter 2 *continued*

A Survey of Probability Concepts

GOALS

When you have completed this chapter, you will be able to:

FIVE

Compare two or more observations that are on different probability distributions.

SIX

Use the normal distribution to approximate the binomial probability distribution.

Characteristics of a Normal Probability Distribution

- The normal curve is *bell-shaped* and has a single peak at the exact center of the distribution.
- The arithmetic mean, median, and mode of the distribution are equal and located at the peak.
- Half the area under the curve is above the peak, and the other half is below it.

Characteristics of a Normal Probability Distribution

- The normal probability distribution is symmetrical about its mean.
- The normal probability distribution is asymptotic - the curve gets closer and closer to the x-axis but never actually touches it.

Characteristics of a Normal Distribution

Normal curve is symmetrical

Theoretically, curve extends to infinity

Mean, median, and mode are equal

The Standard Normal Probability Distribution

- A normal distribution with a mean of 0 and a standard deviation of 1 is called the standard normal distribution.
- Z value: The distance between a selected value, designated X, and the population mean μ, divided by the population standard deviation, σ

$$Z = \frac{X - \mu}{\sigma}$$

EXAMPLE 1

- The monthly incomes of recent MBA graduates in a large corporation are normally distributed with a mean of \$2000 and a standard deviation of \$200. What is the Z value for an income of \$2200? An income of \$1700?
- For X=\$2200, Z=(2200-2000)/200=1.

EXAMPLE 1 *continued*

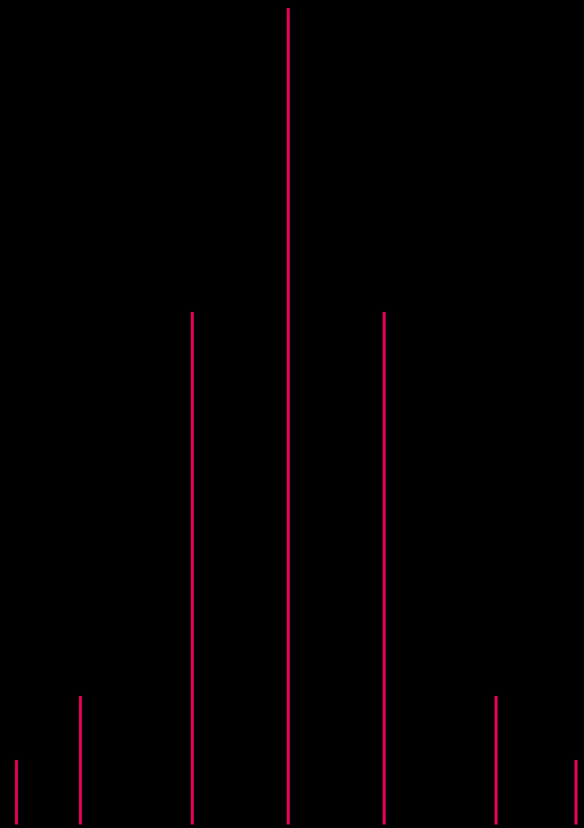
- For X=$1700, Z =(1700-2000)/200= -1.5
- A Z value of 1 indicates that the value of $2200 is 1 standard deviation above the mean of $2000, while a Z value of $1700 is 1.5 standard deviation below the mean of $2000.

Areas Under the Normal Curve

- About 68 percent of the area under the normal curve is within one standard deviation of the mean. $\mu \pm 1\sigma$
- About 95 percent is within two standard deviations of the mean. $\mu \pm 2\sigma$
- 99.74 percent is within three standard deviations of the mean. $\mu \pm 3\sigma$

Areas Under the Normal Curve

Between:
1.68.26%
2.95.44%
3.99.74%



EXAMPLE 2

- The daily water usage per person in New Providence, New Jersey is normally distributed with a mean of 20 gallons and a standard deviation of 5 gallons.
- About 68% of the daily water usage per person in New Providence lies between what two values?

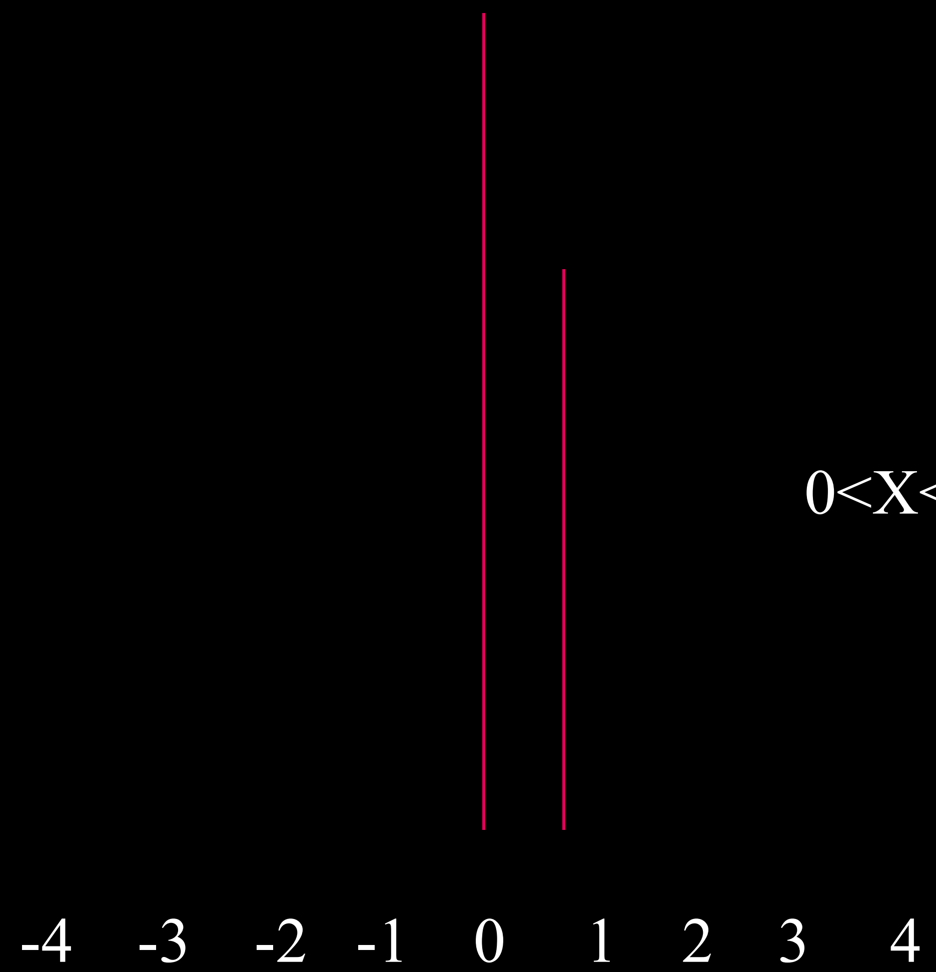
$$\mu \pm 1\sigma = 20 \pm 1(5).$$

- That is, about 68% of the daily water usage will lie between 15 and 25 gallons.

EXAMPLE 3

- What is the probability that a person from New Providence selected at random will use less than 20 gallons per day?
- The associated Z value is Z=(20-20)/5=0. Thus, P(X<20)=P(Z<0)=.5
- What percent uses between 20 and 24 gallons?
- The Z value associated with X=20 is Z=0 and with X=24, Z=(24-20)/5=.8. Thus, P(20<X<24)=P(0<Z<.8)=28.81%

EXAMPLE 3



P(0<Z<.8)
=.2881

0<X<.8

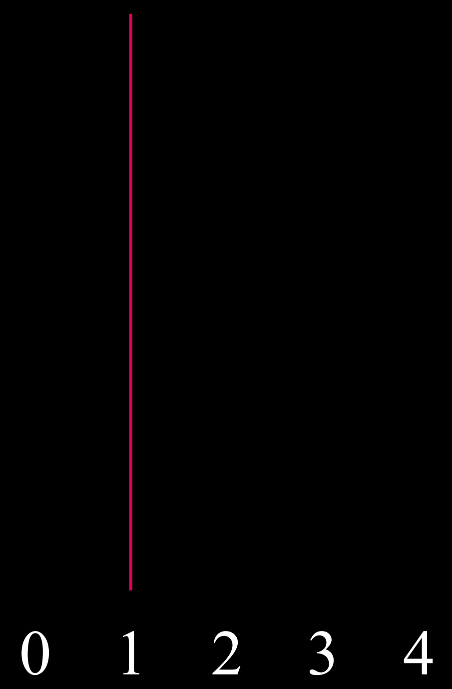
EXAMPLE 3 *continued*

- What percent of the population uses between 18 and 26 gallons?
- The Z value associated with X=18 is Z=(18-20)/5= -.4, and for X=26, Z=(26-20)/5=1.2. Thus P(18<X<26) =P(-.4<Z<1.2)=.1554+.3849=.5403

EXAMPLE 4

- Professor Mann has determined that the final averages in his statistics course is normally distributed with a mean of 72 and a standard deviation of 5. He decides to assign his grades for his current course such that the top 15% of the students receive an A. What is the lowest average a student can receive to earn an A?
- Let X be the lowest average. Find X such that $P(X > X) = .15$. The corresponding Z value is 1.04. Thus we

EXAMPLE 4



EXAMPLE 5

- The amount of tip the servers in an exclusive restaurant receive per shift is normally distributed with a mean of $80 and a standard deviation of $10. Shelli feels she has provided poor service if her total tip for the shift is less than $65. What is the probability she has provided poor service?
- Let X be the amount of tip. The Z value associated with X=65 is Z= (65-80)/10=
-1.5. Thus P(X<65)=P(Z<-1.5)=.5-

The Normal Approximation to the Binomial

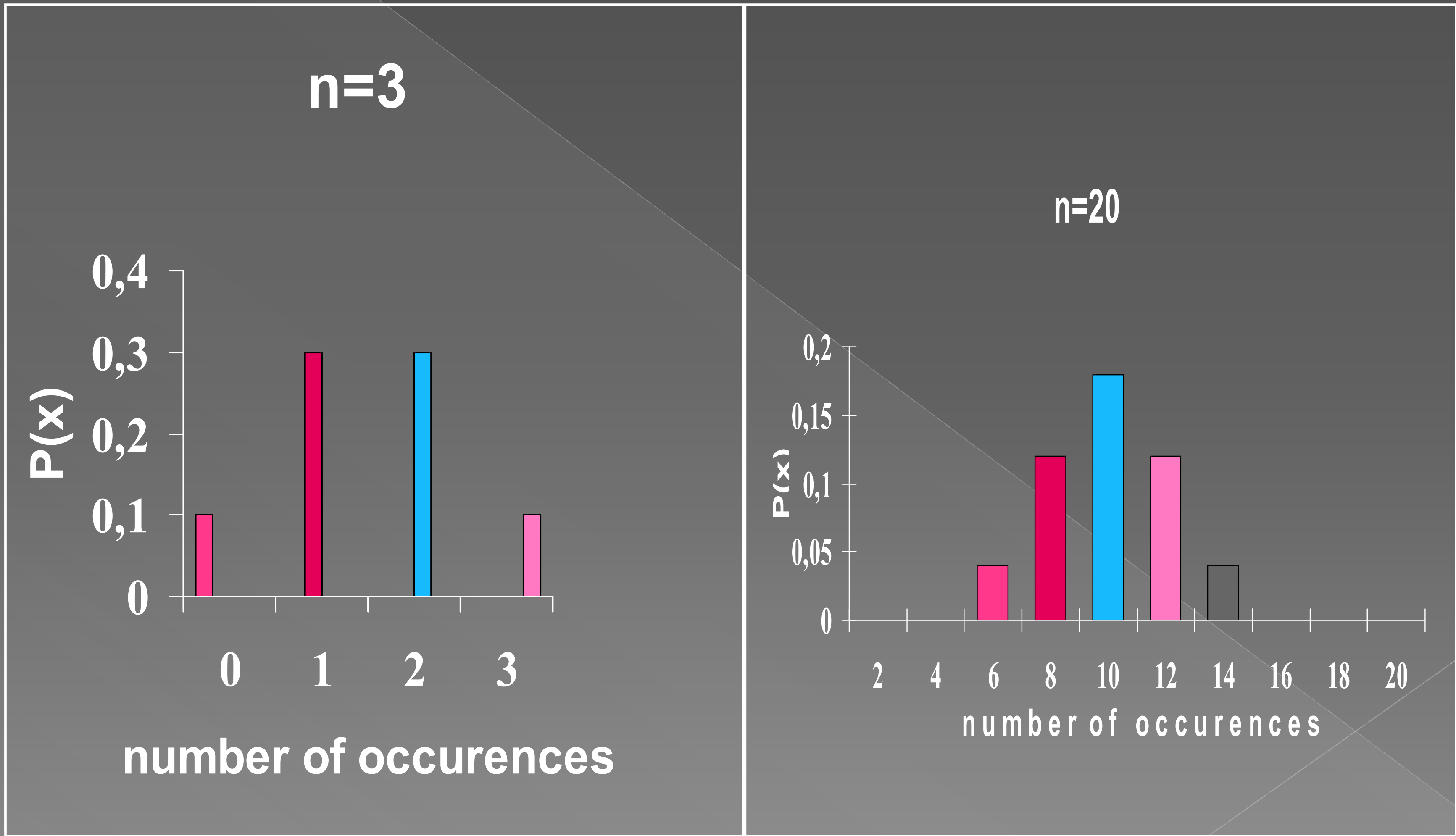
- Using the normal distribution (a continuous distribution) as a substitute for a binomial distribution (a discrete distribution) for large values of *n* seems reasonable because as *n* increases, a binomial distribution gets closer and closer to a normal distribution.
- The normal probability distribution is generally deemed a good approximation to the binomial probability distribution when $n\pi$ and $n(1-\pi)$ are both greater than 5.

The Normal Approximation

continued

- Recall for the binomial experiment:
  - There are only two mutually exclusive outcomes (success or failure) on each trial.
  - A binomial distribution results from counting the number of successes.
  - Each trial is independent.
  - The probability is fixed from trial to trial, and the number of trials *n* is also fixed.

Binomial Distribution for an n of 3 and 20, where π =.50



Continuity Correction Factor

- The value .5 subtracted or added, depending on the problem, to a selected value when a binomial probability distribution (a discrete probability distribution) is being approximated by a continuous probability distribution (the normal distribution).

EXAMPLE 6

- A recent study by a marketing research firm showed that 15% of American households owned a video camera. A sample of 200 homes is obtained.
- Of the 200 homes sampled how many would you expect to have video cameras?

$$\mu = n\pi = (.15)(200) = 30$$

EXAMPLE 6

- What is the variance?

$$\sigma^2 = n\pi(1-\pi) = (30)(1-.15) = 25.5$$

- What is the standard deviation?

$$\sigma = \sqrt{25.5} = 5.0498$$

- What is the probability that less than 40 homes in the sample have video cameras? We need P(X<40)=P(X≤39). So, using the normal approximation, P(X≤39.5) ≈ P[Z ≤ (39.5-30)/5.0498] = P(Z ≤ 1.8812) ≈ P(Z<1.88)=.5+.4699+.9699

EXAMPLE 6

P(Z=1.88)
.5+.4699
=.9699