Statistics 2
Chapter 5

Tests of Hypothesis
Small Samples

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Tests of Hypothesis Small Samples

GOALS

When you have completed this chapter, you will be able to:

ONE

Describe the characteristics of Student's *t*-distribution.

TWO

Understand the difference between dependent and independent samples.

THREE

Understand the assumptions necessary to conduct a test of hypothesis regarding a population mean, when the number of observations is small.

FOUR

Conduct a test of hypothesis regarding one population mean.

Chapter 5 *continued*

Tests of Hypothesis Small Samples

GOALS

When you have completed this chapter, you will be able to:

FIVE

Conduct a test of hypothesis regarding the difference in the means of two independent samples.

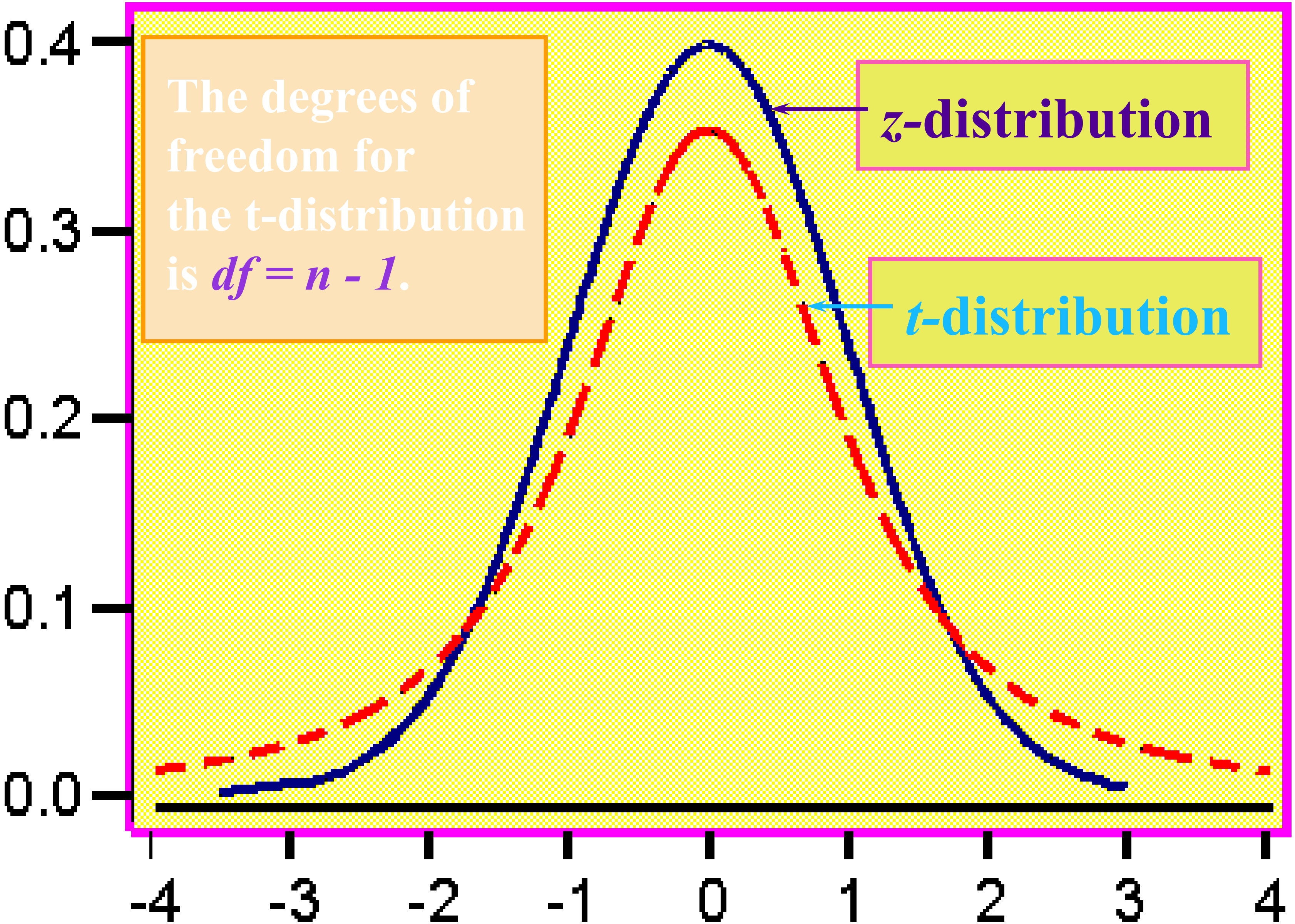
SIX

Conduct a test of hypothesis regarding the mean difference between paired observations.

Characteristics of Student's *t*-Distribution

- The *t*-distribution has the following properties:
  - It is continuous, bell-shaped, and symmetrical about zero like the z-distribution.
  - There is a family of *t*-distributions sharing a mean of zero but having different standard deviations.
  - The t-distribution is more spread out and flatter at the center than the z-distribution, but approaches the z-distribution as the sample size gets larger.

9-3



The degrees of freedom for the t-distribution is *df = n - 1*.
z-distribution
t-distribution
0.4
0.3
0.2
0.1
0.0
-4
-3
-2
-1
0
1
2
3
4

Testing for the Population Mean: Small Sample, Population Standard Deviation Unknown

- The test statistic for the one sample case is given by:

$$t = \frac{\overline{X} - \mu}{s / \sqrt{n}}$$

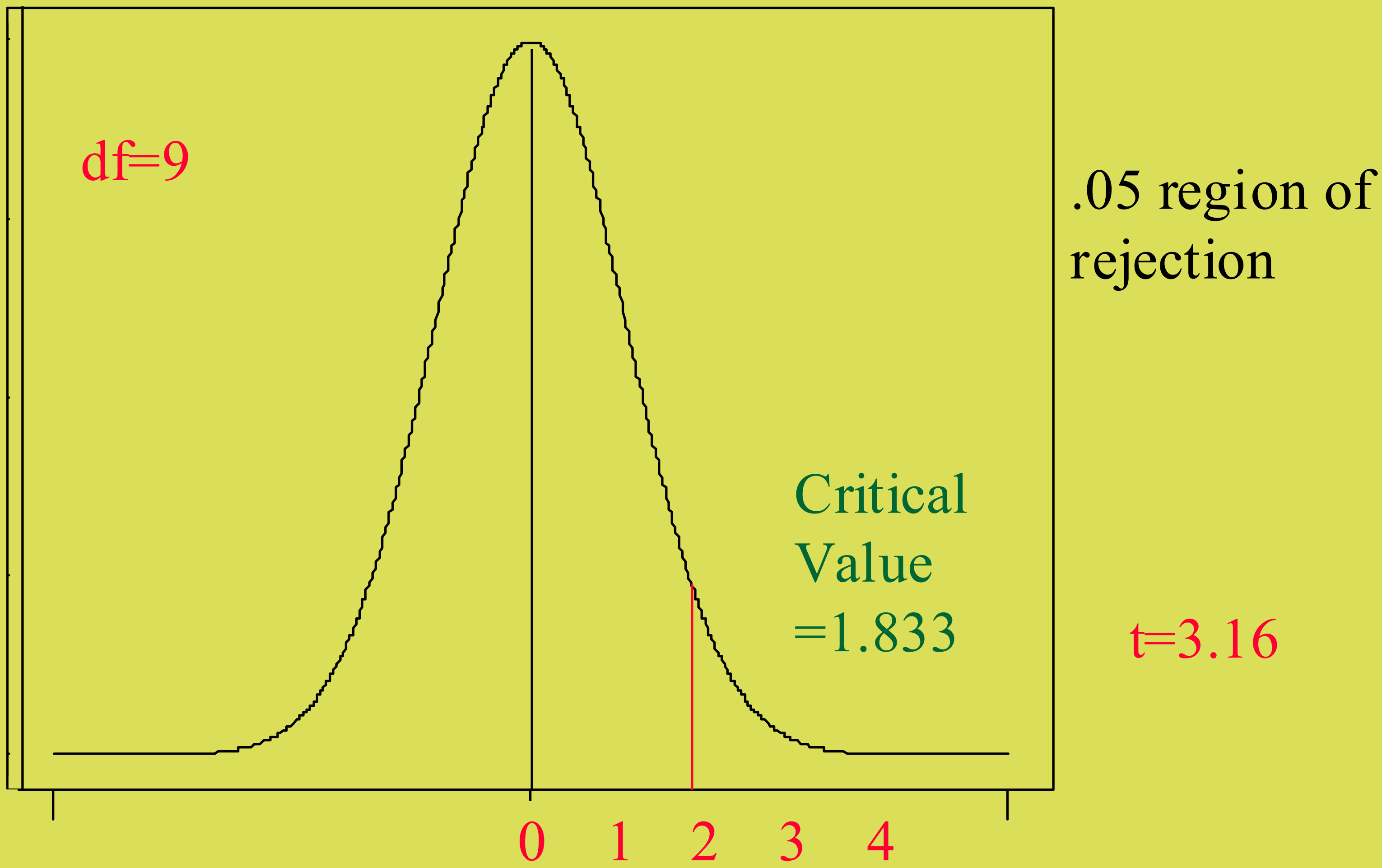
EXAMPLE 1

- The current rate for producing 5 amp fuses at Neary Electric Co. is 250 per hour. A new machine has been purchased and installed that, according to the supplier, will increase the production rate. A sample of 10 randomly selected hours from last month revealed the mean hourly production on the new machine was 256, with a sample standard deviation of 6 per hour. At the .05 significance level can Neary conclude that the new machine is faster?

EXAMPLE 1 *continued*

- Step 1: $H_0: \mu \leq 250 \qquad H_1: \mu > 250$
- Step 2: H_0 is rejected if t >1.833, df=9
- Step 3: $t = [256 - 250]/[6/\sqrt{10}] = 3.16$
- Step 4: H_0 is rejected. The new machine is faster.

Display of the Rejection Region, Critical Value, and the computed Test Statistic



NOTE

- For a two-tail test using the *t*-distribution, you will reject the null hypothesis when the value of the test statistic is greater than
- $t_{n-1,\alpha/2}$ or if it is less than -$t_{n-1,\alpha/2}$
- For a left-tail test using the *t*-distribution, you will reject the null hypothesis when the value of the test statistic is less than -$t_{n-1,\alpha/2}$

Comparing Two Population Means

- To conduct this test, three assumptions are required:
  - The populations must be normally or approximately normally distributed.
  - The populations must be independent.
  - The population variances must be equal.

Pooled Sample Variance and Test Statistic

- Pooled Sample Variance:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

- Test Statistic:

$$t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

EXAMPLE 2

- A recent EPA study compared the highway fuel economy of domestic and imported passenger cars. A sample of 15 domestic cars revealed a mean of 33.7 mpg with a standard deviation of 2.4 mpg. A sample of 12 imported cars revealed a mean of 35.7 mpg with a standard deviation of 3.9. At the .05 significance level can the EPA conclude that the mpg is higher on the imported cars? *(Let subscript 1*

EXAMPLE 2 *continued*

- Step 1: $H_0: \mu_2 \leq \mu_1 \qquad H_1: \mu_2 > \mu_1$
- Step 2: H_0 is rejected if t<-1.708, df=25
- Step 3: t=1.64 (Verify.)
- Step 4: H_0 is not rejected. There is insufficient sample evidence to claim a higher mpg on the imported cars.

Hypothesis Testing Involving Paired Observations

- Independent samples are samples that are not related in any way.
- Dependent samples are samples that are paired or related in some fashion.
  - For example, if you wished to buy a car you would look at the *same* car at two (or more) *different* dealerships and compare the prices.
- Use the following test when the samples are dependent:

Hypothesis Testing Involving Paired Observations

$$t = \frac{\bar{d}}{s_d / \sqrt{n}}$$

- where $\bar{d}$ is the average of the differences
- s_d is the standard deviation of the differences
- n is the number of pairs (differences)

EXAMPLE 3

- An independent testing agency is comparing the daily rental cost for renting a compact car from Hertz and Avis. A random sample of eight cities is obtained and the following rental information obtained. At the .05 significance level can the testing agency conclude that there is a difference in the rental charged?

EXAMPLE 3 *continued*

City	Hertz ($)	Avis ($)
Atlanta	42	40
Chicago	56	52
Cleveland	45	43
Denver	48	48
Honolulu	37	32
Kansas City	45	48
Miami	41	39
Seattle	46	50

EXAMPLE 3 *continued*

- Step 1: $H_0 : \mu_d = 0 \qquad H_1 : \mu_d \neq 0$
- Step 2: H_0 is rejected if t<-2.365 or t>2.365
- Step 3: $t = (1.00) / [3.162 / \sqrt{8}] = .89$
- Step 4: H_0 is not rejected. There is no difference in the charge.