MANAGERIAL ECONOMIC CHAPTER 7

Production Economics

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Production Economics

Chapter 7

- Managers must decide not only **what** to produce for the market, but also **how** to produce it in the *most efficient* or *least cost* manner.
- Economics offers a widely accepted tool for judging whether or not the production choices are least cost.
- A *production function* relates the most that can be produced from a given set of inputs. This allows the manager to measure the marginal product of each input.

1. Production Economics: In the Short Run

- Short Run Production Functions:
  - Max output, from **a n y** set of inputs
  - Q = f (X1, X2, X3, X4, ...)

FIXED IN SR VARIABLE IN SR

–

Q = f (K, L) for **two** input case, where K as **Fixed**

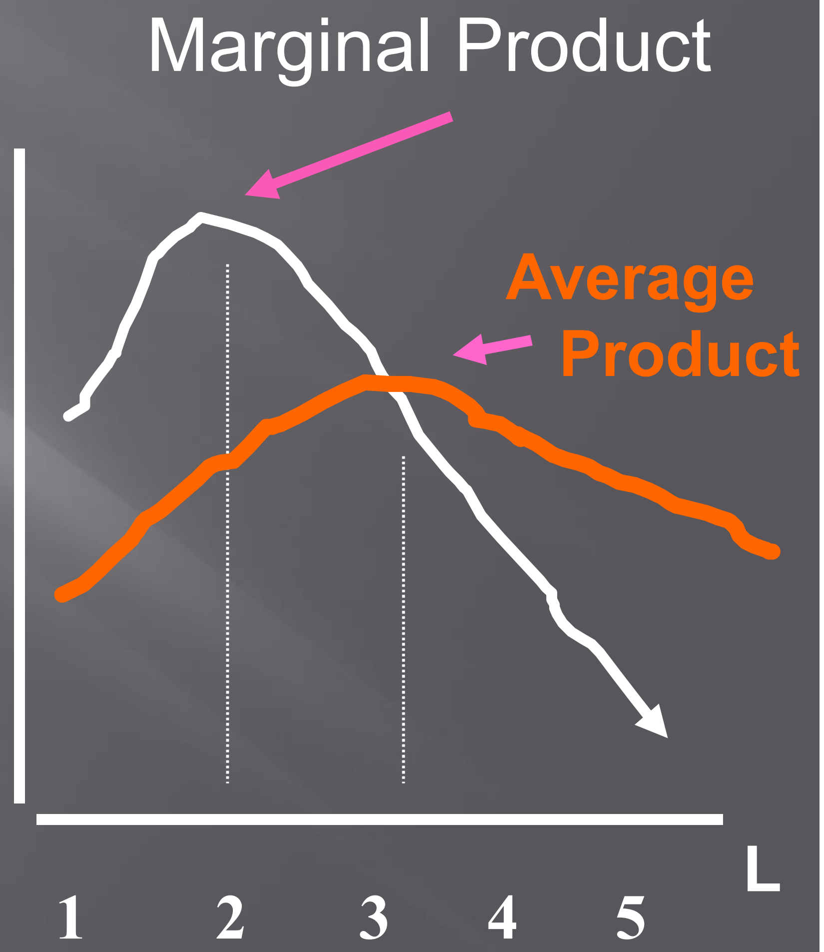
- **Average Product** = Q / L
  - output per labor
- **Marginal Product** = ∂ Q / ∂ L = dQ / dL
  - output attributable to last unit of labor applied
- Similar to profit functions, the Peak of MP occurs before the Peak of average product
- When MP = AP, we're at the peak of the AP curve

Production Elasticities

- The *production elasticity* for any input, X, $E_X = MP_X / AP_X = (\Delta Q/\Delta X) / (Q/X) = (\Delta Q/\Delta X) \cdot (X/Q)$, which is identical in form to other elasticities.
- When $MP_L > AP_L$, then the labor elasticity, $E_L > 1$. A 1 percent increase in labor will increase output by **more** than 1 percent.
- When $MP_L < AP_L$, then the labor elasticity, $E_L < 1$. A 1 percent increase in labor will increase output by **less** than 1 percent.

Short Run Production Function Numerical Example

L	Q	MP	AP
0	0	—	—
1	20	20	20
2	46	**26**	23
3	70	24	**23.33**
4	92	22	23
5	110	18	22



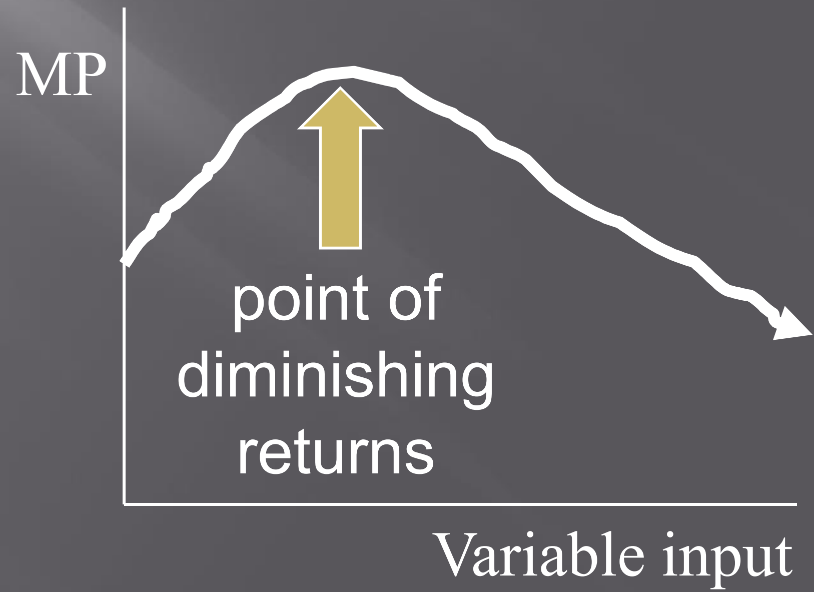
Labor Elasticity is greater then one, for labor use up through L = 3 units

- **When MP > AP, then AP is RISING**
  - IF YOUR MARGINAL GRADE IN THIS CLASS IS HIGHER THAN YOUR AVERAGE GRADE POINT AVERAGE, THEN YOUR G.P.A. IS RISING
- **When MP < AP, then AP is FALLING**
  - IF THE MARGINAL WEIGHT ADDED TO A TEAM IS LESS THAN THE AVERAGE WEIGHT, THEN AVERAGE TEAM WEIGHT DECLINES
- **When MP = AP, then AP is at its MAX**
  - IF THE NEW HIRE IS JUST AS EFFICIENT AS THE AVERAGE EMPLOYEE, THEN AVERAGE PRODUCTIVITY DOESN'T CHANGE

Law of Diminishing Returns

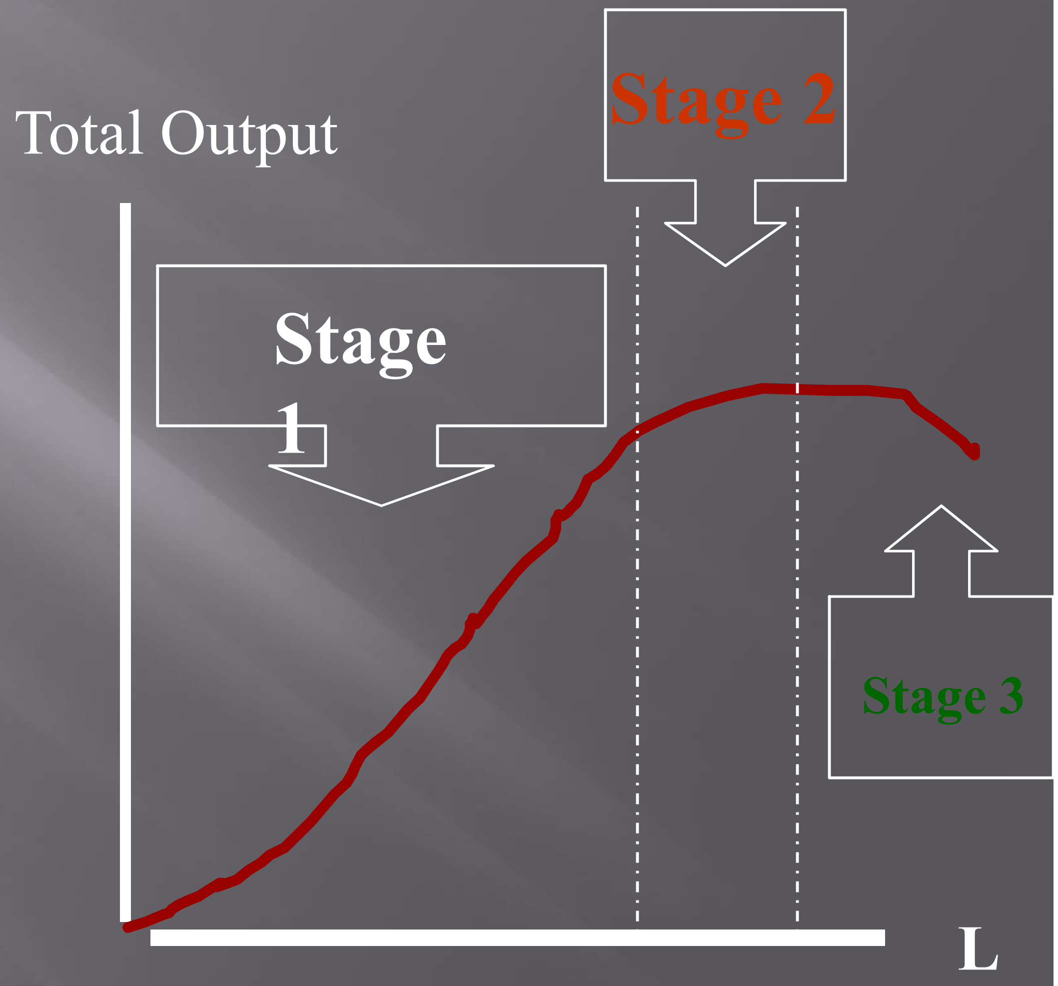
INCREASES IN ONE FACTOR OF PRODUCTION,
HOLDING ONE OR OTHER FACTORS FIXED,
AFTER SOME POINT,
MARGINAL PRODUCT DIMINISHES.

A SHORT RUN LAW



Three stages of production

- **Stage 1**: average product rising.
- **Stage 2**: average product declining (but marginal product positive).
- **Stage 3:** marginal product is negative, or total product is declining.



Optimal Employment of a Factor

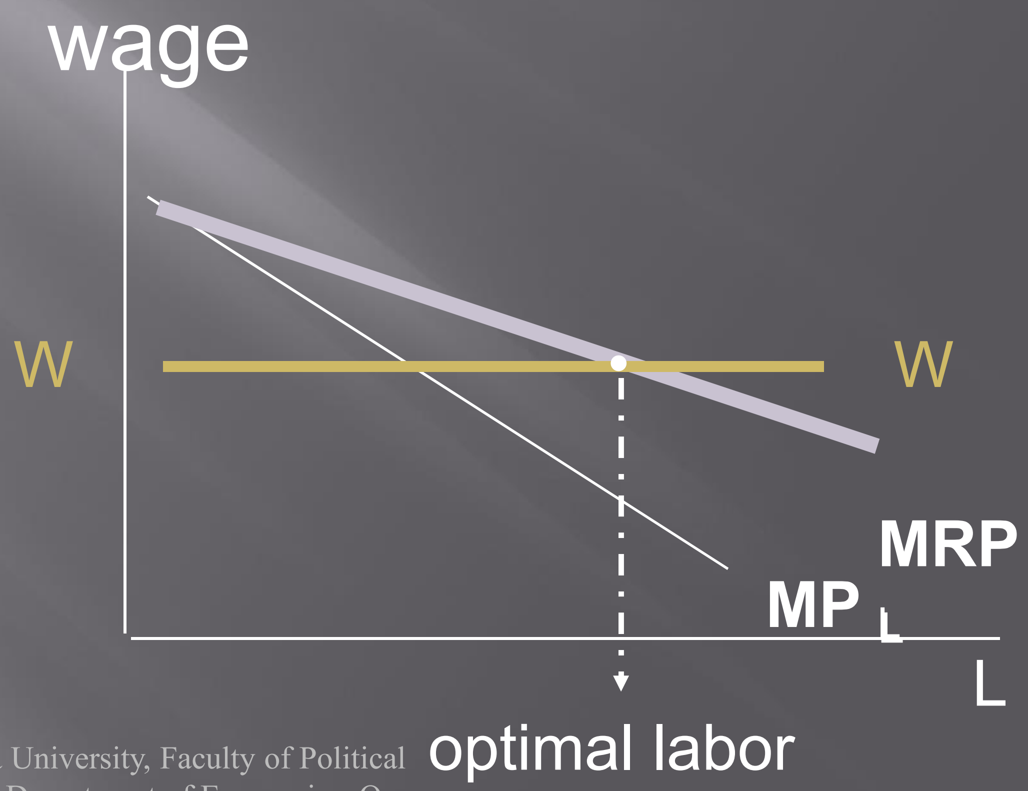
- HIRE, IF GET MORE **REVENUE** THAN COST
- HIRE if

  $\Delta TR/\Delta L > \Delta TC/\Delta L$
- HIRE if

  $MRP_L > MFC_L$
- AT OPTIMUM,

  $$MRP_L = W$$

$$MRP_L \equiv MP_L \bullet P_Q = W$$



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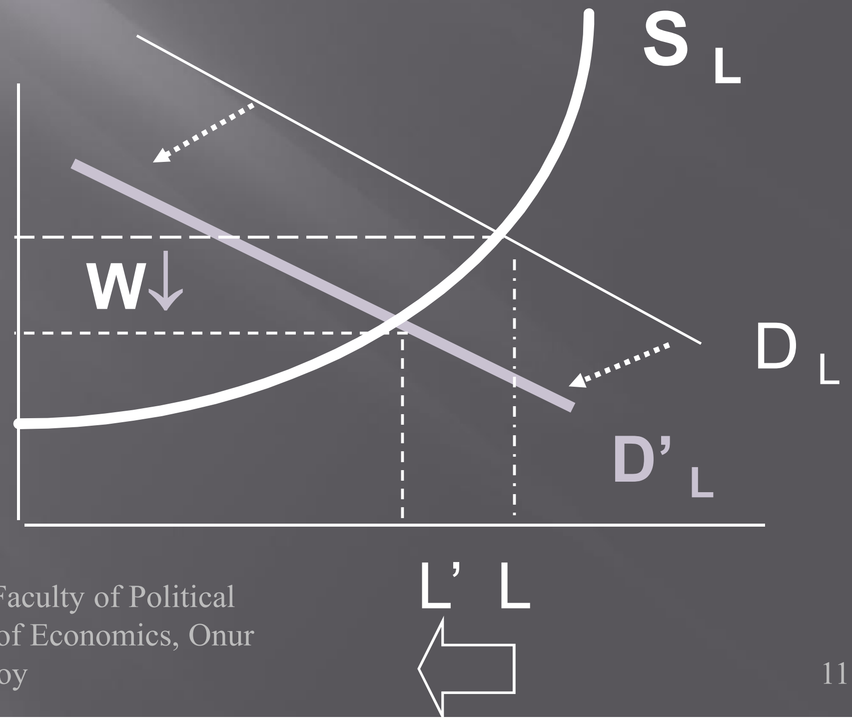
MRP_L is the Demand for Labor

- If Labor is MORE productive, demand for labor increases
- If Labor is LESS productive, demand for labor decreases
- Suppose an EARTHQUAKE destroys capital→

- MP_L declines with less capital, wages and labor are HURT



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2. Long Run Production Functions

- All inputs are variable
  - greatest output from any set of inputs
- Q = f(K, L) is two input example
- MP of capital and MP of labor are the derivatives of the production function
  - $MP_L = \partial Q / \partial L = \Delta Q / \Delta L$
- MP of labor declines as more labor is applied. Also MP of capital declines as more capital is applied.

Homogeneous Functions of Degree n

- A function is homogeneous of degree-n
  - if multiplying all inputs by λ, increases the dependent variable by λ^{n}
  - $Q = f(K, L)$
  - So, $f(\lambda K, \lambda L) = \lambda^{n} \cdot Q$
- Homogenous of degree 1 is CRS.
- Cobb-Douglas Production Functions are homogeneous of degree $\alpha + \beta$

Cobb-Douglas Production Functions:

- $Q = A \cdot K^{\alpha} \cdot L^{\beta}$ is a Cobb-Douglas Production Function
- **IMPLIES:**
  - **Can be IRS, DRS or CRS:**

    if $\alpha + \beta = 1$, then CRS

    if $\alpha + \beta < 1$, then DRS

    if $\alpha + \beta > 1$, then IRS
- **Coefficients are elasticities**

  α is the capital elasticity of output

  β is the labor elasticity of output,

  which are E_K and E_L

Problem

Suppose: $Q = 1.4\, L^{.70} K^{.35}$

- Is the function homogeneous?
- Is the production function constant returns to scale?
- What is the labor elasticity of output?
- What is the capital elasticity of output?
- What happens to Q, if L increases 3% and capital is cut 10%?

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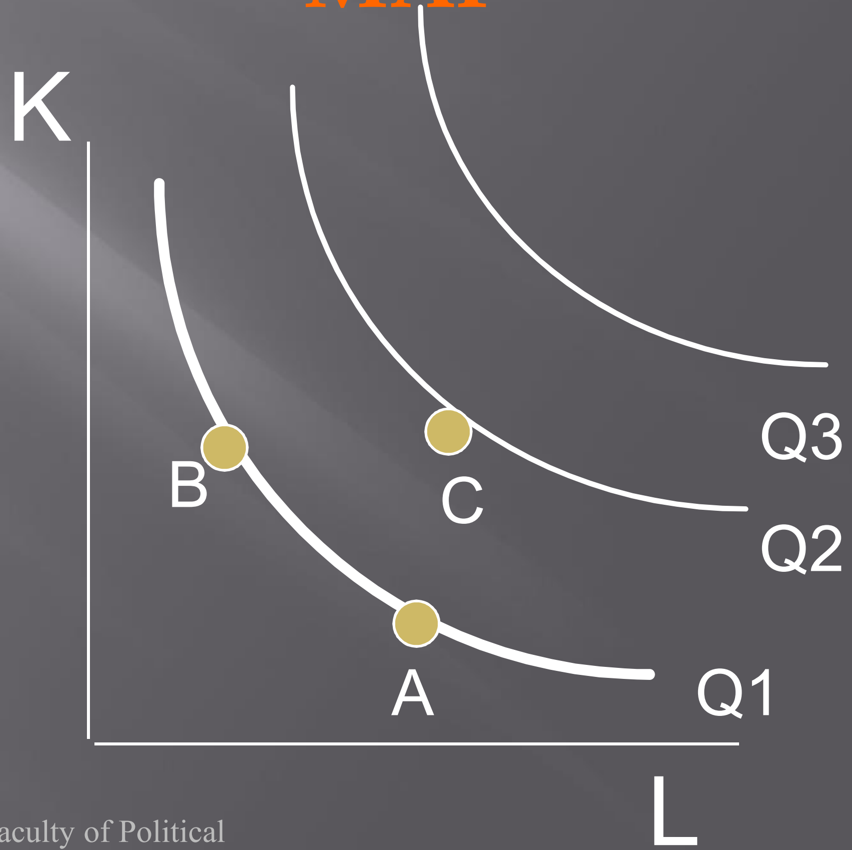
Answers

- Increases in all inputs by λ, increase output by $\lambda^{1.05}$
- Increasing Returns to Scale
- .70
- .35
- $\%\Delta Q = E_{QL} \bullet \%\Delta L + E_{QK} \bullet \%\Delta K$ = .7(+3%) + .35(-10%) = 2.1% -3.5% = **-1.4%**

Isoquants & LR Production Functions

- In the LONG RUN, ALL factors are variable
- Q = f (K, L)
- ISOQUANTS -- locus of input combinations which produces the same output
- SLOPE of ISOQUANT is ratio of Marginal Products

ISOQUANT MAP

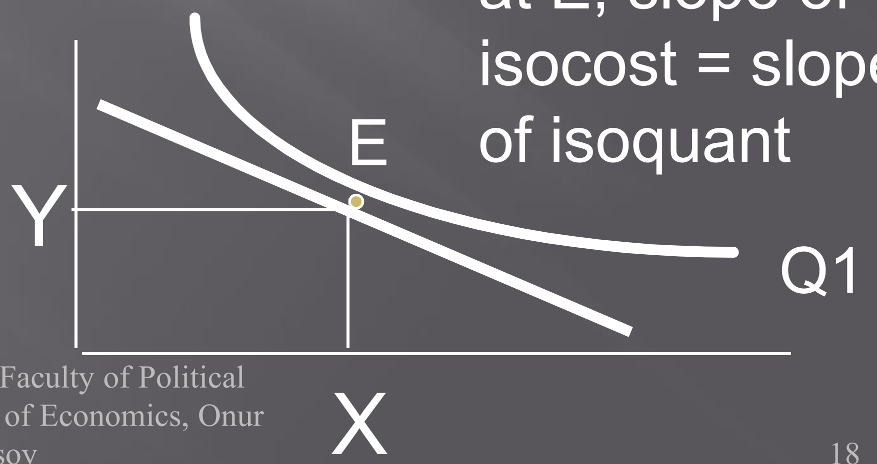


Optimal Input Combinations
in the Long Run

- The Objective is to Minimize Cost for a given Output
- **ISOCOST** lines are the combination of inputs for a given cost
- $C_0 = C_X \cdot X + C_Y \cdot Y$
- $Y = C_0/C_Y - (C_X/C_Y) \cdot X$

- ***Equimarginal Criterion*** Produce where $MP_X/C_X = MP_Y/C_Y$ where marginal products per dollar are equal

at E, slope of isocost = slope of isoquant

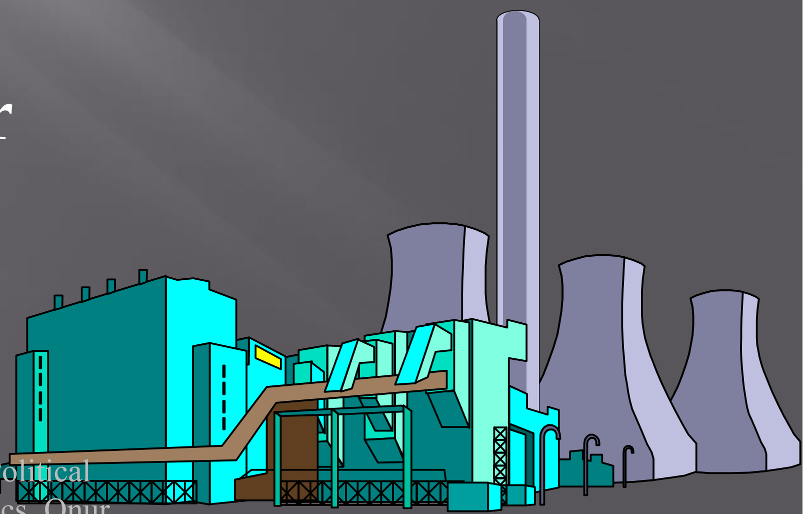


Use of the Efficiency Criterion

- Is the following firm EFFICIENT?
- Suppose that:
  - $MP_L = 30$
  - $MP_K = 50$
  - W = 10 (cost of labor)
  - R = 25 (cost of capital)
- Labor: 30/10 = 3
- Capital 50/25 = 2
- A dollar spent on labor produces 3, and a dollar spent on capital produces 2.
- USE RELATIVELY MORE LABOR
- If spend $1 less in capital, output falls 2 units, but rises 3 units when spent on labor

What Went Wrong With Large-Scale Electrical Generating Plants?

- Large electrical plants had cost advantages in the 1970s and 1980s because of economies of scale
- Competition and purchased power led to an era of deregulation
- Less capital-intensive generating plants appear now to be cheapest



Economies of Scale

- CONSTANT RETURNS TO SCALE (CRS)
  - doubling of all inputs doubles output
- INCREASING RETURNS TO SCALE (IRS)
  - doubling of all inputs MORE than doubles output
- DECREASING RETURNS TO SCALE (DRS)
  - doubling of all inputs DOESN'T QUITE double output

REASONS FOR
Increasing Returns to Scale

- *Specialization in the use of capital and labor*. Labor becomes more skilled at tasks, or the equipment is more specialized, less "a jack of all trades," as scale increases.

- Other advantages include: avoid inherent lumpiness in the size of equipment, quantity discounts, technical efficiencies in building larger volume equipment.

REASONS FOR

DECREASING RETURNS TO SCALE

- ***Problems of coordination and control*** as it is hard to send and receive information as the **scale rises.**
- **Other disadvantages of large size:**
  - slow decision ladder
  - inflexibility
  - capacity limitations on entrepreneurial skills (there are diminishing returns to the C.E.O. which cannot be completely delegated).

Economies of Scope

- FOR MULTI-PRODUCT FIRMS, **COMPLEMENTARY** IN PRODUCTION MAY CREATE SYNERGIES
  - especially common in Vertical Integration of firms
- $TC(Q_1 + Q_2) < TC(Q_1) + TC(Q_2)$

Chemical firm

+

Petroleum firm

= Cost Efficiencies

Statistical Estimation of LR Production Functions

Choice of data sets

- **cross section**
  - output and input measures from a group of firms
  - output and input measures from a group of plants
- **time series**
  - output and input data for a firm over time

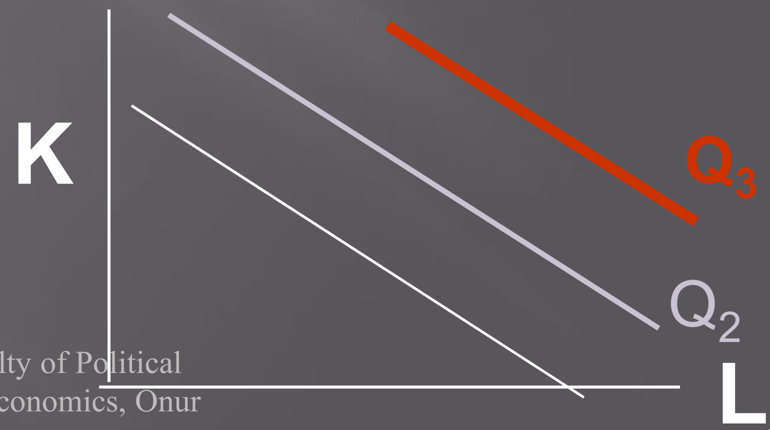
Estimation Complexities

Industries vary -- hence, the appropriate variables for estimation are industry-specific

- single product firms *vs.* multi-product firms
- multi-plant firms
- services *vs.* manufacturing
- measurable output (goods) *vs* non-measurable output (customer satisfaction)

Choice of Functional Form

- Linear ? $Q = a \cdot K + b \cdot L$
  - is CRS
  - marginal product of labor is **constant**, $MP_L = b$
  - can produce with zero labor or zero capital
  - isoquants are straight lines -- perfect **substitutes** in production



- **Multiplicative** -- Cobb Douglas Production Function

$$Q = A \cdot K^{\alpha} \cdot L^{\beta}$$

- IMPLIES
  - Can be CRS, IRS, or DRS
  - $MP_L = \beta \cdot \mathbf{Q/L}$
  - $MP_K = \alpha \cdot \mathbf{Q/K}$
  - Cannot produce with zero L or zero K
  - Log linear -- double log

$$\ln Q = a + \alpha \cdot \ln K + \beta \cdot \ln L$$

  - coefficients are elasticities

CASE: Wilson Company

pages 315-316

- Data on 15 plants that produce fertilizer
  - what sort of data set is this?
  - what functional form should we try?
- Determine if IRS, DRS, or CRS
- Test if coefficients are statistically significant
- Determine labor and capital production elasticities **and** give an economic interpretation of each value

	Output	Capital	Labor	Ln-Output	Ln-Cap	Ln-labor
1	605.3	18891	700.2	6.40572	9.8464	6.55137
2	566.1	19201	651.8	6.33877	9.8627	6.47974
3	647.1	20655	822.9	6.47250	9.9357	6.71283
4	523.7	15082	650.3	6.26092	9.6213	6.47743
5	712.3	20300	859.0	6.56850	9.9184	6.75577
6	487.5	16079	613.0	6.18929	9.6853	6.41837
7	761.6	24194	851.3	6.63542	10.0939	6.74676
8	442.5	11504	655.4	6.09244	9.3505	6.48525
9	821.1	25970	900.6	6.71064	10.1647	6.80306
10	397.8	10127	550.4	5.98595	9.2230	6.31065
11	896.7	25622	842.2	6.79872	10.1512	6.73602
12	359.3	12477	540.5	5.88416	9.4316	6.29249
13	979.1	24002	949.4	6.88663	10.0859	6.85583
14	331.7	8042	575.7	5.80423	8.9924	6.35559
15	1064.9	23972	925.8	6.97064	10.0846	6.83066

Data Set: 15 plants

The linear regression equation is

Output = - 351 + 0.0127 Capital + 1.02 Labor

Predictor	Coef	Stdev	t-ratio	p
Constant	-350.5	123.0	-2.85	0.015
Capital	.012725	.007646	1.66	0.122
Labor	1.0227	0.3134	3.26	0.007

s = 73.63 R-sq = **91.1%** R-sq(adj) = 89.6%

The double-linear regression equation is

LnOutput = - 4.75 + 0.415 Ln-Capital + 1.08 Ln-Labor

Predictor	**Coeff**	**Stdev**	**t-ratio**	**p**
Constant	-4.7547	0.8058	-5.90	0.000
Ln-Capital	0.4152	0.1345	3.09	0.009
Ln-Labor	1.0780	0.2493	4.32	0.001

s = 0.08966 R-sq = **94.8%** R-sq(adj) = 94.0%

Which form fits better--linear or double log?

Are the coefficients significant?

What is the labor and capital elasticities of output?

More Problems

Suppose the following production function is estimated to be:

$$\ln Q = 2.33 + .19 \ln K + .87 \ln L$$

$$R^2 = .97$$

QUESTIONS:

1. Is this constant returns to scale?
2. If L increases 2% what happens to output?
3. What's the MP_L at L = 50, K = 100, & Q = 741

Answers

1.) Take the sum of the coefficients
.19 + .87 = ***1.06*** , which shows
that this production function is **Increasing Returns to Scale**

2.) **Use the Labor Elasticity of Output**
$\%\Delta Q = E_L \cdot \%\Delta L$
$\%\Delta Q = (.87) \cdot (+2\%) = +1.74\%$

3). $MP_L = b\ Q/L = .87 \cdot (741 / 50) = 12.893$

Electrical Generating Capacity

- A cross section of 20 electrical utilities (standard errors in parentheses):
- **Ln Q = -1.54 + .53 Ln K + .65 Ln L**
  (.65) (.12) (.14) **$R^2 = .966$**
- Does this appear to be constant returns to scale?
- If increase labor 10%, what happens to electrical output?

Answers

- No, constant returns to scale. Of course, its increasing returns to scale as sum of coefficients exceeds one.
  - .53 + .65 = 1.18
- If $\%\Delta L = 10\%$, then $\%\Delta Q = E_L \bullet \%\Delta L = .65(10\%) = 6.5\%$

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Lagrangians and Output Maximization: *Appendix 7A*

- Max output to a cost objective. Let r be the cost of capital and w the cost of labor
- Max $\mathcal{L} = A \bullet K^{\alpha} \bullet L^{\beta} - \lambda \{ w \bullet L + r \bullet K - C \}$

$$\left.\begin{array}{l} \mathcal{L}_K: \alpha \bullet A \bullet K^{\alpha-1} \bullet L^{\beta} - r \bullet \lambda = 0 \\ \mathcal{L}_L: \beta \bullet A \bullet K^{\alpha} \bullet L^{\beta-1} - w \bullet \lambda = 0 \end{array}\right\} \quad \begin{array}{l} MP_K = r \\ \sout{MP_L = w} \end{array}$$

$$\mathcal{L}_{\lambda}: C - w \bullet L - r \bullet K = 0$$

- Solution $\alpha\, Q/K \big/ \beta\, Q/L = w / r$
- or $MP_K / r = MP_L / w$

Production and Linear Programming: *Appendix 7B*

- Manufacturers have alternative production processes, some involving mostly labor, others using machinery more intensively.
- The objective is to maximize output from these production processes, given **constraints** on the inputs available, such as **plant capacity** or union labor contract constraints.
- The linear programming techniques are discussed in Web Chapter B.