APPLICATIONS OF MONEY-TIME RELATIONSHIPS

MINIMUM ATTRACTIVE RATE OF RETURN (MARR)

- An interest rate used to convert cash flows into equivalent worth at some point(s) in time
- Usually a policy issue based on:
  - amount, source and cost of money available for investment
  - number and purpose of good projects available for investment
  - amount of perceived risk of investment opportunities and estimated cost of administering projects over short and long run
  - type of organization involved
- MARR is sometimes referred to as hurdle rate

CAPITAL RATIONING

- MARR approach involving opportunity cost viewpoint
- Exists when management decides to restrict the total amount of capital invested, by desire or limit of available capital
- Select only those projects which provide annual rate of return in excess of MARR
- As amount of investment capital and opportunities available change over time, a firm’s MARR will also change

PRESENT WORTH METHOD (PW)

- Based on concept of equivalent worth of all cash flows relative to the present as a base
- All cash inflows and outflows discounted to present at interest -- generally MARR
- PW is a measure of how much money can be afforded for investment in excess of cost
- PW is positive if dollar amount received for investment exceeds minimum required by investors

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- The higher the interest rate and further into future a cash flow occurs, the lower its PW

BOND AS EXAMPLE OF PRESENT WORTH

- The value of a bond, at any time, is the present worth of future cash receipts from the bond
- The bond owner receives two types of payments from the borrower:

  -- periodic interest payments until the bond is retired (based on r);

  -- redemption or disposal payment when the bond is retired (based on *i*);
- The present worth of the bond is the sum of the present values of these two payments at the bond's yield rate

PRESENT WORTH OF A BOND

- For a bond, let

  Z = face, or par value

  C = redemption or disposal price (usually Z)

  r = bond rate (nominal interest) per interest period

  N = number of periods before redemption

  i = bond yield (redemption) rate per period

  V_N = value (price) of the bond N interest periods prior to redemption -- PW measure of merit

  VN = C (P / F, i%, N) + rZ (P / A, *i*%, N)

- Periodic interest payments to owner = rZ for N periods -- an annuity of N payments
- When bond is sold, receive single payment (C), based on the price and the bond yield rate (*i*)

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ANNUAL WORTH METHOD (AW)

- AW is an equal annual series of dollar amounts, over a stated period (N), equivalent to the cash inflows and outflows at interest rate that is generally MARR
- AW is annual equivalent revenues ($\underline{R}$) minus annual equivalent expenses ($\underline{E}$), less the annual equivalent capital recovery (CR)

$$AW\ (\ i\ \%\ ) = \underline{R} - \underline{E} - CR\ (\ i\ \%\ )$$

- AW = PW (A / P, *i* %, N)
- AW = FW (A / F, *i* %, N)
- If AW $\geq$ 0, project is economically attractive
- AW = 0 : annual return = MARR earned

CAPITAL RECOVERY (CR)

- CR is the equivalent uniform annual cost of the capital invested
- CR is an annual amount that covers:
  - Loss in value of the asset
  - Interest on invested capital (i.e., at the MARR)

$$CR(i\%) = I(A/P, i\%, N) - S(A/F, i\%, N)$$

I = initial investment for the project

S = salvage (market) value at the end of the study period

N = project study period

INTERNAL RATE OF RETURN METHOD (IRR)

- IRR solves for the interest rate that equates the equivalent worth of an alternative's cash inflows (receipts or savings) to the equivalent worth of cash outflows (expenditures)
- Also referred to as:
  - investor's method
  - discounted cash flow method
  - profitability index
- IRR is positive for a single alternative only if:
  - both receipts and expenses are present in the cash flow pattern
  - the sum of receipts exceeds sum of cash outflows

INTERNAL RATE OF RETURN METHOD (IRR)

- IRR is i' %, using the following PW formula:

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- If $i' \geq$ MARR, the alternative is acceptable
- To compute IRR for alternative, set net PW = 0

$$PW = \sum_{k=0}^{N} R_k \left(P / F, i' \%, k \right) - \sum_{k=0}^{N} E_k \left(P / F, i' \%, k \right) = 0$$

- i' is calculated on the beginning-of-year unrecovered investment through the life of a project

INTERNAL RATE OF RETURN PROBLEMS

- The IRR method assumes recovered funds, if not consumed each time period, are reinvested at *i* ‘ %, rather than at MARR
- The computation of IRR may be unmanageable
- Multiple IRR’s may be calculated for the same problem
- The IRR method must be carefully applied and interpreted in the analysis of two or more alternatives, where only one is acceptable

THE EXTERNAL RATE OF RETURN METHOD (ERR)

- ERR directly takes into account the interest rate (ε) external to a project at which net cash flows generated over the project life can be reinvested (or borrowed).
- If the external reinvestment rate, usually the firm's MARR, equals the IRR, then ERR method produces same results as IRR method

CALCULATING EXTERNAL RATE OF RETURN (ERR)

1. All net cash outflows are discounted to the present (time 0) at ε % per compounding period.
2. All net cash inflows are discounted to period N at ε %.
3. ERR -- the equivalence between the discounted cash inflows and cash outflows -- is determined.

The absolute value of the present equivalent worth of the net cash outflows at ε % is used in step 3.

- A project is acceptable when i ' % of the ERR method is greater than or equal to the firm's MARR

CALCULATING EXTERNAL RATE OF RETURN (ERR)

$$\sum_{k=0}^{N} E_k \left(P / F, \varepsilon\ \%, k \right) \left(F / P, i'\ \%, N \right)$$

$$=$$

$$\sum_{k=0}^{N} R_k \left(F / P, \varepsilon\ \%, N - k \right)$$

CALCULATING EXTERNAL RATE OF RETURN (ERR)

$$\sum_{k=0}^{N} E_k \, (P / F, \varepsilon \%, k)(F / P, i' \%, N)$$

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R_k = excess of receipts over expenses in period k

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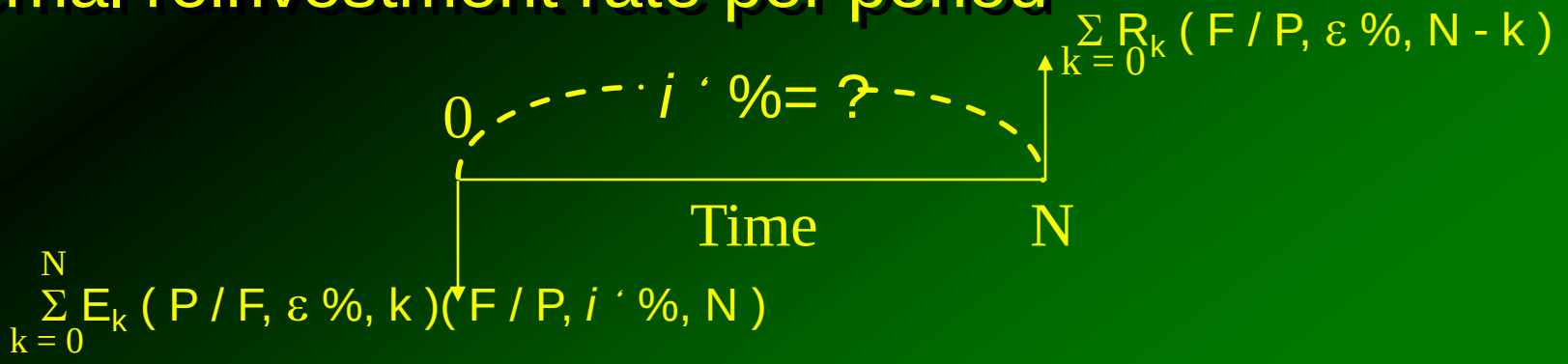

ERR ADVANTAGES

- ERR has two advantages over IRR:

1. It can usually be solved for directly, rather than by trial and error.
2. It is not subject to multiple rates of return.