EQUIVALENCE CALCULATIONS INVOLVING MULTIPLE INTEREST

- All compounding of interest takes place once per time period (e.g., a year), and to this point cash flows also occur once per time period.
- Consider an example where a series of cash outflows occur over a number of years.
- Consider that the value of the outflows is unique for each of a number (i.e., first three) years.
- Consider that the value of outflows is the same for the last four years.
- Find a) the present equivalent expenditure; b) the future equivalent expenditure; and c) the annual equivalent expenditure

PRESENT EQUIVALENT EXPENDITURE

- Use P_0 = F(P / F, i%, N) for each of the unique years:

  -- F is a series of unique outflow for year 1 through year 3;

  -- i is common for each calculation;

  -- N is the year in which the outflow occurred;

  -- Multiply the outflow times the associated table value;

  -- Add the three products together;

- Use A (P / A,i%,N - *j*) (P / F, i%, *j*) -- deferred annuity -- for the remaining (common outflow) years:

  -- A is common for years 4 through 7;

  -- i remains the same;

  -- N is the final year;

  -- j is the last year a unique outflow occurred;

  -- multiply the common outflow value times table values;

  -- add this to the previous total for the present equivalent expenditure.

RELATING A UNIFORM GRADIENT OF CASH FLOWS TO FUTURE EQUIVALENTS

- Find F when given G:

RELATING A UNIFORM GRADIENT OF CASH FLOWS TO FUTURE EQUIVALENTS

- Find F when given G:
- Find the future equivalent value when given the uniform gradient amount

RELATING A UNIFORM GRADIENT OF CASH FLOWS TO FUTURE EQUIVALENTS

- Find F when given G:
- Find the future equivalent value when given the uniform gradient amount
- $$F = G\left[\frac{(1+i)^{N-1}-1}{i} + \frac{(1+i)^{N-2}-1}{i} + \ldots + \frac{(1+i)^{1}-1}{i}\right]$$

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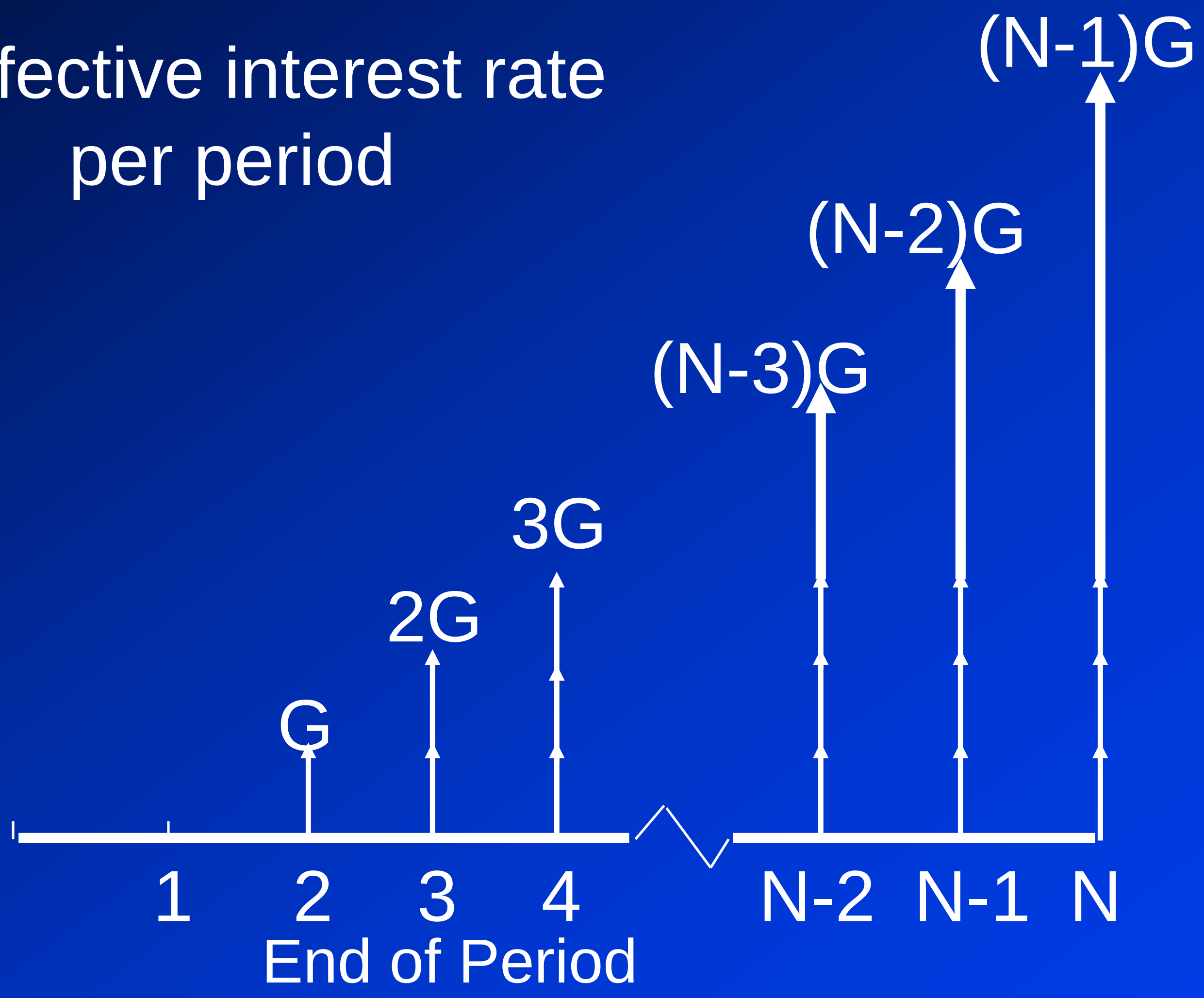
- Functionally represented as (G/ i) (F/A,i%,N) - (NG/ i)
- Usually more practical to deal with annual and present equivalents, rather than future equivalent values

Cash Flow Diagram for a Uniform Gradient Increasing by G Dollars per period

i = effective interest rate per period



RELATING A UNIFORM GRADIENT OF CASH FLOWS TO ANNUAL AND PRESENT EQUIVALENTS

- Find A when given G:

RELATING A UNIFORM GRADIENT OF CASH FLOWS TO ANNUAL AND PRESENT EQUIVALENTS

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RELATING A UNIFORM GRADIENT OF CASH FLOWS TO ANNUAL AND PRESENT EQUIVALENTS

- Find A when given G:
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- $$A = G \left[\frac{1}{i} - \frac{N}{(1 + i)^{N} - 1} \right]$$

RELATING A UNIFORM GRADIENT OF CASH FLOWS TO ANNUAL AND PRESENT EQUIVALENTS

- Find A when given G:
- Find the annual equivalent value when given the uniform gradient amount
- $$A = G \left[\frac{1}{i} - \frac{N}{(1+i)^N - 1} \right]$$
- Functionally represented as A = G (A / G, i%,N)

RELATING A UNIFORM GRADIENT OF CASH FLOWS TO ANNUAL AND PRESENT EQUIVALENTS

- Find A when given G:
- Find the annual equivalent value when given the uniform gradient amount
- $$A = G \left[\frac{1}{i} - \frac{N}{(1+i)^N - 1} \right]$$
- Functionally represented as A = G (A / G, i%,N)
- The value shown in [ ] is the gradient to uniform series conversion factor and is presented in column 9 of Appendix C (represented in the above parenthetical expression).

RELATING A UNIFORM GRADIENT OF CASH FLOWS TO ANNUAL AND PRESENT EQUIVALENTS

- Find P when given G:

RELATING A UNIFORM GRADIENT OF CASH FLOWS TO ANNUAL AND PRESENT EQUIVALENTS

- Find P when given G:
- Find the present equivalent value when given the uniform gradient amount

RELATING A UNIFORM GRADIENT OF CASH FLOWS TO ANNUAL AND PRESENT EQUIVALENTS

- Find P when given G:
- Find the present equivalent value when given the uniform gradient amount
- $$P = G\left\{\frac{1}{i}\left[\frac{(1+i)^N - 1}{i(1+i)^N} - \frac{N}{(1+i)^N}\right]\right\}$$

RELATING A UNIFORM GRADIENT OF CASH FLOWS TO ANNUAL AND PRESENT EQUIVALENTS

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- Functionally represented as P = G (P / G, i%,N)
- The value shown in{ } is the gradient to present equivalent conversion factor and is presented in column 8 of Appendix C (represented in the above parenthetical expression).

RELATING GE0METRIC SEQUENCE OF CASH FLOWS TO PRESENT AND ANNUAL EQUIVALENTS

- Projected cash flow patterns changing at an average rate of f each period;

RELATING GE0METRIC SEQUENCE OF CASH FLOWS TO PRESENT AND ANNUAL EQUIVALENTS

- Projected cash flow patterns changing at an average rate of *f* each period;
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- Projected cash flow patterns changing at an average rate of *f* each period;
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- $A_k = (A_{k-1})(1+f), 2 \le k \le N$

RELATING GE0METRIC SEQUENCE OF CASH FLOWS TO PRESENT AND ANNUAL EQUIVALENTS

- Projected cash flow patterns changing at an average rate of *f* each period;
- Resultant end-of-period cash-flow pattern is referred to as a <u>geometric gradient series</u>;
- A_1 is cash flow at end of period 1
- $A_k = (A_{k-1})(1 + f), 2 \leq k \leq N$
- $A_N = A_1 (1 + f)^{N-1}$

RELATING GE0METRIC SEQUENCE OF CASH FLOWS TO PRESENT AND ANNUAL EQUIVALENTS

- Projected cash flow patterns changing at an average rate of *f* each period;
- Resultant end-of-period cash-flow pattern is referred to as a geometric gradient series;
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- $A_N = A_1 (1 + f)^{N-1}$
- $f = (A_k - A_{k-1}) / A_{k-1}$

RELATING GE0METRIC SEQUENCE OF CASH FLOWS TO PRESENT AND ANNUAL EQUIVALENTS

- Projected cash flow patterns changing at an average rate of *f* each period;
- Resultant end-of-period cash-flow pattern is referred to as a geometric gradient series;
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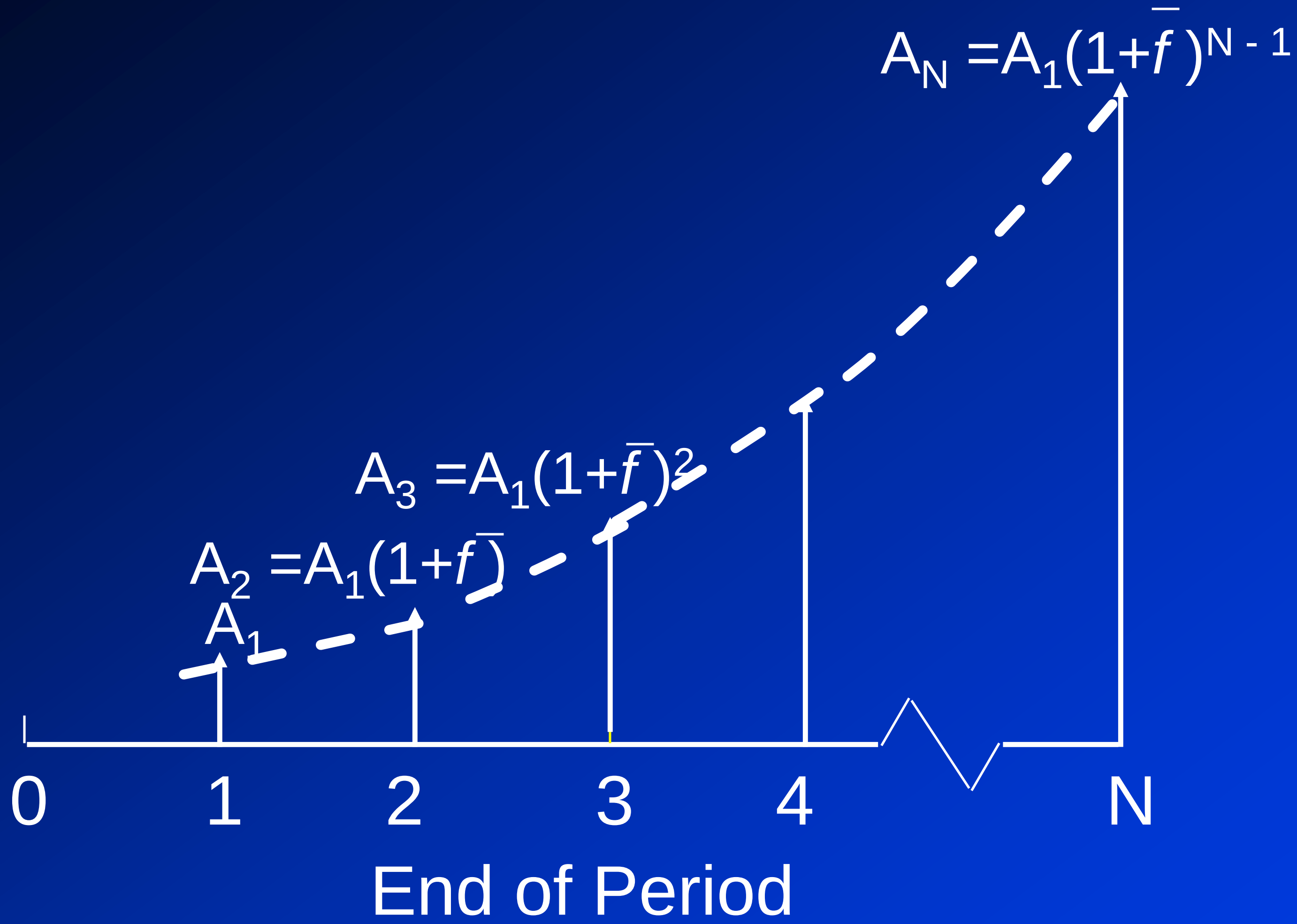
- $A_N = A_1 (1 + f)^{N-1}$
- $f = (A_k - A_{k-1}) / A_{k-1}$
- *f* may be either positive *or* negative



Cash-flow diagram for a Geometric Sequence of Cash Flows

RELATING A GEOMETRIC SEQUENCE OF CASH FLOWS TO ANNUAL AND PRESENT EQUIVALENTS

- Find P when given A:

RELATING A GEOMETRIC SEQUENCE OF CASH FLOWS TO ANNUAL AND PRESENT EQUIVALENTS

- Find P when given A:
- Find the present equivalent value when given the annual equivalent value ($i \neq \bar{f}$)

RELATING A GEOMETRIC SEQUENCE OF CASH FLOWS TO ANNUAL AND PRESENT EQUIVALENTS

- Find P when given A:
- Find the present equivalent value when given the annual equivalent value ($i \neq \bar{f}$)

$$P = \frac{A_1}{(1 + \bar{f})} \left(P / A, \frac{1 + i}{1 + \bar{f}} - 1, N \right)$$

RELATING A GEOMETRIC SEQUENCE OF CASH FLOWS TO ANNUAL AND PRESENT EQUIVALENTS

- Find P when given A:
- Find the present equivalent value when given the annual equivalent value ($i \neq \bar{f}$)

$$P = \frac{A_1[1 - (1+i)^{-N} (1+\bar{f})^{N}]}{1 - \bar{f}}$$

which may also be written as

$$P = \frac{A_1[1 - (P/F,i\%,N) (F/P,\bar{f}\%,N)]}{i - \bar{f}}$$

RELATING A GEOMETRIC SEQUENCE OF CASH FLOWS TO ANNUAL AND PRESENT EQUIVALENTS

- Note that the foregoing is mathematically equivalent to the following ($i \neq \bar{f}$):

$$P = \frac{A_1}{1+\bar{f}} \left(P/A \; \frac{1+i}{1+\bar{f}} - 1, N \right)$$

RELATING A GEOMETRIC SEQUENCE OF CASH FLOWS TO ANNUAL AND PRESENT EQUIVALENTS

- The foregoing may be functionally represented as A = P (A / P, i%,N)
- The year zero "base" of annuity, increasing at constant rate *f* % is A_0 = P (A / P, f %, N)
- The future equivalent of this geometric gradient is F = P (F / P, i%, N)

RELATING A GEOMETRIC SEQUENCE OF CASH FLOWS TO ANNUAL AND PRESENT EQUIVALENTS

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RELATING A GEOMETRIC SEQUENCE OF CASH FLOWS TO ANNUAL AND PRESENT EQUIVALENTS

- Find P when given A:
- Find the present equivalent value when given the annual equivalent value ($i = \bar{f}$)

$P = A_1 N (1+i)^{-1}$ *which may be written as*

RELATING A GEOMETRIC SEQUENCE OF CASH FLOWS TO ANNUAL AND PRESENT EQUIVALENTS

- Find P when given A:
- Find the present equivalent value when given the annual equivalent value ($i = \bar{f}$)

$$P = A_1 N\ (P/F, i\%, 1)$$

Functionally represented as $A = P\ (A / P,\ i\%, N\ )$

RELATING A GEOMETRIC SEQUENCE OF CASH FLOWS TO ANNUAL AND PRESENT EQUIVALENTS

- Find P when given A:
- Find the present equivalent value when given the annual equivalent value ($i = \bar{f}$)

  $P = A_1N$ (i+i)-1 *which may be written as*

  $P = A_1N$ (P/F,i%,1)

  Functionally represented as A = P (A / P, i%,N)
- The year zero “base” of annuity, increasing at constant rate *f* % is A_0 = P (A / P, f %, N)
- The future equivalent of this geometric gradient is F = P (F / P, i%, N)