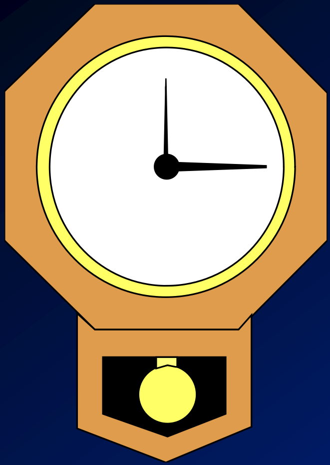
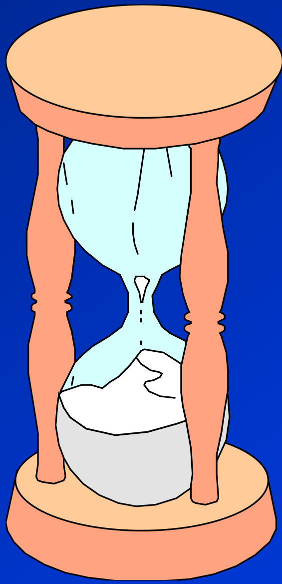
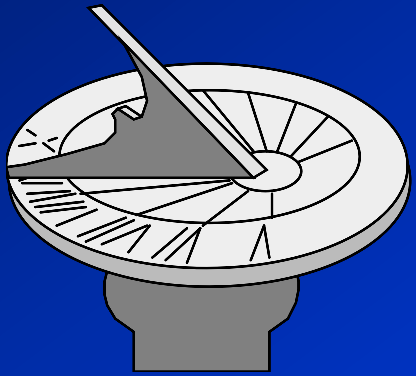
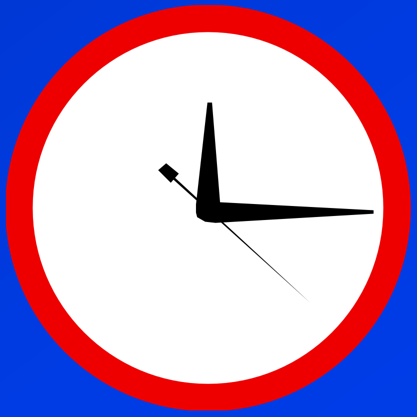




MONEY-TIME RELATIONSHIPS AND EQUIVALENCE





$ $ $ MONEY $ $ $

- **<u>Medium of Exchange</u>** --

  Means of payment for goods or services;

  What sellers accept and buyers pay ;

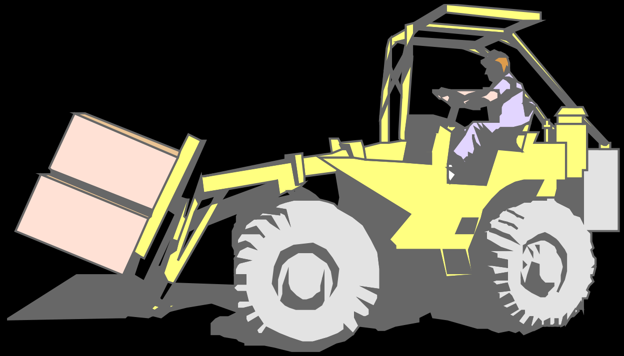
- **<u>Store of Value</u>** --

  A way to transport buying power from one time period to another;

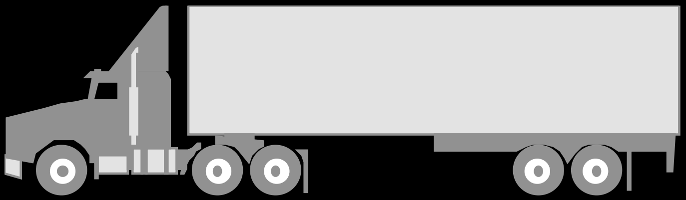
- **<u>Unit of Account</u>** --

  A precise measurement of value or worth;

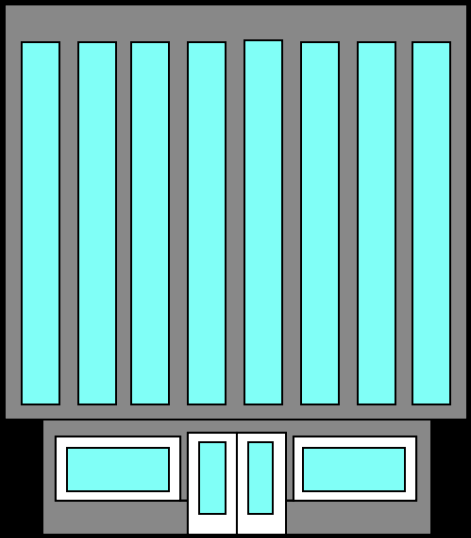
  Allows for tabulating debits and credits;

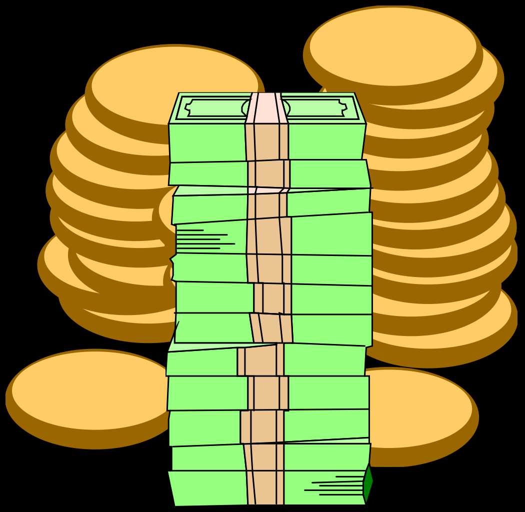


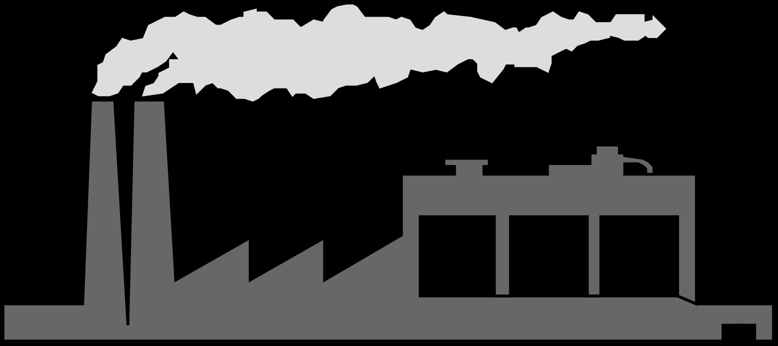
CAPITAL



Wealth in the form of money or property that can be used to produce more wealth.







KINDS OF CAPITAL

- Equity capital is that owned by individuals who have invested their money or property in a business project or venture in the hope of receiving a profit.
- Debt capital, often called borrowed capital, is obtained from lenders (e.g., through the sale of bonds) for investment.

Financing	Definition	Instrument	Description

Financing	Definition	Instrument	Description
• Debt financing	• Borrow money	• Bond	• Promise to pay principle & interest;

Financing	Definition	Instrument	Description
• Debt financing	• Borrow money	• Bond	• Promise to pay principle & interest;
• Equity financing	• Sell partial ownership of company;	• Stock	Exchange money for shares of stock as proof of partial ownership

INTEREST

The fee that a borrower pays to a lender for the use of his or her money.

INTEREST RATE

The percentage of money being borrowed that is paid to the lender on some time basis.

HOW INTEREST RATE IS DETERMINED

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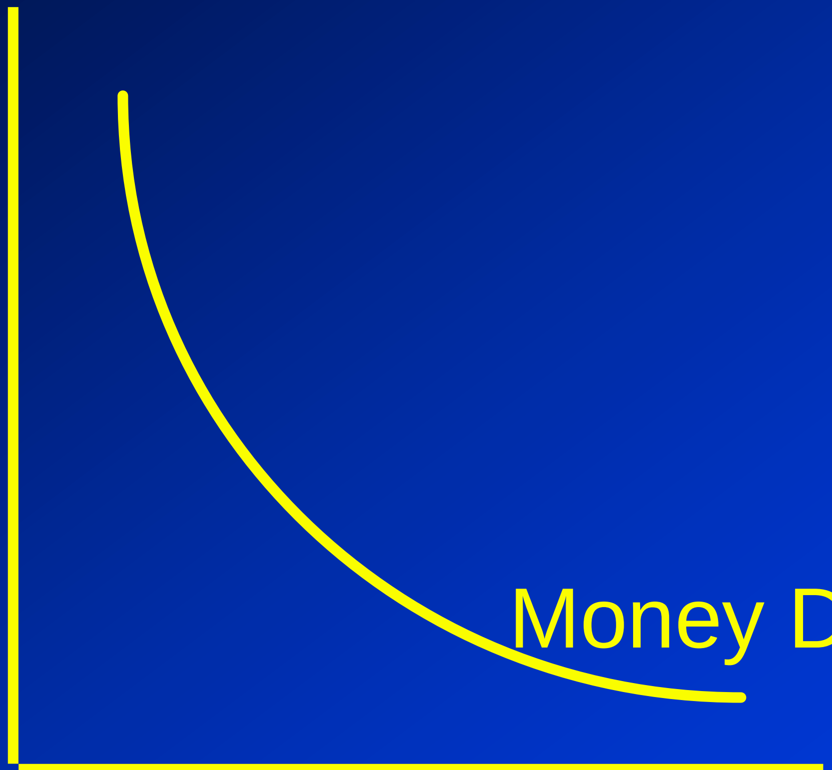
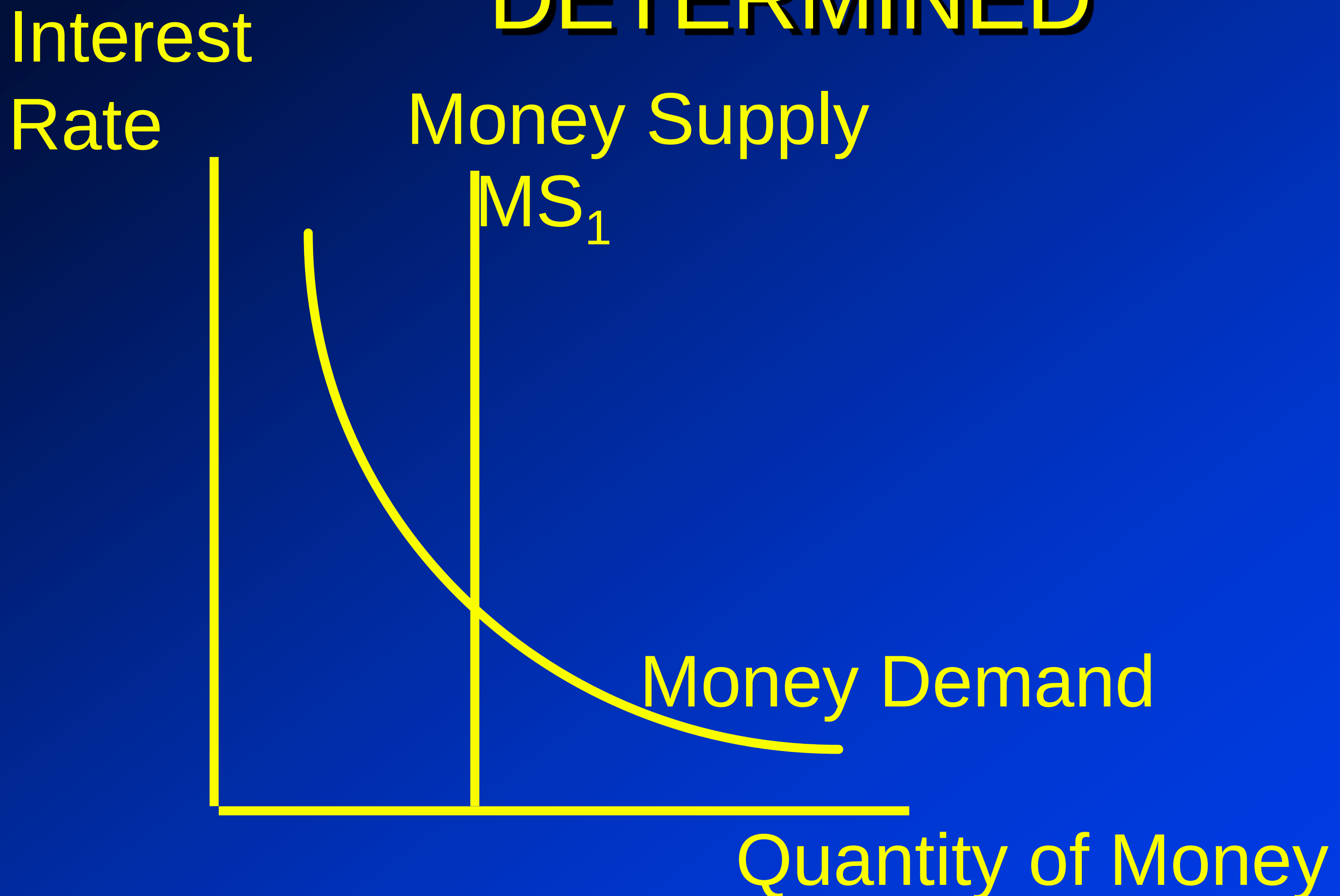


HOW INTEREST RATE IS DETERMINED



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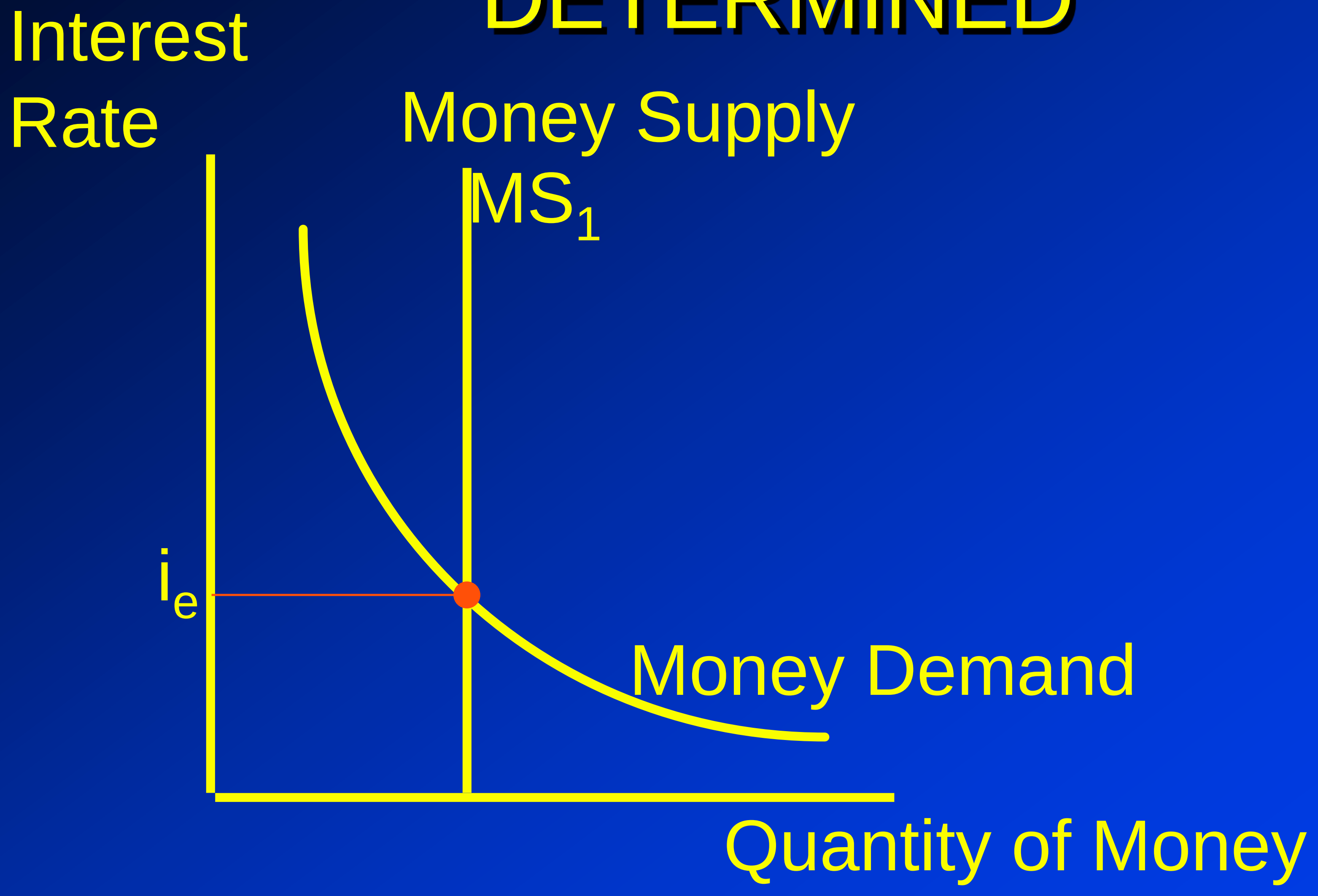

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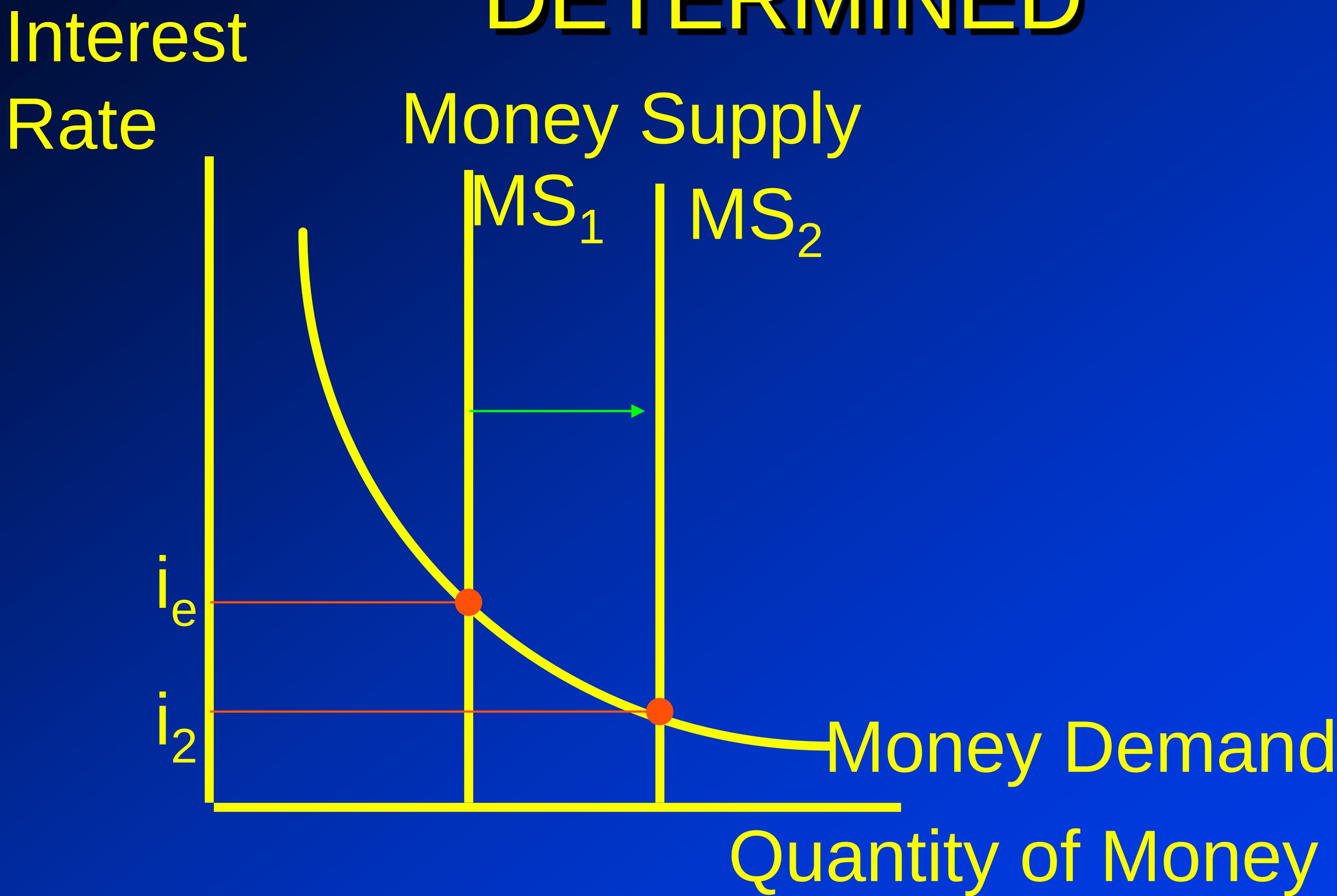

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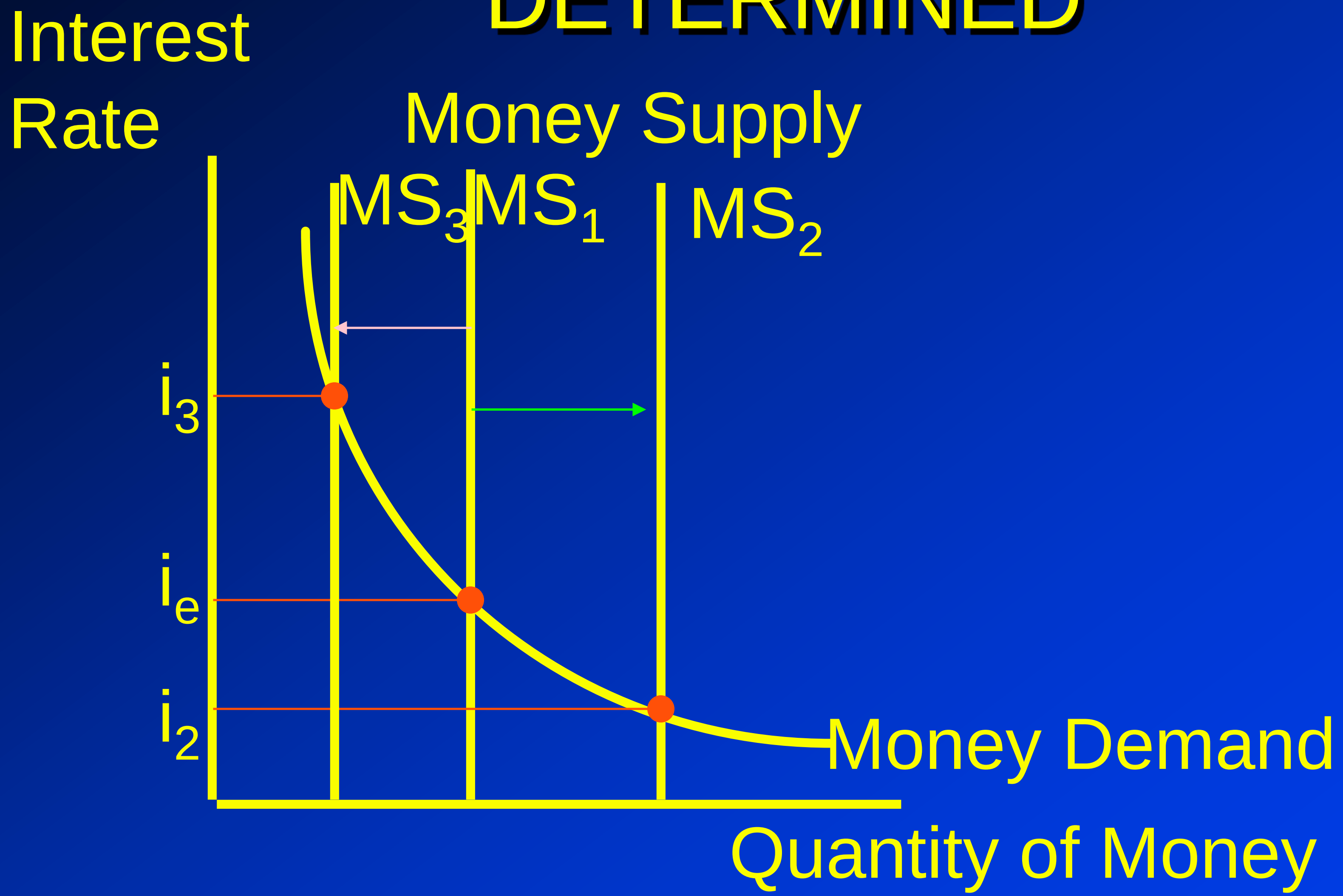


SIMPLE INTEREST

- The total interest earned or charged is linearly proportional to the initial amount of the loan (principal), the interest rate and the number of interest periods for which the principal is committed.
- When applied, total interest "I" may be found by

$$I = (P) (N) (i), \text{ where}$$

  – P = principal amount lent or borrowed
  – N = number of interest periods (e.g., years)
  – i = interest rate per interest period

COMPOUND INTEREST

- Whenever the interest charge for any interest period is based on the remaining principal amount plus any accumulated interest charges up to the *beginning* of that period.

Period	Amount Owed Beginning of period	Interest Amount for Period (@ 10%)	Amount Owed at end of period
1	$1,000	$100	$1,100
2	$1,100	$110	$1,210
3	$1,210	$121	$1,331

ECONOMIC EQUIVALENCE

- Established when we are <u>indifferent</u> between a future payment, or a series of future payments, and a present sum of money .
- Considers the comparison of alternative options, or proposals, by reducing them to an equivalent basis, depending on:
  - interest rate;
  - amounts of money involved;
  - timing of the affected monetary receipts and/or expenditures;
  - manner in which the interest , or profit on invested capital is paid and the initial capital is recovered.

ECONOMIC EQUIVALENCE FOR FOUR REPAYMENT PLANS OF AN $8,000 LOAN

- Plan #1: $2,000 of loan principal plus 10% of BOY principal paid at the end of year; interest paid at the end of each year is reduced by $200 (i.e., 10% of remaining principal)

Year	Amount Owed at beginning of Year (BOY)	Interest Accrued for Year	Total Money owed at end of Year	Principal Payment	Total end of Year Payment
1	$8,000	$800	$8,800	$2,000	$2,800
2	$6,000	$600	$6,600	$2,000	$2,600
3	$4,000	$400	$4,400	$2,000	$2,400
4	$2,000	$200	$2,200	$2,000	$2,200

Total interest paid ($2,000) is 10% of total dollar-years ($20,000)

ECONOMIC EQUIVALENCE FOR FOUR REPAYMENT PLANS OF AN $8,000 LOAN

- Plan #2: $0 of loan principal paid until end of fourth year; $800 interest paid at the end of each year

Year	Amount Owed at beginning of Year (BOY)	Interest Accrued for Year	Total Money owed at end of Year	Principal Payment	Total end of Year Payment
1	$8,000	$800	$8,800	$0	$800
2	$8,000	$800	$8,800	$0	$800
3	$8,000	$800	$8,800	$0	$800
4	$8,000	$800	$8,800	$8,000	$8,800

Total interest paid ($3,200) is 10% of total dollar-years ($32,000)

ECONOMIC EQUIVALENCE FOR FOUR REPAYMENT PLANS OF AN $8,000 LOAN

- Plan #3: $2,524 paid at the end of each year; interest paid at the end of each year is 10% of amount owed at the beginning of the year.

Year	Amount Owed at beginning of Year (BOY)	Interest Accrued for Year	Total Money owed at end of Year	Principal Payment	Total end of Year Payment
1	$8,000	$800	$8,800	$1,724	$2,524
2	$6,276	$628	$6,904	$1,896	$2,524
3	$4,380	$438	$4,818	$2,086	$2,524
4	$2,294	$230	$2,524	$2,294	$2,524

Total interest paid ($2,096) is 10% of total dollar-years ($20,950)

ECONOMIC EQUIVALENCE FOR FOUR REPAYMENT PLANS OF AN $8,000 LOAN

- Plan #4: No interest and no principal paid for first three years. At the end of the fourth year, the original principal plus accumulated (compounded) interest is paid.

Year	Amount Owed at beginning of Year (BOY)	Interest Accrued for Year	Total Money owed at end of Year	Principal Payment	Total end of Year Payment
1	$8,000	$800	$8,800	$0	$0
2	$8,800	$880	$9,680	$0	$0
3	$9,680	$968	$10,648	$0	$0
4	$10,648	$1,065	$11,713	$8,000	$11,713

Total interest paid ($3,713) is 10% of total dollar-years ($37,128)

CASH FLOW DIAGRAMS / TABLE NOTATION

i = effective interest rate per interest period

N = number of compounding periods (e.g., years)

P = present sum of money; the equivalent value of one or more cash flows at the present time reference point

F = future sum of money; the equivalent value of one or more cash flows at a future time reference point

A = end-of-period cash flows (or equivalent end-of-period values) in a uniform series continuing for a specified number of periods, starting at the end of the first period and continuing through the last period

G = uniform gradient amounts -- used if cash flows increase by a constant amount in each period

CASH FLOW DIAGRAM NOTATION

1

1 2 3 4 5 = N

1

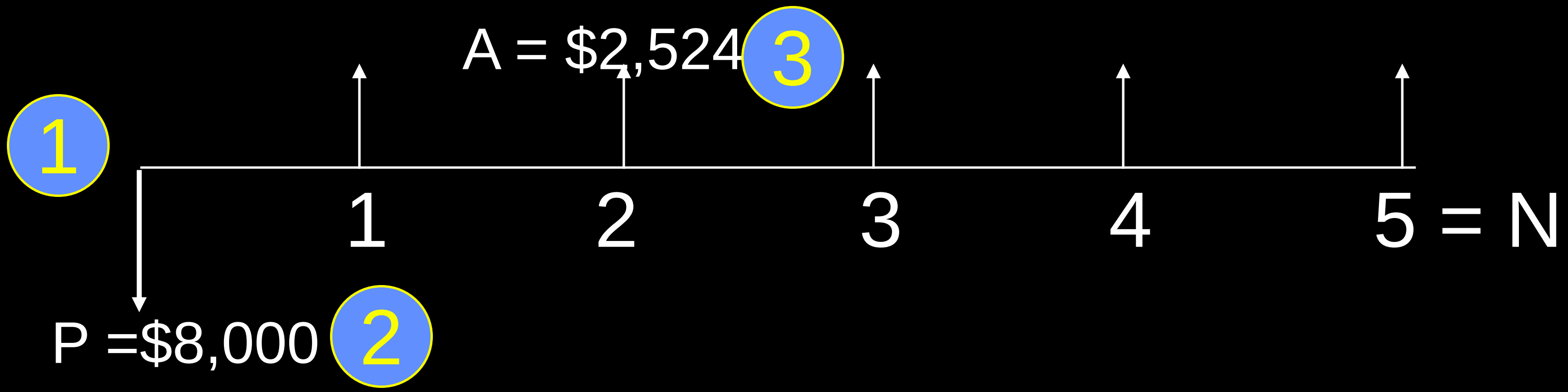
Time scale with progression of time moving from left to right; the numbers represent time periods (e.g., years, months, quarters, etc...) and may be presented within a time interval or at the end of a time interval.

CASH FLOW DIAGRAM NOTATION

1 Time scale with progression of time moving from left to right; the numbers represent time periods (e.g., years, months, quarters, etc...) and may be presented within a time interval or at the end of a time interval.

2 Present expense (cash outflow) of $8,000 for lender.

CASH FLOW DIAGRAM NOTATION



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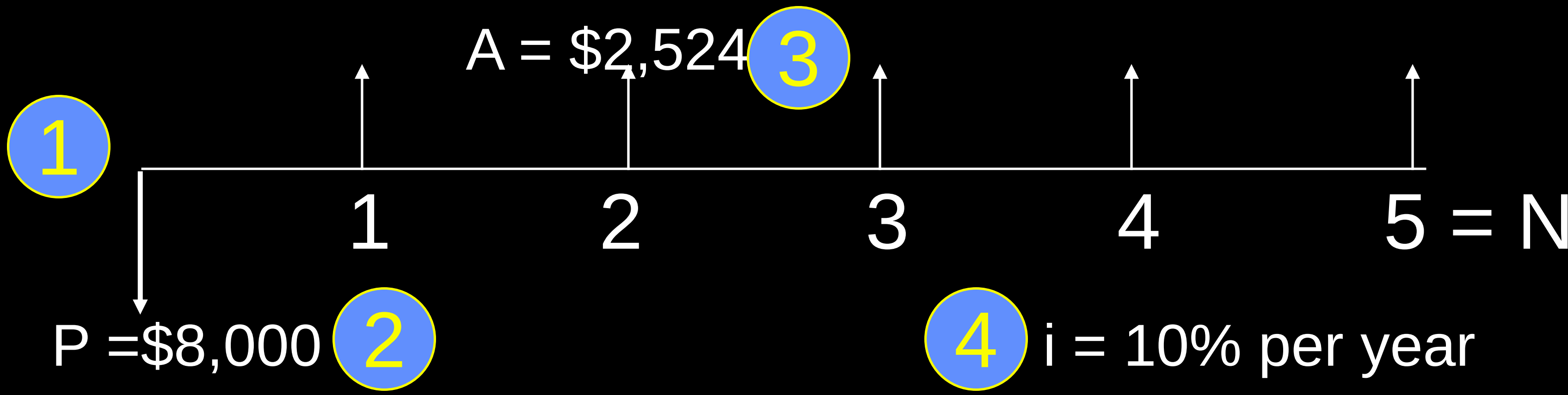
CASH FLOW DIAGRAM NOTATION



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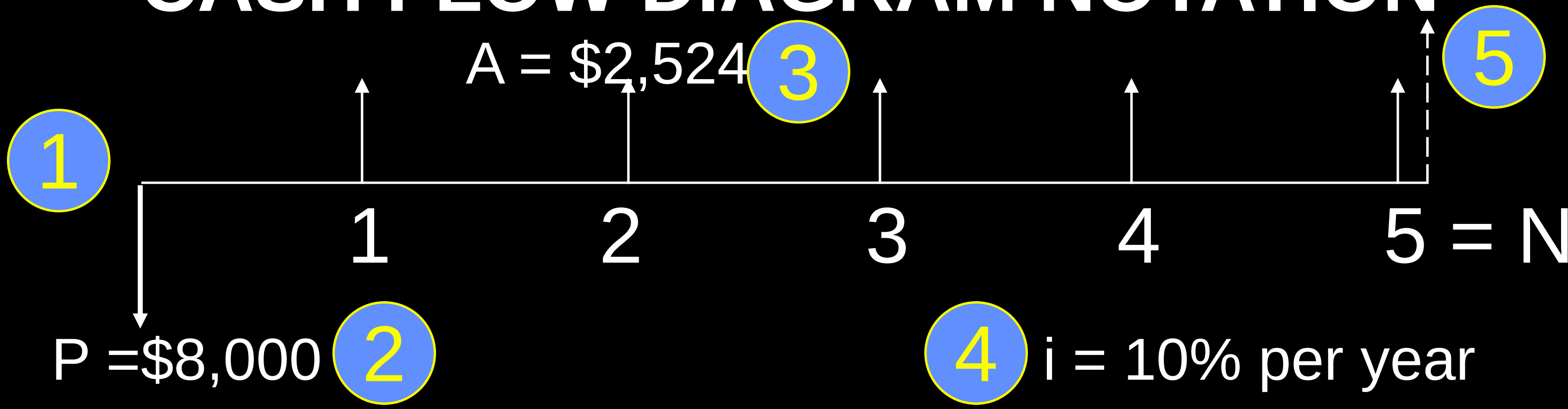
4. Interest rate of loan.

CASH FLOW DIAGRAM NOTATION



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2 Present expense (cash outflow) of $8,000 for lender.

3 Annual income (cash inflow) of $2,524 for lender.

4 Interest rate of loan.

5 Dashed-arrow line indicates amount to be determined.

INTEREST FORMULAS FOR ALL OCCASIONS

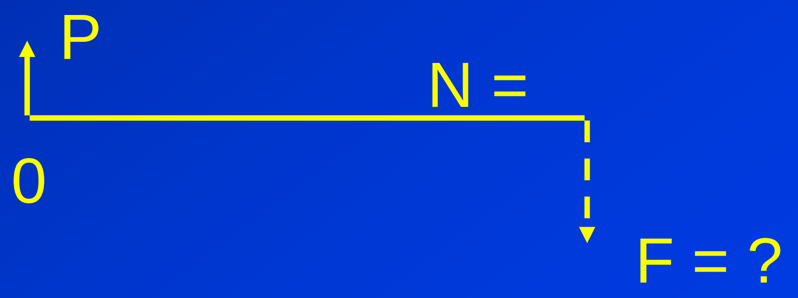
- relating present and future values of single cash flows;
- relating a uniform series (annuity) to present and future equivalent values;
  - for discrete compounding and discrete cash flows;
  - for deferred annuities (uniform series);
- equivalence calculations involving multiple interest;
- relating a uniform gradient of cash flows to annual and present equivalents;
- relating a geometric sequence of cash flows to present and annual equivalents;

INTEREST FORMULAS FOR ALL OCCASIONS

- relating nominal and effective interest rates;
- relating to compounding more frequently than once a year;
- relating to cash flows occurring less often than compounding periods;
- for continuous compounding and discrete cash flows;
- for continuous compounding and continuous cash flows;

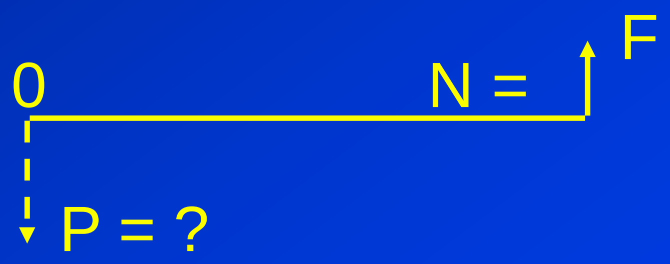
RELATING PRESENT AND FUTURE EQUIVALENT VALUES OF <u>SINGLE CASH FLOWS</u>

- Finding F when given P:
- Finding future value when given present value
- $F = P(1+i)^N$
  - $(1+i)^N$ single payment compound amount factor
  - functionally expressed as $F = (F/P, i\%, N)$
  - predetermined values of this are presented in column 2 of Appendix C of text.



RELATING PRESENT AND FUTURE EQUIVALENT VALUES OF SINGLE CASH FLOWS

- Finding P when given F:
- Finding present value when given future value
- $P = F\,[1 / (1 + i)]^{N}$
  - $(1+i)^{-N}$ single payment present worth factor
  - functionally expressed as $P = F\,(P / F, i\%, N)$
  - predetermined values of this are presented in column 3 of Appendix C of text;



RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding F given A:

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding F given A:
- Finding future equivalent income (inflow) value given a series of uniform equal Payments

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding F given A:
- Finding future equivalent income (inflow) value given a series of uniform equal Payments
- $$F = A \left[\frac{(1+i)^N - 1}{i} \right]$$

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding F given A:
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- $$F = A\left[\frac{(1+i)^N - 1}{i}\right]$$
  - – uniform series compound amount factor in [ ]

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

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- $$F = A \left[\frac{(1+i)^N - 1}{i} \right]$$
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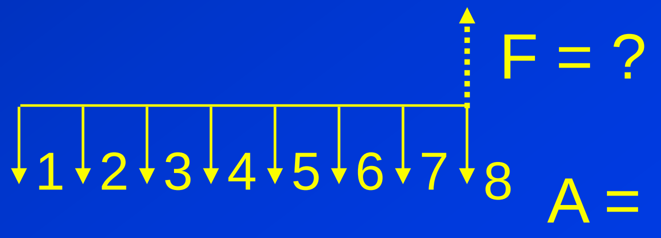


$$(F/A,i\%,N) = (P/A,i,N)(F/P,i,N)$$

$$(F/A,i\%,N) = \sum_{k=1}^{N} (F/P,i,N-k)$$

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding P given A:

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding P given A:
- Finding present equivalent value given a series of uniform equal receipts

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding P given A:
- Finding present equivalent value given a series of uniform equal receipts
- $$P = A \left[\frac{(1+i)^N - 1}{i(1+i)^N} \right]$$

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding P given A:
- Finding present equivalent value given a series of uniform equal receipts
- $$P = A \left[\frac{(1+i)^N - 1}{i(1+i)^N} \right]$$
  - uniform series present worth factor in [ ]

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

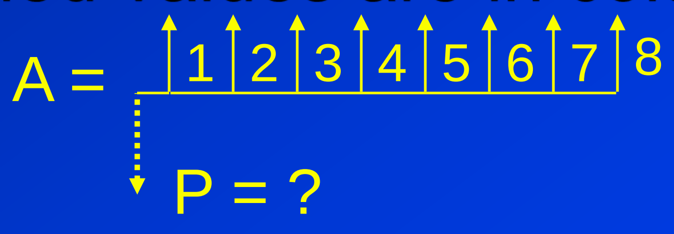
- Finding P given A:
- Finding present equivalent value given a series of uniform equal receipts
- $$P = A \left[\frac{(1+i)^N - 1}{i(1+i)^N} \right]$$
  - uniform series present worth factor in [ ]
  - functionally expressed as P = A (P / A,i%,N)

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding P given A:
- Finding present equivalent value given a series of uniform equal receipts
- $$P = A \left[\frac{(1+i)^N - 1}{i(1+i)^N} \right]$$
  - uniform series present worth factor in [ ]
  - functionally expressed as P = A (P / A,i%,N)
  - predetermined values are in column 5 of Appendix C of text

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding P given A:
- Finding present equivalent value given a series of uniform equal receipts
- $$P = A \left[\frac{(1+i)^N - 1}{i(1+i)^N} \right]$$
  - uniform series present worth factor in [ ]
  - functionally expressed as P = A (P / A,i%,N)
  - predetermined values are in column 5 of Appendix C of text



$$(P/A, i\%, N) = \sum_{k=1}^{N} (P/F, i, k)$$

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding A given F:

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding A given F:
- Finding amount A of a uniform series when given the equivalent future value

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding A given F:
- Finding amount A of a uniform series when given the equivalent future value

$$A = F \left[\frac{i}{(1+i)^N - 1} \right]$$

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding A given F:
- Finding amount A of a uniform series when given the equivalent future value

$$A = F \left[\frac{i}{(1+i)^N - 1} \right]$$

  – sinking fund factor in [ ]

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding A given F:
- Finding amount A of a uniform series when given the equivalent future value

$$A = F \left[\frac{i}{(1+i)^N - 1} \right]$$

  - – sinking fund factor in [ ]
  - – functionally expressed as A = F (A / F,i%,N)

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding A given F:
- Finding amount A of a uniform series when given the equivalent future value

$$A = F \left[\frac{i}{(1+i)^N - 1} \right]$$

  - sinking fund factor in [ ]
  - functionally expressed as A = F (A / F,i%,N)
  - predetermined values are in column 6 of Appendix C of text

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding A given F:
- Finding amount A of a uniform series when given the equivalent future value

$$A = F \left[\frac{i}{(1+i)^{N} - 1} \right]$$

  - sinking fund factor in [ ]
  - functionally expressed as A = F (A / F,i%,N)
  - predetermined values are in column 6 of Appendix C of text

$$(A/F, i\%, N) = 1/(F/A, i\%, N)$$

$$(A/F, i\%, N) = (A/P, i\%, N) - i$$

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding A given P:

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding A given P:
- Finding amount A of a uniform series when given the equivalent present value

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding A given P:
- Finding amount A of a uniform series when given the equivalent present value

$$A = P \left[\frac{i\,(1+i)^N}{(1+i)^N - 1} \right]$$

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding A given P:
- Finding amount A of a uniform series when given the equivalent present value

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  – capital recovery factor in [ ]

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding A given P:
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  - capital recovery factor in [ ]
  - functionally expressed as A = P (A / P,i%,N)

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding A given P:
- Finding amount A of a uniform series when given the equivalent present value

$$A = P \left[\frac{i\,(1+i)^N}{(1+i)^N - 1} \right]$$

  - capital recovery factor in [ ]
  - functionally expressed as A = P (A / P,i%,N)
  - predetermined values are in column 7 of Appendix C of text

RELATING A UNIFORM SERIES (ORDINARY ANNUITY) TO PRESENT AND FUTURE EQUIVALENT VALUES

- Finding A given P:
- Finding amount A of a uniform series when given the equivalent present value

$$A = P\left[\frac{i\,(1+i)^N}{(1+i)^N - 1}\right]$$

  - capital recovery factor in [ ]
  - functionally expressed as A = P (A / P,i%,N)
  - predetermined values are in column 7 of Appendix C of text

$$(A / P,i\%,N) = 1 / (P / A,i\%,N)$$

RELATING A UNIFORM SERIES (DEFERRED ANNUITY) TO PRESENT / FUTURE EQUIVALENT VALUES

- If an annuity is deferred *j* periods, where $j < N$
- And finding P given A for an ordinary annuity is expressed by:

$$P = A\,(P / A, i\%, N)$$

- This is expressed for a deferred annuity by:

$A\,(P / A, i\%, N - j)$ at end of period *j*

- This is expressed for a deferred annuity by:

$A\,(P / A, i\%, N - j)\,(P / F, i\%, j)$

as of time 0 (time present)