

## Chapter 7 Energy and Energy Transfer

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## Energy is ability <br> to <br> do work

Energy is a quantity that can be converted from one form to another but cannot be created or destroyed (total amount of energy always same).

Kinetic energy K is energy associated with the state of motion of an object. For an object of mass $m$ whose speed $v$ is well below the speed of light,

$$
K=\frac{1}{2} m v^{2}
$$

The SI unit of kinetic energy (and every other type of energy) is the joule (J), named for James Prescott Joule, an English scientist of the 1800s.

$$
1 J=1 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}^{2}
$$

## + 7.3. Work

## Physicist's Definition of Work


(a)

(b)

(c)

In analyzing forces to determine the work they do, we must consider the vector nature of forces.

## Work done by constant force:



The work W done on a system by an agent exerting a constant force on the system is the product of the magnitude $\mathbf{F}$ of the force, the magnitude $\Delta \mathbf{r}$ of the displacement of the point of application of the force, and $\cos \theta$, where $\theta$ is the angle between the force and displacement vectors:

## $W=F \Delta r \cos \theta=\vec{F} . \Delta \vec{r}$

The SI unit of work is Newton.meter (N.m). This combination of units is used so frequently that it has been given a name of its own: the joule (J)

## + 7.3. Work

The sign of the work also depends on the direction of F relative to $\Delta \mathrm{r}$.
(a) Force $\overrightarrow{\boldsymbol{F}}$ has a component in direction of displacement:
$W=F_{\|} s=(F \cos \phi) s$
Work is positive.

(b) Force $\overrightarrow{\boldsymbol{F}}$ has a component opposite to direction of displacement: $W=F_{\|} s=(F \cos \phi) s$
Work is negative (because $F \cos \phi$ is negative for $90^{\circ}<\phi<180^{\circ}$ ).

(c) Force $\overrightarrow{\boldsymbol{F}}$ (or force component $F_{\perp}$ ) is perpendicular to direction of displacement: The force (or force component) does no work on the object.


## Work is an energy transfer:

If $W$ is the work done on a system and $W$ is positive, energy is transferred to the system;



The work done is positive The block speeds ups


The work done is negative The block slow down


The work done is zero The block's speed doesn't change.

Consider a particle with mass m moving along the x -axis under the action of a constant net force with magnitude F directed along the positive x -axis.


From timeless velocity equation for constant acceleration motion:

$$
\begin{gathered}
v_{f}^{2}=v_{i}^{2}+2 a \Delta x \\
a=\frac{v_{f}^{2}-v_{i}^{2}}{2 \Delta x}
\end{gathered}
$$

When we multiply this equation by m :

When we multiply this equation by $m$ :

$$
\begin{aligned}
& F=m a=m \frac{v_{f}^{2}-v_{i}^{2}}{2 \Delta x} \\
& F \Delta x=W=\frac{1}{2} m v_{f}^{2}-\frac{1}{2} m v_{i}^{2}
\end{aligned}
$$

The product $F \Delta x$ is the work done by the net force $F$ and thus is equal to the total work done by all the forces acting on the particle:

$$
W=\frac{1}{2} m v_{f}^{2}-\frac{1}{2} m v_{i}^{2}=K_{f}-K_{i}=\Delta K
$$

This is work kinetic energy theorem:
The work done by the net force on a particle equals the change in the particle's kinetic energy.

## + 7.4. Work and Kinetic Energy Theorem

## Example: 1

Two iceboats hold a race on a frictionless horizontal lake. The two iceboats have masses m and 2 m . The iceboats have identical sails, so the wind exerts the same constant force on each iceboat. They start from rest and cross the finish line a distance s away. Which iceboat crosses the finish line with greater kinetic energy?


## + 7.5. Work Done by Gravitational Force

An object is thrown upward with initial speed $\mathrm{v}_{0}$.

The only force acting on the object is gravitational force $\mathbf{F}_{\mathbf{g}}$

The work done by gravitational force during a displacement d is:

$$
W=-m g\left(y_{B}-y_{A}\right)=-m g d
$$

Since the work done is negative, the object's speed decreasing.

This work is equal the kinetic energy change of particle according to work-kinetic energy theorem:

$$
W=-m g d=K_{B}-K_{A}
$$

## + 7.5. Work Done by Gravitational Force

An object is thrown upward with initial speed $\mathrm{v}_{0}$.


To find the maximum point that object can reach:

$$
\begin{gathered}
W=-m g h=K_{B}-K_{A} \\
-m g h=0-\frac{1}{2} m v_{o}^{2} \\
h=\frac{v_{o}^{2}}{2 g}
\end{gathered}
$$



$$
\Delta K=K_{f}-K_{i}=W_{a}+W_{g}
$$

## + 7.6. Work Done by a General Variable Force

Consider a particle being displaced along the $x$ axis under the action of a force that varies with position. In such a situation, we cannot use $\mathrm{W}=\mathrm{F} \Delta x \cos \theta$ to calculate the work done by the force because this relationship applies only when $F$ is constant in magnitude and direction.


## + 7.6. Work Done by a General Variable Force

We can approximate that area with the area of these strips.


However, if we imagine that the particle undergoes a very small displacement $\Delta x$, shown in Figure, the $x$ component $F_{x}$ of the force is approximately constant over this small interval; for this small displacement, we can approximate the work done by the force as

$$
\Delta W_{j} \approx F_{x, j} \Delta x
$$

We can do better with more, narrower strips.


However, if we imagine that the particle undergoes a very small displacement $\Delta x$, shown in Figure, the $x$ component $F_{x}$ of the force is approximately constant over this small interval; for this small displacement, we can approximate the work done by the force as

$$
\Delta W_{j} \approx F_{x, j} \Delta x
$$

For the best, take the limit of strip widths going to zero.


$$
W \approx \sum \Delta W_{j} \approx \sum_{x_{i}}^{x_{s}} F_{x, j} \Delta x
$$

$$
W=\lim _{\Delta x \rightarrow 0} \sum_{x_{i}}^{x_{s}} F_{x} \Delta x
$$

The work done by $F_{x}$ as the particle moves from $x_{i}$ to $x_{f}$ can be expressed as

$$
W=\int_{x_{i}}^{x_{f}} F_{x} d x
$$

Work is equal to the area under the curve

## + 7.6. Work Done by a General Variable Force

Work-Kinetic Energy Theorem with a Variable Force
Work done by variable force

$$
W=\int_{x_{i}}^{x_{s}} F_{x} d x
$$

By using Newton's 2.Law

$$
\begin{aligned}
& W=\int_{x_{i}}^{x_{f}} m a d x=\int_{x_{i}}^{x_{f}} m \frac{d v}{d t} d x=\int_{x_{i}}^{x_{f}} m \frac{d v}{d x} \frac{d x}{d t} d x=\int_{v_{i}}^{v_{f}} m v d v \\
& W=\frac{1}{2} m v_{f}^{2}-\frac{1}{2} m v_{i}^{2}
\end{aligned}
$$

We obtain the work-kinetic energy theorem again.

## + 7.6. Work Done by a General Variable Force

A model of a common physical system for which the force varies with position is spring-mass system:


Hooke's law:

$$
\vec{F}_{s}=-k \vec{d}
$$

where d is the position of the block relative to its equilibrium $(\mathrm{x}=0$ ) position and k is a positive constant called the force constant or the spring constant of the spring.

The negative sign in this equation signifies that the force exerted by the spring is always directed opposite to the displacement from equilibrium.

A model of a common physical system for which the force varies with position is spring-mass system:


In one dimension we can rewrite the Hooke's law:

$$
F_{s}=-k x
$$


(b)

(c)

Because the spring force always acts toward the equilibrium position ( $x=0$ ), it is sometimes called a restoring force.

Work done done by the spring force on the block when it undergoes an arbitrary displacement from $\mathrm{x}_{\mathrm{i}}$ to $\mathrm{x}_{\mathrm{f}}$.

$$
\begin{gathered}
W=\int_{x_{i}}^{x_{f}} F_{x} d x \\
W=\int_{x_{i}}^{x_{f}}-k x d x \\
W=\frac{1}{2} k x_{i}^{2}-\frac{1}{2} k x_{f}^{2}
\end{gathered}
$$

Now let us consider the work done on the spring by an external agent that stretches the spring very slowly from $x_{i}$ to $x_{f}$.


$$
\begin{gathered}
W=\int_{x_{i}}^{x_{f}} F_{x} d x \\
W=\int_{x_{i}}^{x_{f}}+k x d x \\
W=\frac{1}{2} k x_{f}^{2}-\frac{1}{2} k x_{i}^{2}
\end{gathered}
$$

Notice that this is the negative of the work done by the spring as expressed

The time rate at which work is done by a force is said to be the power due to the force. If a force does an amount of work W in an amount of time $\Delta \mathrm{t}$, the average power due to the force during that time interval is:

$$
P_{\text {avg. }}=\frac{\Delta W}{\Delta t}
$$

The instantaneous power $P$ is the instantaneous time rate of doing work, which we can write as

$$
P=\frac{d W}{d t}
$$

The SI unit of power is the joule per second. . This unit is used so often that it has a special name, the watt (W), after James Watt

$$
1 \mathrm{watt}=1 \mathrm{~W}=1 \mathrm{~J} / \mathrm{s}
$$

By using the definition of work, we can rewrite the instantaneous power as:

$$
P=\frac{d W}{d t}=\frac{\vec{F} \cdot d \vec{r}}{d t}=\vec{F} \cdot \vec{v}
$$

