Statistics 2
Chapter 4

Tests of Hypothesis
Large Samples

Chapter 4

Tests of Hypothesis Large Samples

GOALS

When you have completed this chapter, you will be able to:

ONE

Define a hypothesis and hypothesis testing.

TWO

Describe the five step hypothesis testing procedure.

THREE

Distinguish between a one-tailed and a two-tailed test of hypothesis.

FOUR

Conduct a test of hypothesis about a population mean and a population proportion.

Chapter 4 *continued*

Tests of Hypothesis Large Samples

GOALS

When you have completed this chapter, you will be able to:

FIVE

Conduct a test of hypothesis about the difference between two population means and two population proportions.

SIX

Define Type I and Type II errors.

SEVEN

Compute the probability of a Type II error.

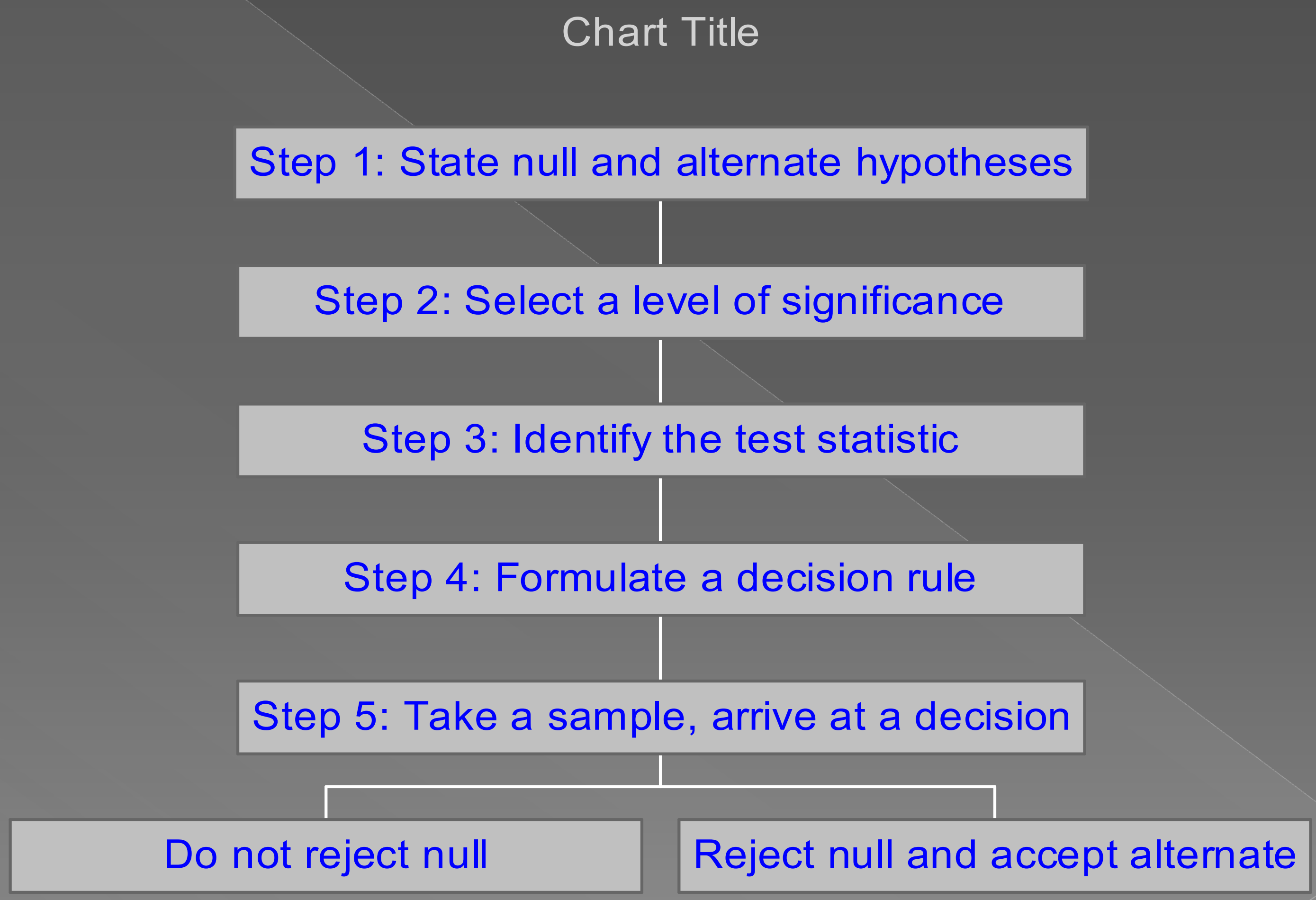
What is a Hypothesis?

- Hypothesis: A statement about the value of a population parameter developed for the purpose of testing.
- Examples of hypotheses made about a population parameter are:
  - The mean monthly income for systems analysts is $3, 625.
  - Twenty percent of all juvenile offenders are caught and sentenced to prison.

What is Hypothesis Testing?

- Hypothesis testing: A procedure, based on sample evidence and probability theory, used to determine whether the hypothesis is a reasonable statement and should not be rejected, or is unreasonable and should be rejected.

Hypothesis Testing



Definitions

- Null Hypothesis H_0: A statement about the value of a population parameter.
- Alternative Hypothesis H_1: A statement that is accepted if the sample data provide evidence that the null hypothesis is false.
- Level of Significance: The probability of rejecting the null hypothesis when it is actually true.
- Type I Error: Rejecting the null hypothesis when it is actually true.

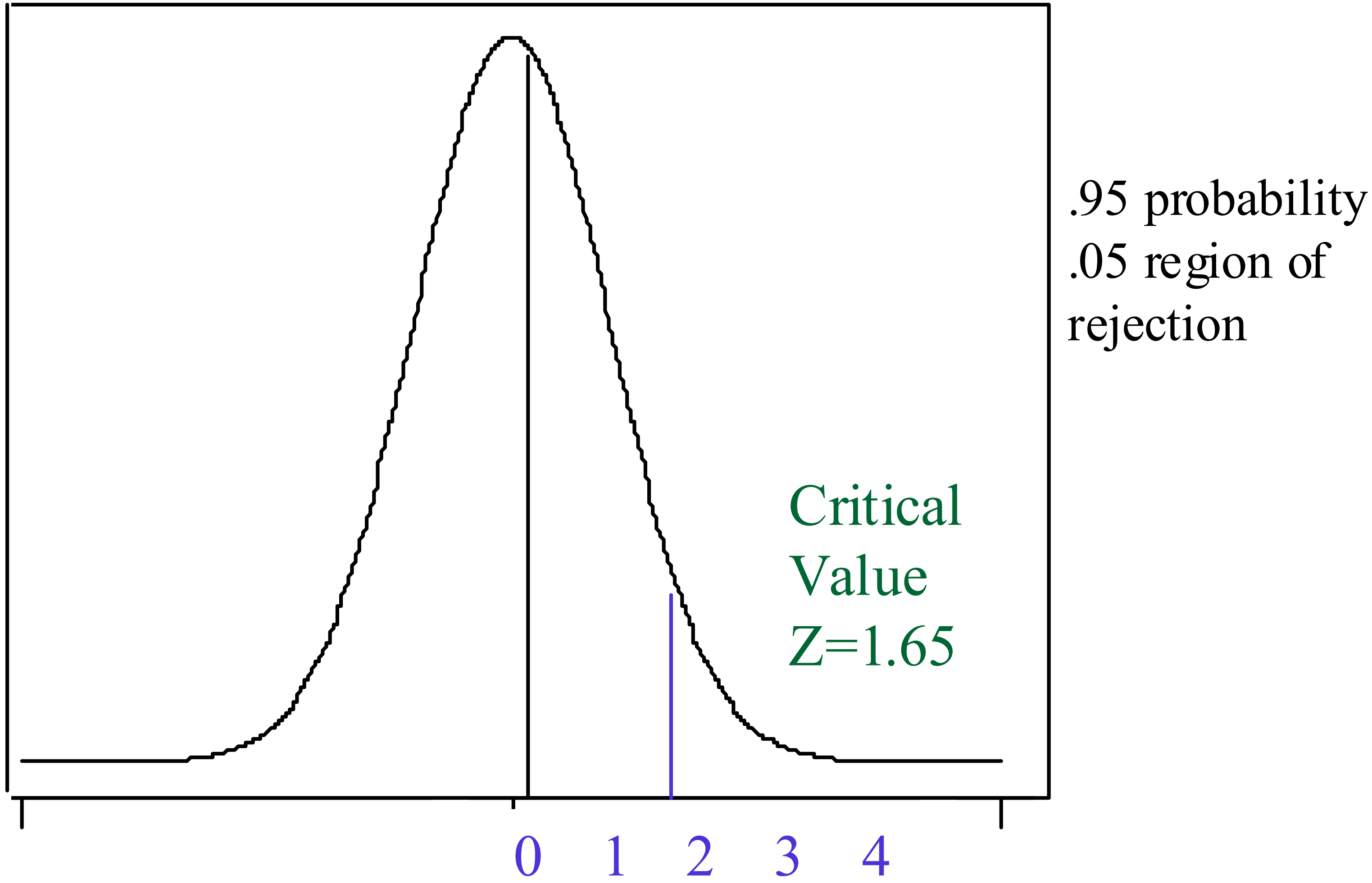
Definitions

- Type II Error: Accepting the null hypothesis when it is actually false.
- Test statistic: A value, determined from sample information, used to determine whether or not to reject the null hypothesis.
- Critical value: The dividing point between the region where the null hypothesis is rejected and the region where it is not rejected.

One-Tailed Tests of Significance

- A test is one-tailed when the alternate hypothesis, H_1 , states a direction, such as:
  - H_0 : The mean income of females is less than or equal to the mean income of males.
  - H_1 : The mean income of females is greater than males.

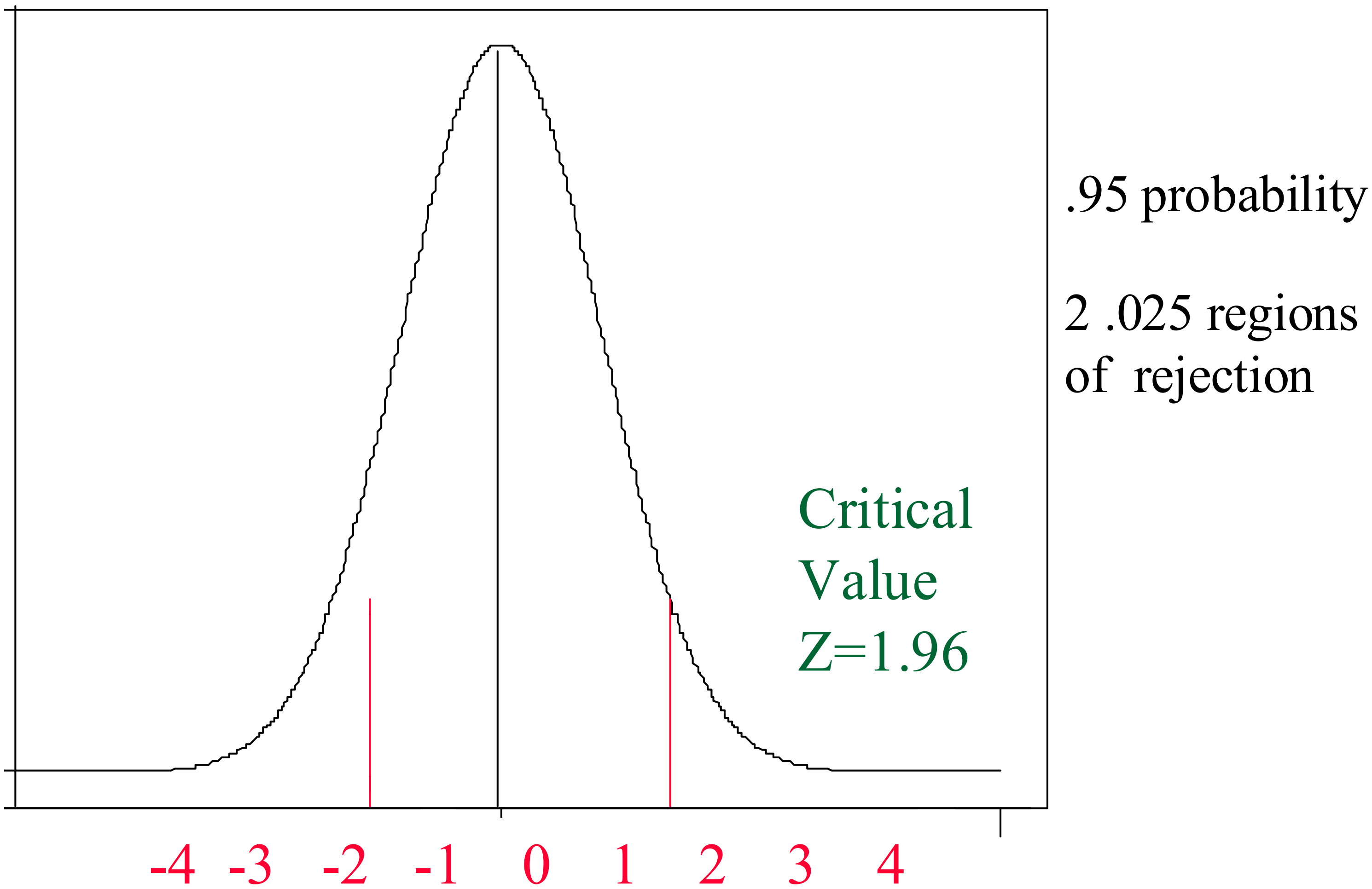
Sampling Distribution for the Statistic Z for a One-Tailed Test, .05 Level of Significance



Two-Tailed Tests of Significance

- A test is two-tailed when no direction is specified in the alternate hypothesis H_1 , such as:
  - H_0 : The mean income of females is equal to the mean income of males.
  - H_1 : The mean income of females is not equal to the mean income of the males.

Sampling Distribution for the Statistic Z for a Two-Tailed Test, .05 Level of Significance



Testing for the Population Mean: Large Sample, Population Standard Deviation Known

- When testing for the population mean from a large sample and the population standard deviation is known, the test statistic is given by:

$$z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}}$$

EXAMPLE 1

- The processors of Fries' Catsup indicate on the label that the bottle contains 16 ounces of catsup. A sample of 36 bottles is selected hourly and the contents weighed. Last hour a sample of 36 bottles had a mean weight of 16.12 ounces with a standard deviation of .5 ounces. At the .05 significance level is the process out of control?

EXAMPLE 1 *continued*

- Step 1: State the null and the alternative hypotheses:
- Step 2: State the decision rule:

$$H_0 : \mu = 16 \qquad H_1 : \mu \neq 16$$

- Step 3: Compute the value of the test statistic:

$$H_0 \text{ is rejected if } z < -1.96 \text{ or } z > 1.96$$

- Step 4: Decide on H_0 : H_0 is not rejected because 1.44 is less than the critical value of 1.96

$$z = [16.12 - 16]/[.5/\sqrt{36}] = 1.44$$

p-Value in Hypothesis Testing

- p-Value: the probability, assuming that the null hypothesis is true, of getting a value of the test statistic at least as extreme as the computed value for the test.
- If the p-value is smaller than the significance level, H_0 is rejected.
- If the p-value is larger than the significance level, H_0 is not rejected.

Computation of the p-Value

- One-Tailed Test: p-Value = P{z $\geq$ absolute value of the computed test statistic value}
- Two-Tailed Test: p-Value = 2P{z $\geq$ absolute value of the computed test statistic value}
- From EXAMPLE 1, z= 1.44, and since it was a two-tailed test, then p-Value = 2P {z $\geq$ 1.44} = 2(.5-.4251) = .1498. Since .1498 > .05, do not reject H_0.

Testing for the Population Mean: Large Sample, Population Standard Deviation Unknown

- Here σ is unknown, so we estimate it with the sample standard deviation s.
- As long as the sample size n $\geq$ 30, z can be approximated with:

$$z = \frac{\overline{X} - \mu}{s / \sqrt{n}}$$

EXAMPLE 2

- Roder's Discount Store chain issues its own credit card. Lisa, the credit manager, wants to find out if the mean monthly unpaid balance is more than $400. The level of significance is set at .05. A random check of 172 unpaid balances revealed the sample mean to be $407 and the sample standard deviation to be $38. Should Lisa conclude that the population mean is greater than $400, or is it reasonable to assume that the

EXAMPLE 2 *continued*

- Step 1: $H_0: \mu \leq 400 \qquad H_1: \mu > 400$
- Step 2: H_0 is rejected if z>1.645
- Step 3: $z = [407 - 400]/[38/\sqrt{172}] = 2.42$
- Step 4: H_0 is rejected. Lisa can conclude that the mean unpaid balance is greater than $400.

Hypothesis Testing: Two Population Means

- Assume the parameters for the two populations are: $\mu_1, \mu_2, \sigma_1, \text{and } \sigma_2$
- For large samples the test statistic is:

$$z = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{s_1^{\,2}}{n_1} + \frac{s_2^{\,2}}{n_2}}}$$

Hypothesis Testing: Two Population Means

- When $\sigma_1 \text{ and } \sigma_2$ are unknown but the sample sizes $n_1 \text{ and } n_2$ are greater than or equal to 30, the test statistic is

$$z = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{s_1^{\,2}}{n_1} + \frac{s_2^{\,2}}{n_2}}}$$

EXAMPLE 3

- A study was conducted to compare the mean years of service for those retiring in 1979 with those retiring last year at Delong Manufacturing Co. At the .01 significance level can we conclude that the workers retiring last year gave more service based on the following sample data? *Note:* Let pop #1= last year.

Characteristic	1979	Last Year
Sample Mean	25.6	30.4
Sample Standard Deviation	2.9	3.6
Sample Size	40	45

EXAMPLE 3 *continued*

- Step 1: $H_0: \mu_2 \leq \mu_1 \qquad H_1: \mu_2 > \mu_1$
- Step 2: Reject H_0 if z>2.33
- Step 3: $$z = \frac{30.4 - 25.6}{\sqrt{\frac{3.6^2}{45} + \frac{2.9^2}{40}}} = 6.80$$
- Step 4: Since z = 6.80>2.33, H_0 is rejected. Those retiring last year had more years of service.

Tests Concerning Proportion

- Proportion: A fraction or percentage that indicates the part of the population or sample having a particular trait of interest.
- The sample proportion is denoted by $\bar{p}$ where

$$\bar{p} = \frac{\text{number of successes in the sample}}{\text{number sampled}}$$

Test Statistic for Testing a Single Population Proportion

$$z = \frac{p - \pi}{\sqrt{\dfrac{\pi(1-\pi)}{n}}}$$

$$\pi \equiv \text{population proportion}$$

$$p \equiv \text{sample proportion}$$

EXAMPLE 4

- In the past, 15% of the mail order solicitations for a certain charity resulted in a financial contribution. A new solicitation letter that has been drafted is sent to a sample of 200 people and 45 responded with a contribution. At the .05 significance level can it be concluded that the new letter is more effective?

EXAMPLE 4 *continued*

- Step 1: $H_0: p \leq .15 \qquad H_1: p > .15$
- Step 2: H_0 is rejected if z>1.645
- Step 3:

$$z = \frac{\frac{45}{200} - .15}{\sqrt{\frac{(.15)(.85)}{200}}} = 2.97$$

- Step 4: Since z = 2.97 >1.645, H_0 is rejected. The new letter is more effective.

A Test Involving the Difference Between Two Population Proportions

- The test statistic in this case is :

$$z = \frac{p_1 - p_2}{\sqrt{\frac{p_c(1-p_c)}{n_1} + \frac{p_c(1-p_c)}{n_2}}}$$

A Test Involving the Difference Between Two Population Proportions *continued*

- $\bar{p}_c$ is the weighted mean of the two sample proportions, computed by:

$$\bar{p}_c = \frac{Total\ number\ of\ successes}{Total\ number\ in\ samples} = \frac{X_1 + X_2}{n_1 + n_2}$$

EXAMPLE 5

- Are unmarried workers more likely to be absent from work than married workers? A sample of 250 married workers showed 22 missed more than 5 days last year, while a sample of 300 unmarried workers showed 35 missed more than five days. Use a .05 significance level. *Note:* let pop #1= unmarried workers.

EXAMPLE 5 *continued*

- Step 1: $H_0: p_2 \le p_1 \qquad H_1: p_2 > p_1$
- Step 2: H_0 is rejected if z >1.645
- Step 3:

$$\overline{p} = \frac{22+35}{250+300} = .1036$$

$$z = \frac{.1167 - .0880}{\sqrt{\frac{.1036(1-.1036)}{300} + \frac{.1036(1-.1036)}{250}}}$$

EXAMPLE 5 *continued*

- Step 4: H_0 is not rejected. There is no difference in the proportion of married and unmarried workers missing more than 5 days of work.
- the p-Value = P{z>1.1} = .1357