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- Find the present equivalent value given a future value and a varying interest rate over the period of the loan
- $$P = \frac{F_N}{\prod_{k + 1}^{N} (1 + i_k)}$$

NOMINAL AND EFFECTIVE INTEREST RATES

- Nominal Interest Rate - r - For rates compounded more frequently than one year, the stated annual interest rate.
- Effective Interest Rate - i - For rates compounded more frequently than one year, the actual amount of interest paid.
- $i = (1 + r / M)^M - 1 = (F / P, r / M, M) - 1$
  – M - the number of compounding periods per year
- Annual Percentage Rate - APR - percentage rate per period times number of periods.
  – $APR = r \times M$

COMPOUNDING MORE OFTEN THAN ONCE A YEAR

Single Amounts

- Given nominal interest rate and total number of compounding periods, P, F or A can be determined by

$$F = P\ (\ F\ /\ P,\ i\%,\ N\ )$$

$$i\% = (\ 1 + r\ /\ M\ )^{M} - 1$$

Uniform and / or Gradient Series

- Given nominal interest rate, total number of compounding periods, and existence of a cash flow at the end of each period, P, F or A may be determined by the formulas and tables for uniform annual series and uniform gradient series.

CASH FLOWS LESS OFTEN THAN COMPOUNDING PERIODS

- Find A, given i, k and X, where:
  - i is the effective interest rate per interest period
  - k is the period at the end of which cash flow occurs
  - X is the uniform cash flow amount

  Use: A = X (A / F,i%, k)

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  one unit of principal = $[\,1 + (r\,/\,M\,)\,]^{M}$
- Given $M\,/\,r = p$, $[\,1 + (r\,/\,M\,)\,]^{M} = [1 + (1/p)\,]^{rp}$

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- Given $\lim_{p \to \infty} [1 + (1 / p)]^{p} = e^{1} = 2.71828$
- $(F / P, r\%, N) = e^{rN}$
- $i = e^{r} - 1$

CONTINUOUS COMPOUNDING AND DISCRETE CASH FLOWS

Single Cash Flow

- Finding F given P
- Finding future equivalent value given present value
- $F = P (e^{rN})$
- Functionally expressed as (F / P, r%, N)
- e^{rN} is continuous compounding compound amount
- Predetermined values are in column 2 of appendix D of text

CONTINUOUS COMPOUNDING AND DISCRETE CASH FLOWS

Single Cash Flow

- Finding P given F
- Finding present equivalent value given future value
- $P = F\,(e^{-rN})$
- Functionally expressed as (P / F, r%, N)
- e^{-rN} is continuous compounding present equivalent
- Predetermined values are in column 3 of appendix D of text

CONTINUOUS COMPOUNDING AND DISCRETE CASH FLOWS

Uniform Series

- Finding F given A
- Finding future equivalent value given a series of uniform equal receipts
- $F = A\,(e^{rN} - 1)/(e^{r} - 1)$
- Functionally expressed as (F / A, r%, N)
- $(e^{rN} - 1)/(e^{r} - 1)$ is continuous compounding compound amount
- Predetermined values are in column 4 of appendix D of text

CONTINUOUS COMPOUNDING AND DISCRETE CASH FLOWS

Uniform Series

- Finding P given A
- Finding present equivalent value given a series of uniform equal receipts
- $P = A (e^{rN} - 1) / (e^{rN}) (e^{r} - 1)$
- Functionally expressed as (P / A, r%, N)
- $(e^{rN} - 1) / (e^{rN}) (e^{r} - 1)$ is continuous compounding present equivalent
- Predetermined values are in column 5 of appendix D of text

CONTINUOUS COMPOUNDING AND DISCRETE CASH FLOWS

Uniform Series

- Finding A given F
- Finding a uniform series given a future value
- $A = F\,(e^{r} - 1) / (e^{rN} - 1)$
- Functionally expressed as (A / F, <u>r</u>%, N)
- $(e^{r} - 1) / (e^{rN} - 1)$ is continuous compounding sinking fund
- Predetermined values are in column 6 of appendix D of text

CONTINUOUS COMPOUNDING AND DISCRETE CASH FLOWS

Uniform Series

- Finding A given P
- Finding a series of uniform equal receipts given present equivalent value
- $A = P\ [e^{rN}\ (e^{r} - 1)\ /\ (e^{rN} - 1)\ ]$
- Functionally expressed as (A / P, r%, N)
- $[e^{rN}\ (e^{r} - 1)\ /\ (e^{rN} - 1)\ ]$ is continuous compounding capital recovery
- Predetermined values are in column 7 of appendix D of text

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$$P = \frac{[\,1 + (r / p)\,]^{p} - 1}{r\,[\,1 + (r / p)\,]^{p}}$$

- Given $\lim_{p \to \infty} [\,1 + (r / p)\,]^{p} = e^{r}$
- For one year (P / A, $\underline{r}$%, 1) = $(e^{r} - 1) / re^{r}$

CONTINUOUS COMPOUNDING AND CONTINUOUS CASH FLOWS

- Finding F given $\bar{A}$

CONTINUOUS COMPOUNDING AND CONTINUOUS CASH FLOWS

- Finding F given $\bar{A}$
- Finding the future equivalent given the continuous funds flow

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CONTINUOUS COMPOUNDING AND CONTINUOUS CASH FLOWS

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- Finding P given $\bar{A}$
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- $\left(e^{rN} - 1 \right) / re^{rN}$ is continuous compounding present equivalent
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CONTINUOUS COMPOUNDING AND CONTINUOUS CASH FLOWS

- Finding $\bar{A}$ given F

CONTINUOUS COMPOUNDING AND CONTINUOUS CASH FLOWS

- Finding $\bar{A}$ given F
- Finding the continuous funds flow given the future equivalent

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- Finding $\bar{A}$ given F
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- $\bar{A} = F\,[\,r\,/\,(\,e^{rN} - 1\,)]$

CONTINUOUS COMPOUNDING AND CONTINUOUS CASH FLOWS

- Finding $\bar{A}$ given F
- Finding the continuous funds flow given the future equivalent
- $\bar{A} = F\,[\,r\,/\,(\,e^{rN} - 1\,)]$
- Functionally expressed as ($\bar{A}$ / F, r%, N)

CONTINUOUS COMPOUNDING AND CONTINUOUS CASH FLOWS

- Finding $\bar{A}$ given F
- Finding the continuous funds flow given the future equivalent
- $\bar{A} = F\ [\ r\ /\ (\ e^{rN} - 1\ )]$
- Functionally expressed as ($\bar{A}$ / F, *r*%, N)
- r / (e^{rN} - 1) is continuous compounding sinking fund

CONTINUOUS COMPOUNDING AND CONTINUOUS CASH FLOWS

- Finding $\bar{A}$ given P

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- Finding the continuous funds flow given the present equivalent
- $\bar{A} = P \left[r / \left(e^{rN} - 1 \right) \right]$

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- Finding $\bar{A}$ given P
- Finding the continuous funds flow given the present equivalent
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CONTINUOUS COMPOUNDING AND CONTINUOUS CASH FLOWS

- Finding $\bar{A}$ given P
- Finding the continuous funds flow given the present equivalent
- $\bar{A} = F\ [\ re^{rN} / (\ e^{rN} - 1\ )]$
- Functionally expressed as ($\bar{A}$ / P, r%, N)
- $re^{rN} / (\ e^{rN} - 1\ )$ is continuous compounding capital recovery