Statistics 2
Chapter 14
Decision Making

Chapter 14

An Introduction to Decision Making

GOALS

When you have completed this chapter, you will be able to:

ONE

Define the terms: *state of nature, event, act,* and *payoff.*

TWO

Organize information into a payoff table or a decision tree.

THREE

Determine the expected payoff of an act.

FOUR

Compute opportunity loss and expected opportunity loss.

FIVE

Assess the value of information.

Statistical Decision Theory

- Classical statistics focuses on estimating a parameter, such as the population mean, constructing confidence intervals, or hypothesis testing.
- Statistical Decision Theory (Bayesian statistics) is concerned with determining which decision, from a set of possible decisions, is optimal for a particular set of decisions.

Elements of a Decision

- There are three components to any decision-making situation:
  - 1) the available choices (alternatives or acts)
  - 2) the states of nature, which are not under the control of the decision maker - uncontrollable future events
  - 3) the payoffs - needed for each combination of decision alternative and state of nature

Payoff Table and Expected Payoff

- A Payoff Table is a listing of all possible combinations of decision alternatives and states of nature.
- The *Expected Payoff* or the Expected Monetary Value (EMV) is the expected value of each decision criterion.

Calculating the EMV

- Let A_i be the i^{th} decision alternative.
- Let $P(S_j)$ be the probability of the j^{th} state of nature.
- Let $V(A_i, S_j)$ be the value of the payoff for the combination of decision alternative A_i and state of nature S_j.
- Let EMV (A_i) be the expected monetary value for the decision alternative A_i.

$$EMV(A_i) = \sum[P(S_j) \cdot V(A_i, S_j)]$$

EXAMPLE 1

- The following payoff table (profit) was developed. Let P(S1)=.5, P(S2)=.3, and P(S3)=.2. Compute the EMV for each of the alternatives.

Alternative	S1	S2	S3
A1	50	70	100
A2	40	80	90
A3	90	70	60

EXAMPLE 1 *continued*

- EMV (A1) =(.5)(50)+(.3)(70)+(.2)(100)=66
- EMV (A2) =(.5)(40)+(.3)(80)+(.2)(90)=62
- EMV (A3) =(.5)(90)+(.3)(70)+(.2)(60)=78
- What decision would you recommend?
- Choose alternative A3 since it gives the largest expected monetary value or expected payoff.

Opportunity Loss

- Opportunity Loss or *Regret* is the loss due to the fact that the exact state of nature is not known at the time a decision is made. An example would be the profit that may be lost by an investor in purchasing a stock when the market behavior is not known.
- The opportunity loss is computed by taking the difference between the optimal decision for each state of nature and the other decision alternatives.

EXAMPLE 1 *continued*

- OPPORTUNITY LOSS TABLE

Alternative	S1	S2	S3
A1	40	10	0
A2	50	0	10
A3	0	10	40

Expected Opportunity Loss

- Let A_i be the i^{th} decision alternative.
- Let $P(S_j)$ be the probability of the j^{th} state of nature.
- Let $R(A_i,S_j)$ be the value of the regret for the combination of decision alternative A_i and state of nature S_j.
- Let $EOL(A_i)$ be the expected opportunity loss for the decision alternative A_i.

$$EOL(A_i) = P(S_j)R(A_i, S_j)$$

EXAMPLE 1 *continued*

- EOL (A1) =(.5)(40)+(.3)(10)+(.2)(0)=23
- EOL (A2) =(.5)(50)+(.3)(0)+(.2)(10)=27
- EOL (A3) =(.5)(0)+(.3)(10)+(.2)(40)=11
- What decision would you make based on the lowest expected opportunity loss?
- Choose alternative A3 since it gives the smallest expected opportunity loss
- Note: This decision is the same when using the highest expected payoff. These two approaches will always lead to the same decision.

Maximin, Maximax, and Minimax Regret Strategies

- Maximin Strategy - maximizes the minimum gain (pessimistic strategy)
- Maximax Strategy - maximizes the maximum gain (optimistic strategy)
- Minimax Regret Strategy - minimizes the maximum opportunity loss

EXAMPLE 1 *continued*

- Under the maximin strategy, what profit are you expecting? From the initial payoff table, the profit will be $60.
- Under the maximax strategy, what profit are you expecting? From the initial payoff table, the profit will be $100.
- Under the minimax regret strategy, what will be your strategy? From the opportunity loss table, the strategy would be to select A1 or A3 since these minimize the maximum regret.

Value of Perfect Information

- What is the worth of information known in advance before a strategy is employed?
- Expected Value of Perfect Information (EVPI) is the difference between the expected payoff if the state of nature were known and the optimal decision under the conditions of uncertainty.
- From EXAMPLE 1, EVPI= [(.5)(90) + (.3)(80) + (.2)(100)]-78 = 11

Sensitivity Analysis and Decision Trees

- Sensitivity Analysis examines the effects of various probabilities for the states of nature on the expected values for the decision alternatives.
- Decision Trees are useful for structuring the various alternatives. They present a picture of the various courses of action and the possible states of nature.