$$\bar{\nabla}\times\bar{E} = -j\omega\bar{B} = -j\omega\mu\bar{H}$$

$$\bar{\nabla}\times\bar{H} = j\omega\bar{D} + \underbrace{\cancel{\bar{J}}}_{0} = j\omega\epsilon\bar{E}$$

$$\bar{\nabla}\times\bar{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \hat{a}_x\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \hat{a}_y\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)$$

$$\begin{array}{c|ccc|c} & \hat{a}_x & \hat{a}_y & \hat{a}_z & \\ & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} & \\ & E_x & E_y & E_z & \\ -\hat{a}_z\frac{\partial E_x}{\partial y} & \hat{a}_x & \hat{a}_y & \hat{a}_z & \hat{a}_x\frac{\partial E_z}{\partial y} \\ -\hat{a}_x\frac{\partial E_y}{\partial z} & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} & \hat{a}_z\frac{\partial E_y}{\partial x} \\ -\hat{a}_y\frac{\partial E_z}{\partial x} & & & & \hat{a}_y\frac{\partial E_x}{\partial z} \end{array}$$

$$+\hat{a}_z\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

$$= -j\omega\mu\left(H_x\hat{a}_x + H_y\hat{a}_y + H_z\hat{a}_z\right)$$

$$\Rightarrow \hat{a}_x\left(\frac{\partial E_z}{\partial y} + \gamma E_y\right) + \hat{a}_y\left(-\gamma E_x - \frac{\partial E_z}{\partial x}\right) + \hat{a}_z\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

$$= \underbrace{\hat{a}_x\left(-j\omega\mu H_x\right)}_{} + \underbrace{\hat{a}_y\left(-j\omega\mu H_y\right)}_{} + \underbrace{\hat{a}_z\left(-j\omega\mu H_z\right)}_{}$$

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x \qquad -\gamma E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \qquad \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

Benzer şekilde $\bar{\nabla}\times\bar{H} = j\omega\epsilon\bar{E}$ açıkça yazılıp $\frac{\partial}{\partial z} = -\gamma$ konulursa

$$\frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\epsilon E_x \qquad -\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y \qquad \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z$$

Transvers düzlem bileşenlerini (E_x, E_y, H_x, H_y) ; yayılım yönündeki bileşenler cinsinden (E_z, H_z) yazarsak:

$$H_x = -\frac{1}{h^2}\left(\gamma \frac{\partial H_z}{\partial x} - j\omega\epsilon \frac{\partial E_z}{\partial y}\right)$$

$$H_y = -\frac{1}{h^2}\left(\gamma \frac{\partial H_z}{\partial y} + j\omega\epsilon \frac{\partial E_z}{\partial x}\right) \qquad h^2 = \gamma^2 + k^2$$

$$E_x = -\frac{1}{h^2}\left(\gamma \frac{\partial E_z}{\partial x} + j\omega\mu \frac{\partial H_z}{\partial y}\right)$$

$$E_y = -\frac{1}{h^2}\left(\gamma \frac{\partial E_z}{\partial y} - j\omega\mu \frac{\partial H_z}{\partial x}\right)$$

TEM dalgalar için $E_z = 0$; $H_z = 0$ $\Rightarrow E_x, E_y, H_x, H_y = 0$!

$$\left.\begin{array}{l} \text{TE dalgalar için } E_z = 0 \; ; \; H_z \neq 0 \\ \text{TM dalgalar için } E_z \neq 0 \; ; \; H_z = 0 \end{array}\right\}$$

Geçen hafta yaptığımız çözümlerde bulduğumuz E_z ve H_z fonksiyonlarını yerine koyarak E_x, E_y, H_x, H_y hesaplanabilir

TEM Dalgalar :

$$h^2 = \gamma^2 + k^2 = 0 \Rightarrow \gamma^2 = -k^2 \Rightarrow \gamma = jk$$

$$e^{-\gamma z} e^{j\omega t} \longrightarrow \underbrace{e^{-jkz} e^{j\omega t}}$$

TEM dalga, ancak buralarda yayılır !...

1. Açık alanda yayılan dalganın formülü
2. İletim hattı içerisinde yayılan dalganın formülü

TM Dalgalar :
TE

$e^{-\gamma z}$ ifadesinin bir dalga ifade edebilmesi için

γ'nın imajiner olması gerekli

$$h^2 = \gamma^2 + k^2 \rightarrow \gamma = \sqrt{h^2 - k^2} = \sqrt{\underbrace{h^2 - \omega^2 \mu \epsilon}_{<0 \text{ olması gerekli}}}$$

$$h^2 = \omega_c^2 \mu \epsilon$$

↳ cut-off (kesme) frekansı

$\omega < \omega_c \Rightarrow h^2 - \omega^2 \mu \epsilon > 0 \Rightarrow \gamma : \text{reel} \Rightarrow e^{-\gamma z}$ sönümlenen bir fonksiyon

$\omega > \omega_c \Rightarrow h^2 - \omega^2 \mu \epsilon < 0 \Rightarrow \gamma : \text{imajiner} \Rightarrow e^{-\gamma z}$ zaman bölgesinde $\cos(\omega t - \beta z)$ şeklinde bir "dalga"

$\gamma = j\beta$

$$\omega_c = \frac{h}{\sqrt{\mu \epsilon}} \quad \text{veya} \quad f_c = \frac{h}{2\pi \sqrt{\mu \epsilon}} \quad (Hz)$$

$\left(\frac{f}{f_c}\right)^2 > 1$ veya $f > f_c$ $\rightarrow$ $\gamma = j\beta = jk\sqrt{\left(1-\frac{h}{k}\right)^2}$

$$= jk\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\beta = k\sqrt{1-\left(\frac{f_c}{f}\right)^2} \quad (\text{rad/m})$$

(kılavuzlanmış dalganın dalga boyu) $\lambda_g = \frac{2\pi}{\beta} = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$ (m)

$$\lambda = \frac{2\pi}{k} = \frac{1}{f\sqrt{\mu\epsilon}}$$

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2} \qquad \lambda_c : \frac{u}{f_c}$$

(kesme frekansına karşılık gelen dalga boyu)

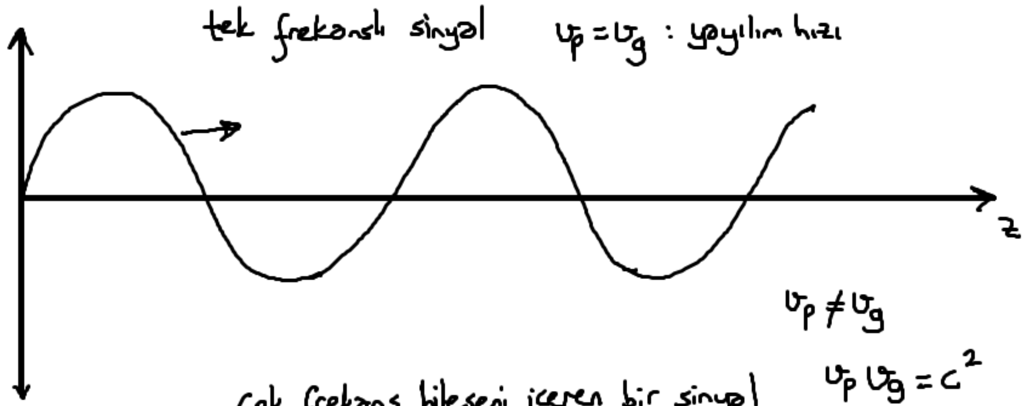
u_p : dalga kılavuzu içerisinde yayılan dalganın faz hızı

$$u_p = \frac{\omega}{\beta} = \frac{u}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\lambda_g}{\lambda}u > u$$

$\downarrow$

frekansa bağlı $\longrightarrow$ tek iletkenli dalga kılavuzları "dispersif" tir.

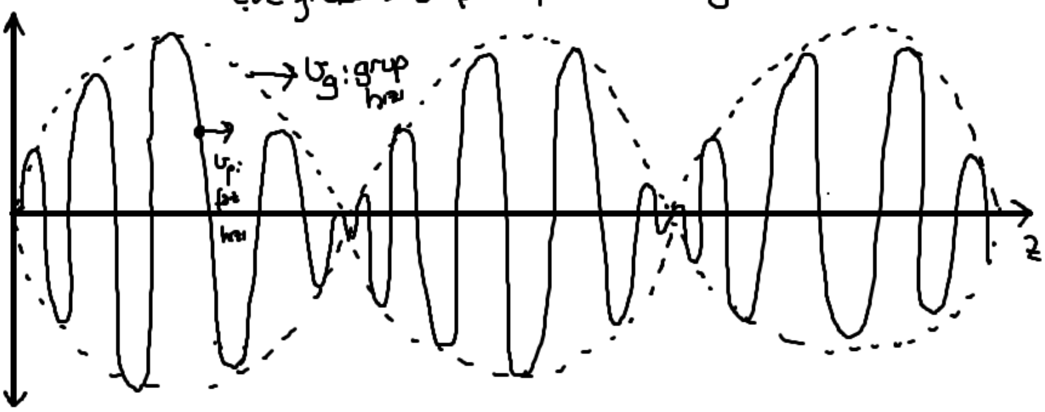
tek frekanslı sinyal $\quad v_p = v_g$: yayılım hızı



$v_p \neq v_g$

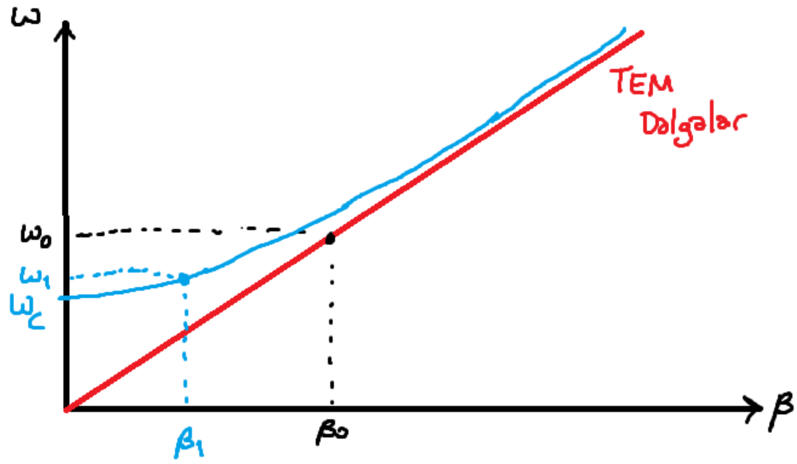
$v_p v_g = c^2$

Çok frekans bileşeni içeren bir sinyal

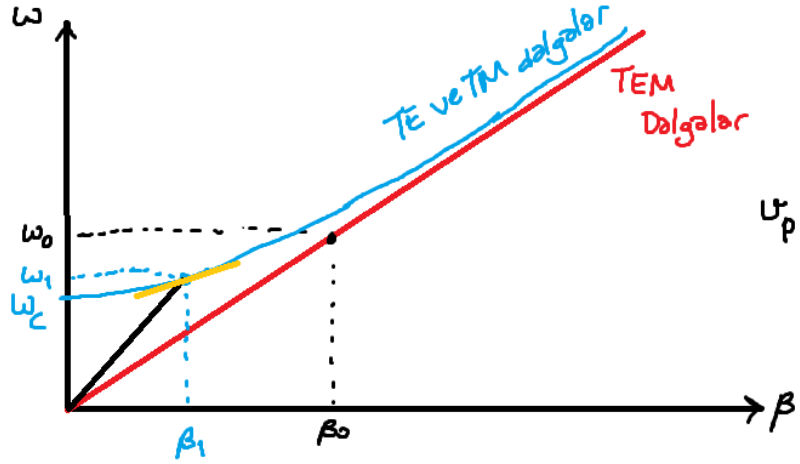


$$v_p = \frac{\omega}{\beta}$$

$$v_g = \frac{d\omega}{d\beta}$$



$$\frac{\omega_1}{\beta_1} \neq \left.\frac{d\omega}{d\beta}\right|_{\omega=\omega_1}$$



$v_p = \frac{\omega_1}{\beta_1}$: siyah doğrunun eğimi

$v_g = \left.\frac{d\omega}{d\beta}\right|_{\omega=\omega_1}$: turuncu doğrunun eğimi