PHY121 Physics I

Chapter 8 Potential Energy and Conversation of Energy

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Chapter 8 Potential Energy and Conversation of Energy

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8.3. Gravitational Potential Energy
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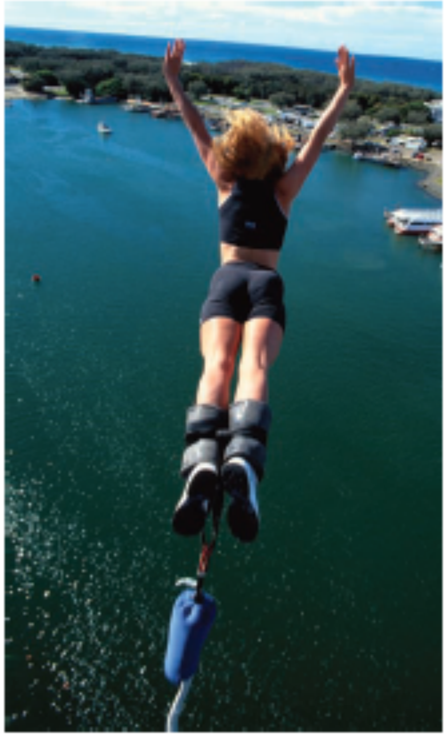
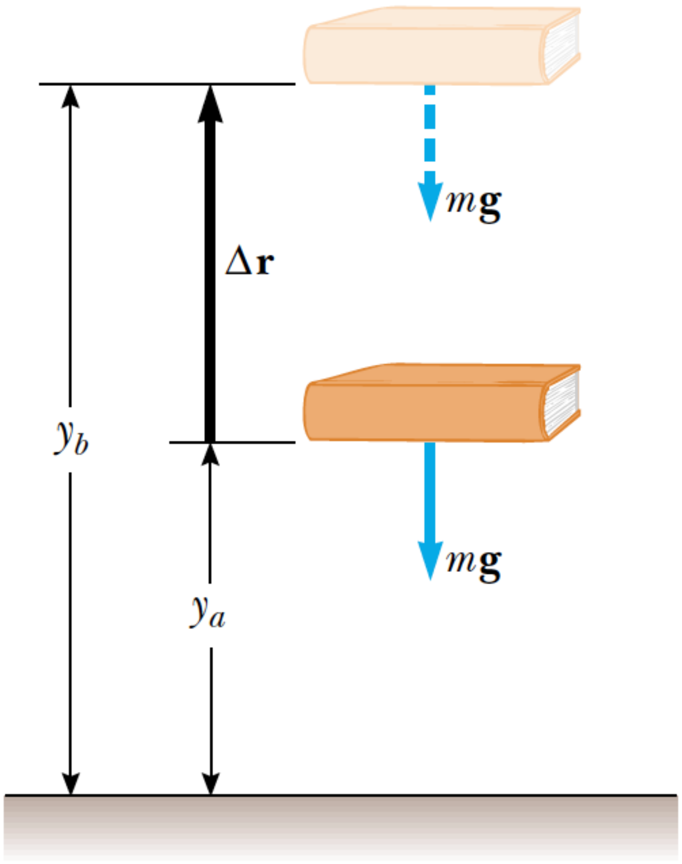
8.6. Work Done on a System by an External Force



8.1. Potential Energy

We do some work on the system by lifting the book slowly through a height $\Delta y = y_b - y_a$, as in Figure.



Because the energy change of the system is not in the form of kinetic energy or internal energy, it must appear as some other form of energy storage.

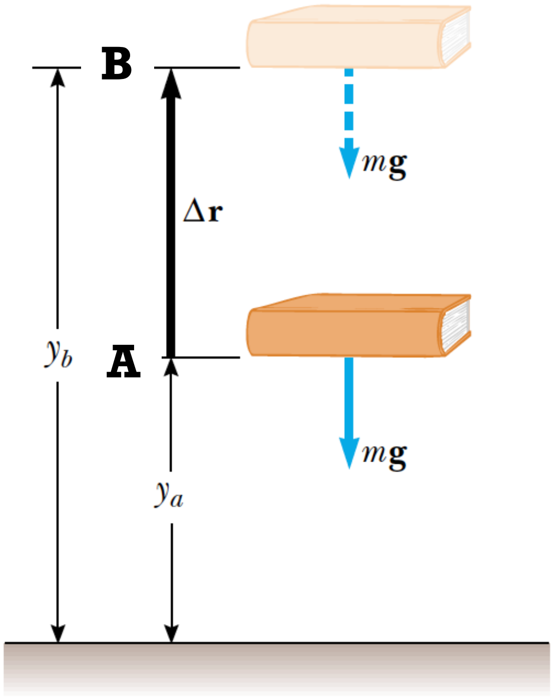
After lifting the book, we could release it and let it fall back to the position y_a and it gains kinetic energy.

While the book was at the yb, it had potential to gain kinetic energy.

We call the energy storage mechanism before we release the book as potential energy.

A potential energy can only be associated with specific types of forces. !!!!

8.1. Potential Energy



The work done by the external agent on the system (object and Earth) as the book undergoes upward displacement $\Delta y = y_b - y_a$:

$$W_{A\to B} = F\Delta r = mg(y_b - y_a)$$

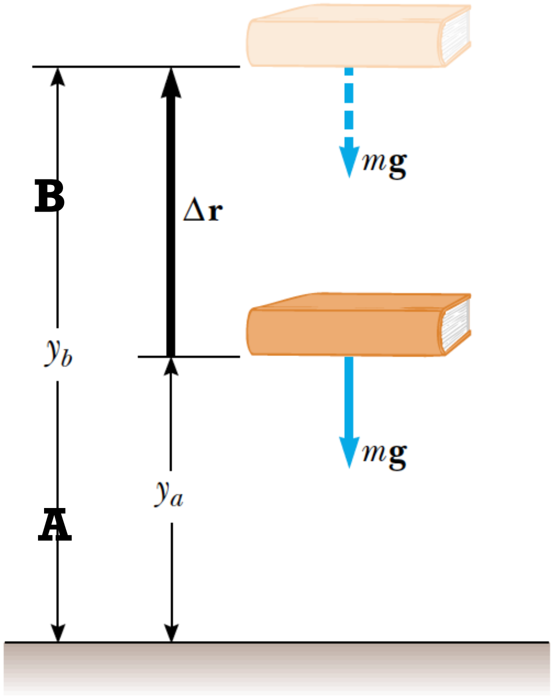
$$W_{A\to B} = mgy_b - mgy_a$$

In previous chapter we found the work kinetic energy theorem:

$$W_{A\to B} = \frac{1}{2}mv_b^2 - \frac{1}{2}mv_a^2 = \Delta K$$

In each equation, the work done on a system equals a difference between the final and initial values of a quantity.

8.1. Potential Energy



$$W_{A\to B} = mgy_b - mgy_a$$

we can identify the quantity mgy as the gravitational potential energy U_g:

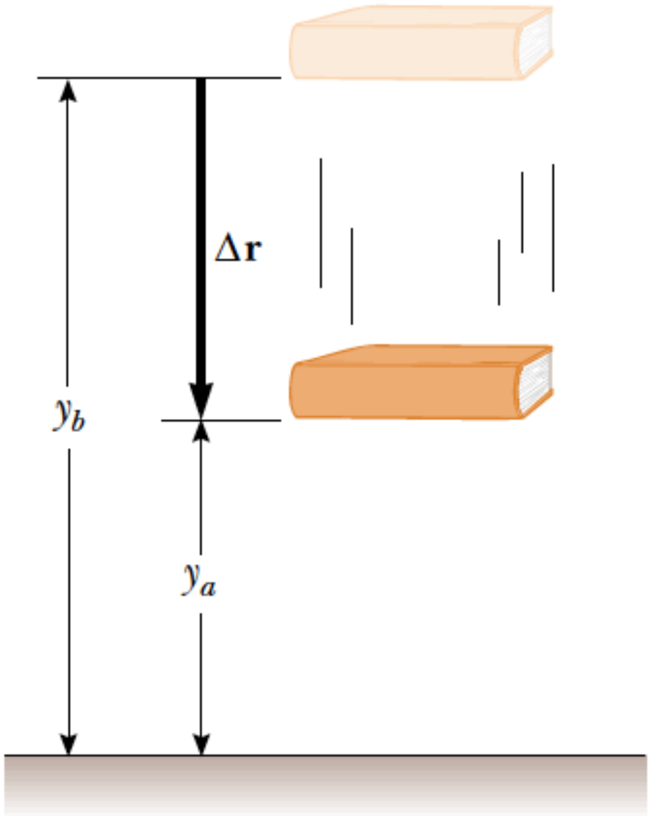
$$U_g = mgy$$

The units of gravitational potential energy are joules, the same as those of work and kinetic energy.

Potential energy, like work and kinetic energy, is a scalar quantity.

8.1. Potential Energy

The work done by gravitational force on the system (object and Earth) as the book undergoes downward displacement $\Delta y = y_b - y_a$:



$$W = (-mg\hat{j}).\left[(y_a - y_b)\hat{j}\right]$$

$$W = mgy_b - mgy_a$$

$$W = -(mgy_a - mgy_b)$$

$$\boxed{W = -\Delta U}$$

For either rise or fall, the change U in gravitational potential energy is defined as being equal to the negative of the work done on the book by the gravitational force.

8.1. Potential Energy

For either rise or fall, the change U in gravitational potential energy is defined as being equal to the negative of the work done on the book by the gravitational force.

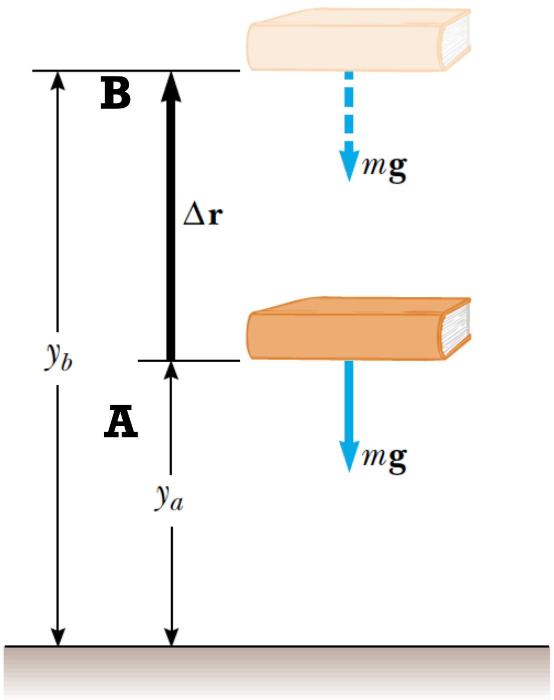
$$W = -\Delta U$$

When the body moves up, y increases, the work done by the gravitational force is negative, and the gravitational potential energy increases

When the body moves down, y decreases, the gravitational force does positive work, and the gravitational potential energy decreases

8.2. Conservative Forces

The work done by gravitational force on the system (object and Earth) as the book undergoes upward displacement $\Delta y = y_b - y_a$:



$$W_{A\to B} = F\Delta r = -mg(y_b - y_a)$$

$$W_{A\to B} = -mgh$$

İf h positice the gravitaional force does a negative work.

The work done on the book alone by the gravitational force as the book falls back to its original height (from B to A) is positive :

$$W_{A\to B} = -W_{B\to A}$$

8.2. Conservative Forces

In a situation in which

$$W_{A\rightarrow B} = -W_{B\rightarrow A}$$

is always true, the other type of energy is a potential energy and the force is said to be a conservative force.

Gravitational force and elastic force are conservative forces.

A force that is not conservative is called a nonconservative force. The kinetic frictional force and drag force are nonconservative forces.

Some properties of conservative forces:

The net work done by a conservative force on a particle moving around any closed path is zero.

The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.

+ 8.2. Conservative Forces

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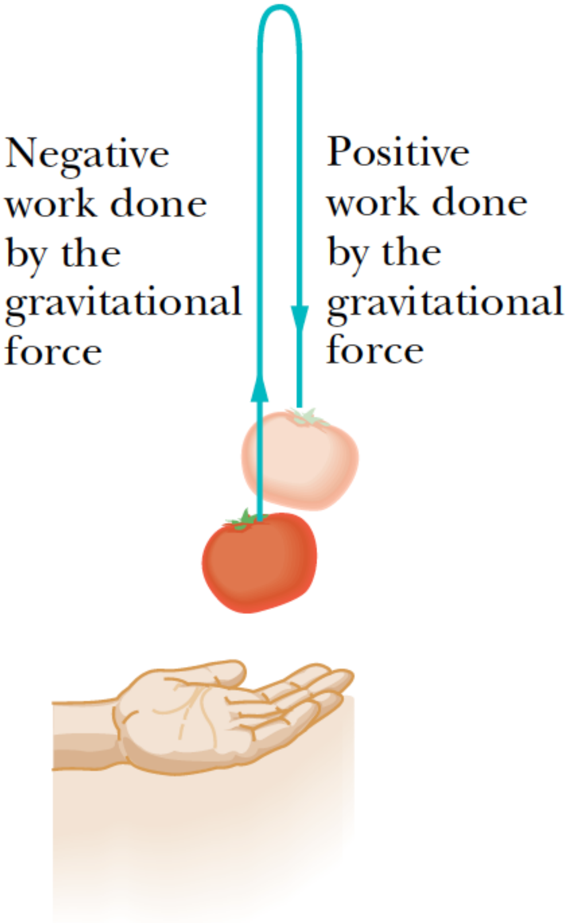
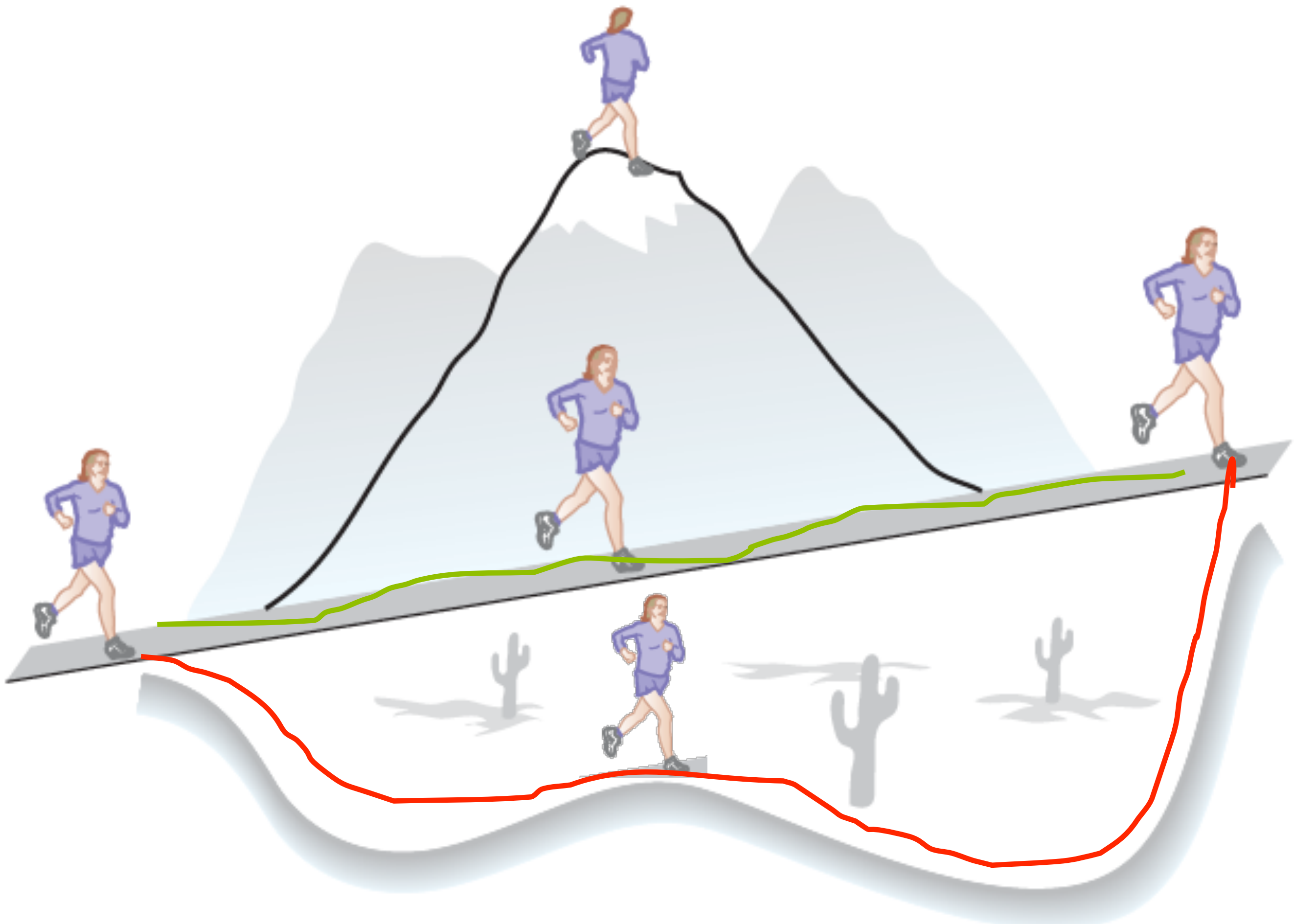


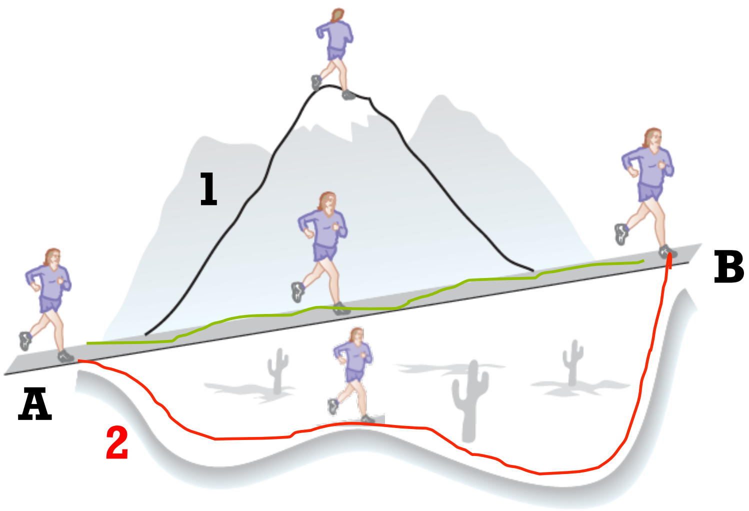
8.2. Conservative Forces

The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.



8.2. Conservative Forces

✓ **The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle..**



If runner run from A to B buy using road no 1 and back to point A by using road nı2:

$$W_{A\to B;1} + W_{B\to A,2} = 0$$

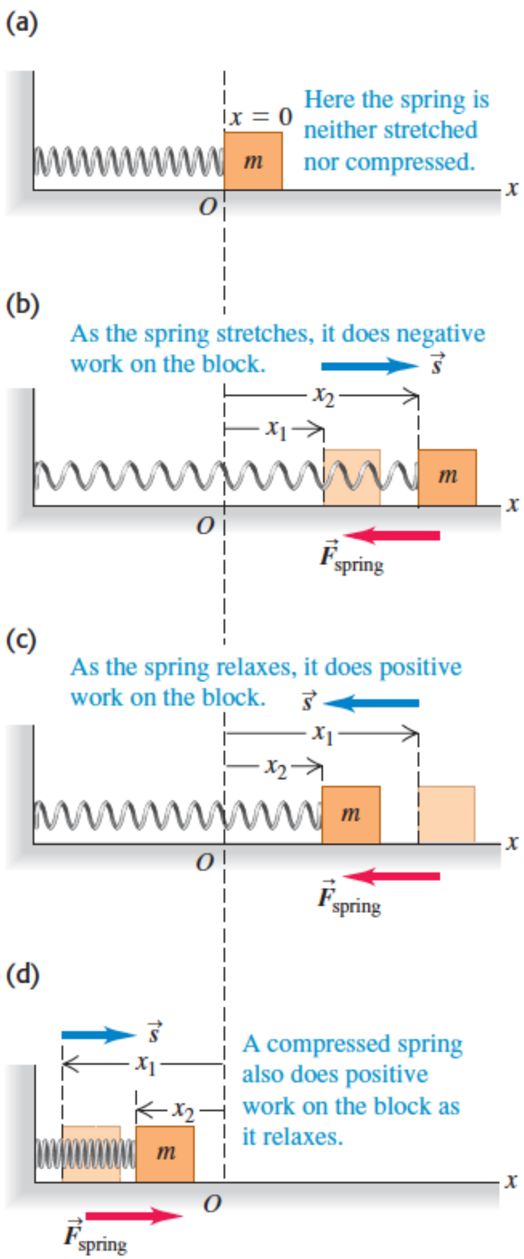
$$W_{A\to B;1} = -W_{B\to A,2}$$

Now we assume that, the runner run from A to B by using the road no 2:

$$W_{A\to B;2} = -W_{B\to A,2}$$

$$W_{A\to B;2} = W_{A\to B;1}$$

8.3. Elastic Potential Energy



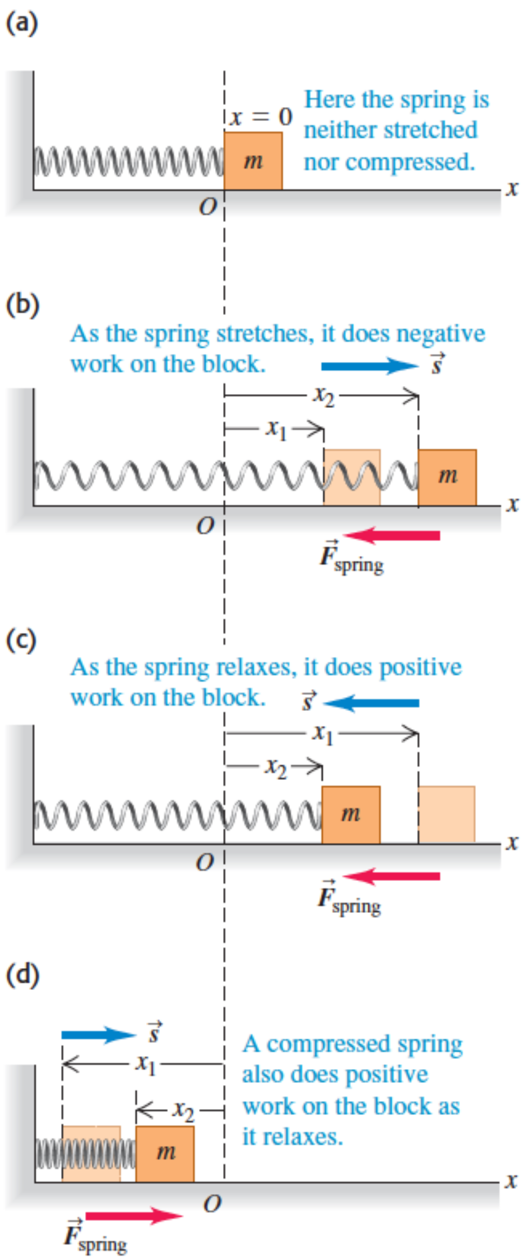
The work done by external agent to compress or stretch the spring is:

$$W_{ext.} = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$

The work done by spring on the block is:

$$W_{el} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

8.3. Elastic Potential Energy



Elastic potential energy is given by:

$$U_{el} = \frac{1}{2}kx^2$$

The work done on the block by the elastic force in terms of the change in elastic potential energy:

$$W_{el} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

$$W_{el} = U_{el;1} - U_{el;2} = -\Delta U_{el}$$

8.4. Conversation of Mechanic Energy

The mechanical energy E_{mec} of a system is the sum of its potential energy U and the kinetic energy K of the objects within it:

$$E_{mec} = K + U$$

When a conservative force does work W on an object within the system, that force transfers energy between kinetic energy K of the object and potential energy U of the system.

From Work-kinetic energy theorem

$$\Delta K = W$$

and from the potential energy discussion:

$$W = -\Delta U$$

8.5. Conversation of Mechanic Energy

$$\Delta K = W \qquad W = -\Delta U$$

Thus:

$$\boxed{\Delta K = -\Delta U}$$

In words, one of these energies increases exactly as much as the other decreases

We can rewrite this eguation

$$K_2 - K_1 = -(U_2 - U_1)$$

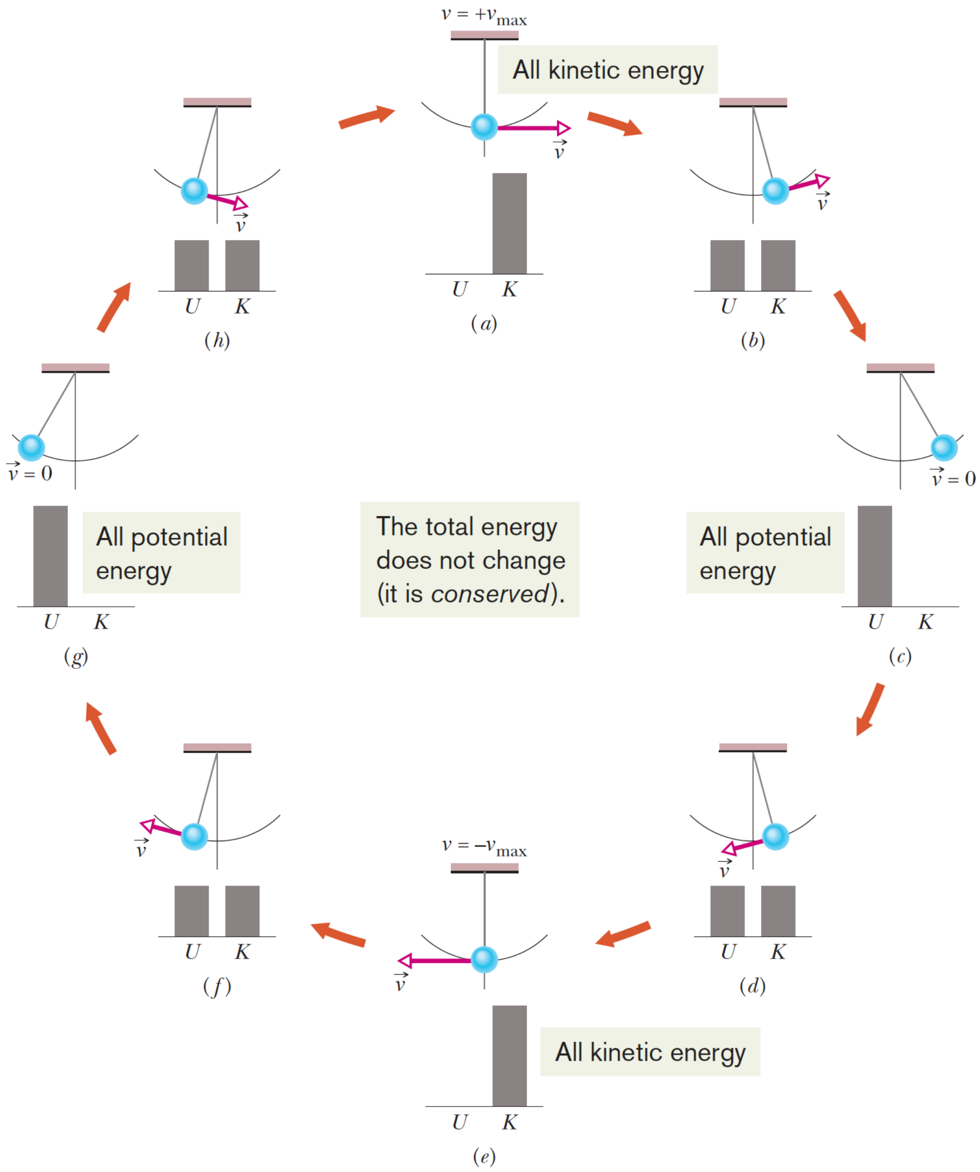
$$\boxed{K_1 + U_1 = K_2 + U_2}$$

This result is called the principle of conservation of mechanical energy

In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

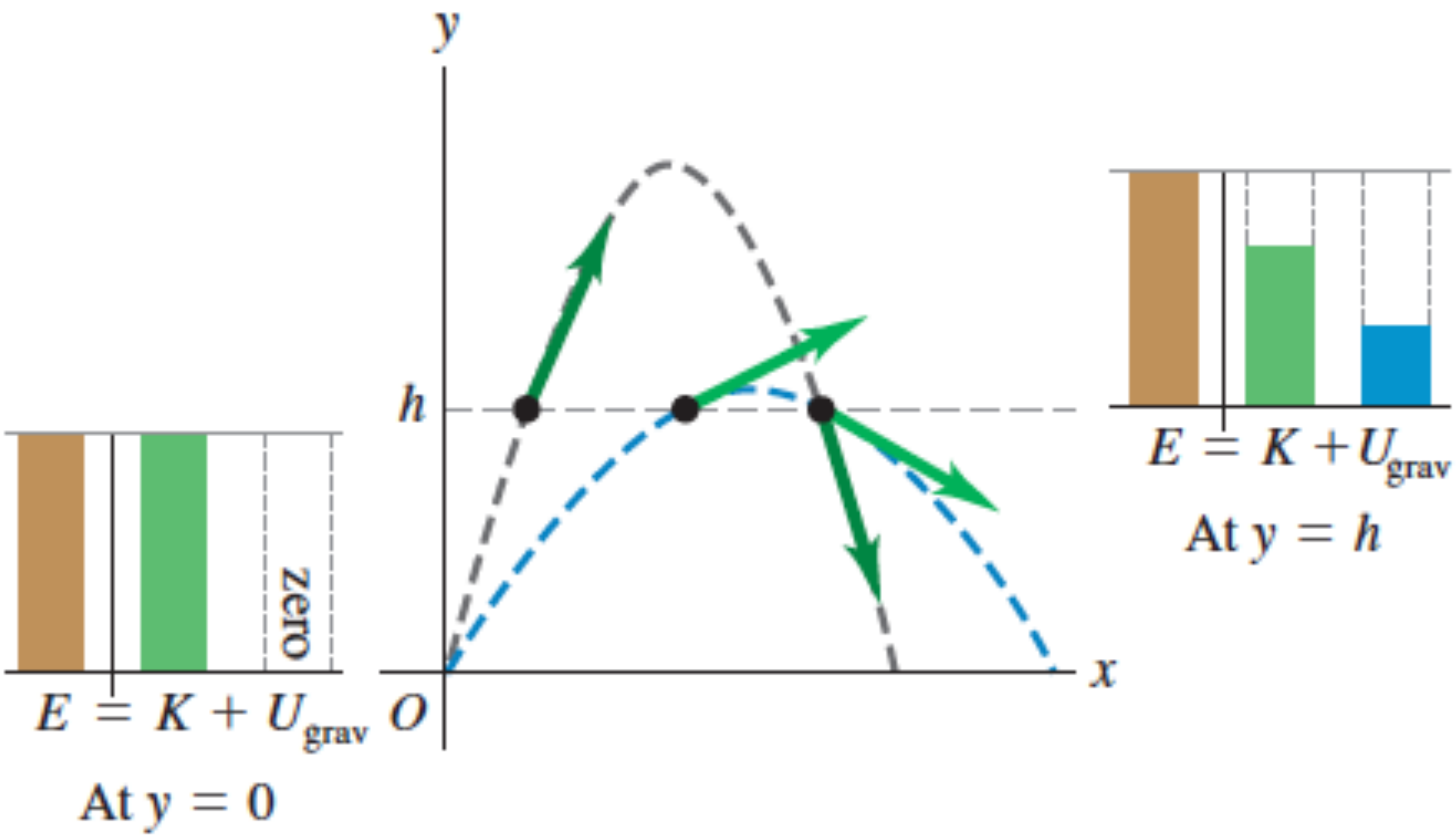
8.5. Conversation of Mechanic Energy

Pendulum:



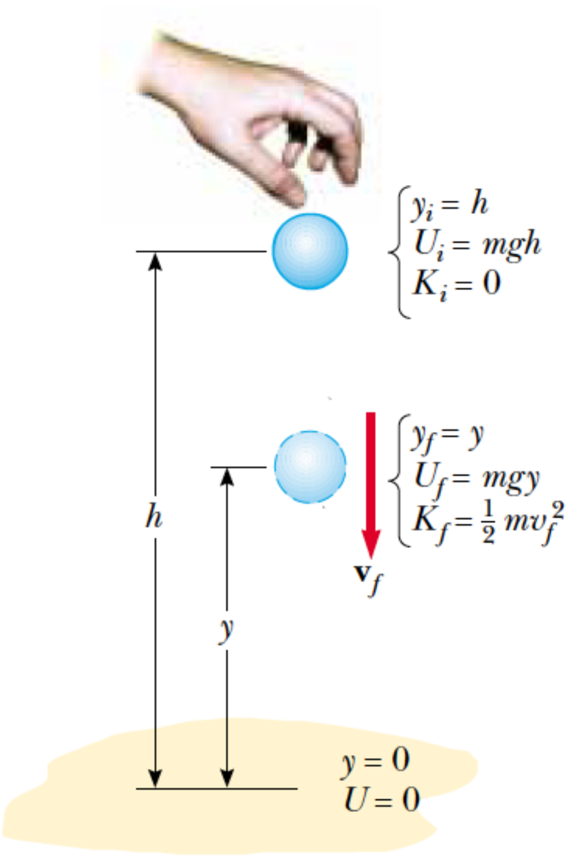
8.5. Conversation of Mechanic Energy

Projectile motion:



8.5. Conversation of Mechanic Energy

Example:1

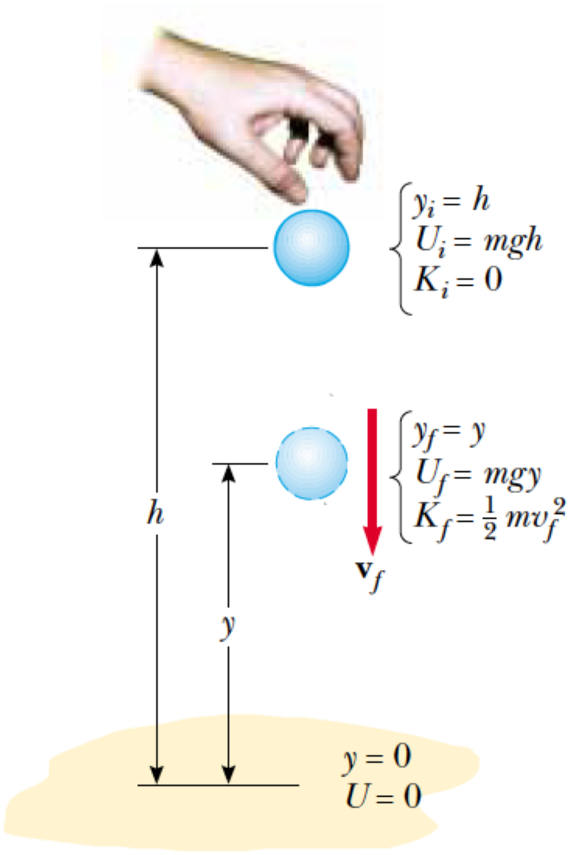


A ball of mass m is dropped from a height h above the ground, as shown in Figure. Neglecting air resistance, determine the speed of the ball when it is at a height y above the ground.

+ 8.5. Conversation of Mechanic Energy

Answer:1

From the conservation of mechanical energy:



$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2}mv_f^2 + mgy = 0 + mgh$$

$$v_f^2 = 2g(h - y)$$

$$v_f = \sqrt{2g(h - y)}$$

8.5. Conversation of Mechanic Energy

Example:2

Determine the speed of the ball in example 1 at y if at the instant of release it already has an initial upward speed v_i at the initial altitude h.

Answer:2

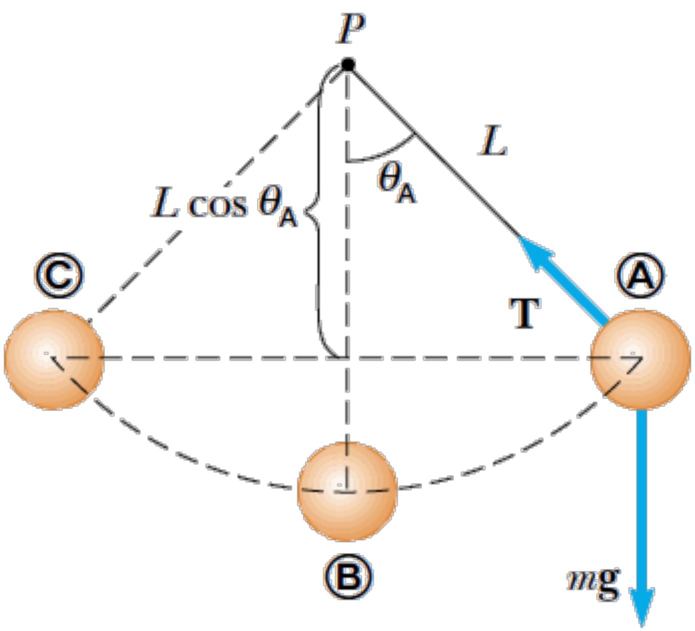
$$\frac{1}{2}mv_f^2 + mgy = \frac{1}{2}mv_i^2 + mgh$$

$$v_f^2 = v_i^2 + 2g(h-y)$$

$$v_f = \sqrt{v_i^2 + 2g(h-y)}$$

8.5. Conversation of Mechanic Energy

Example:3

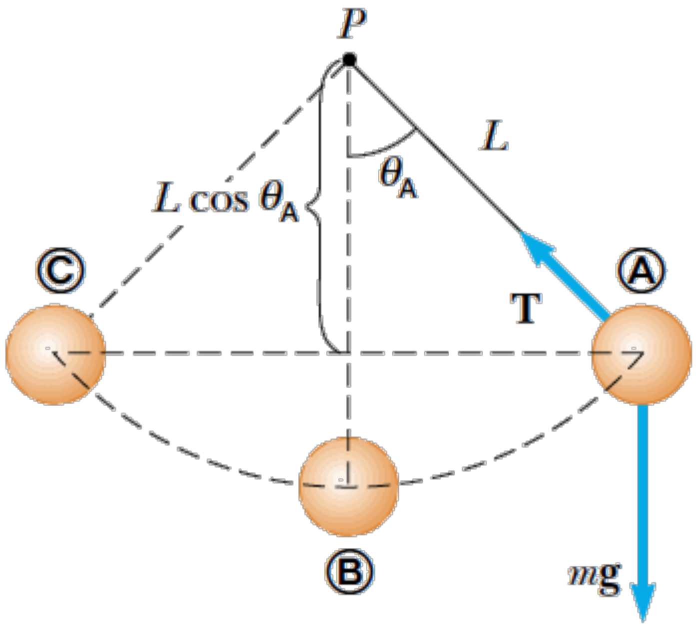


A pendulum consists of a sphere of mass m attached to a light cord of length L, as shown in Figure. The sphere is released from rest at point A when the cord makes an angle θ_A with the vertical, and the pivot at P is frictionless.

a) Find the speed of the sphere when it is at the lowest point B.
b) What is the tension T_B in the cord at point B

+ 8.5. Conversation of Mechanic Energy

Answer:3



If we measure the y coordinates of the sphere from the center of rotation, the coordinate of point A

$$y_A = -L\cos\theta_A$$

and the coordinate of point B

$$y_B = -L$$

The potential energy at these points are:

$$U_A = -mgL\cos\theta_A$$

$$U_B = -mgL$$

8.5. Conversation of Mechanic Energy

Answer:3

From the conversation of energy:

$$K_B + U_B = K_A + U_A$$

$$\frac{1}{2}mv_B^2 - mgL = 0 - mgL\cos\theta_A$$

$$v_B = \sqrt{2gL(1-\cos\theta_A)}$$

8.5. Conversation of Mechanic Energy

Answer:3

$$\sum F_r = mg - T = ma_r = -m\frac{v_B^2}{L}$$

$$T = mg(3 - 2\cos\theta_A)$$

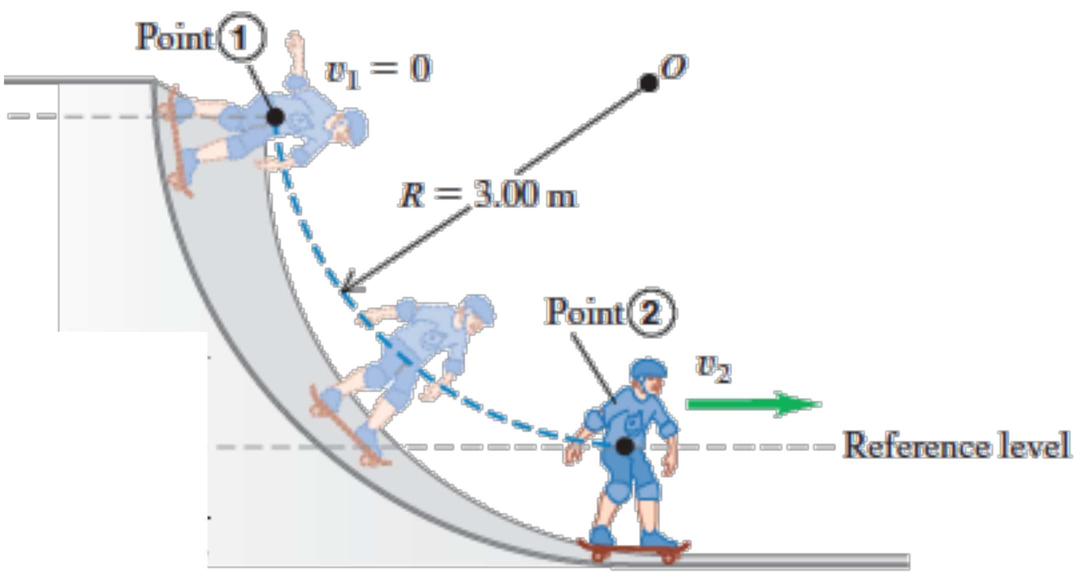
+ 8.5. Conversation of Mechanic Energy

Example:4

A friend of you skateboards from rest down a curved, frictionless ramp. If we treat he and his skateboard as a particle, he moves through a quarter-circle with radius R=3.00 m. He and his skateboard have a total mass of
25.0 kg.

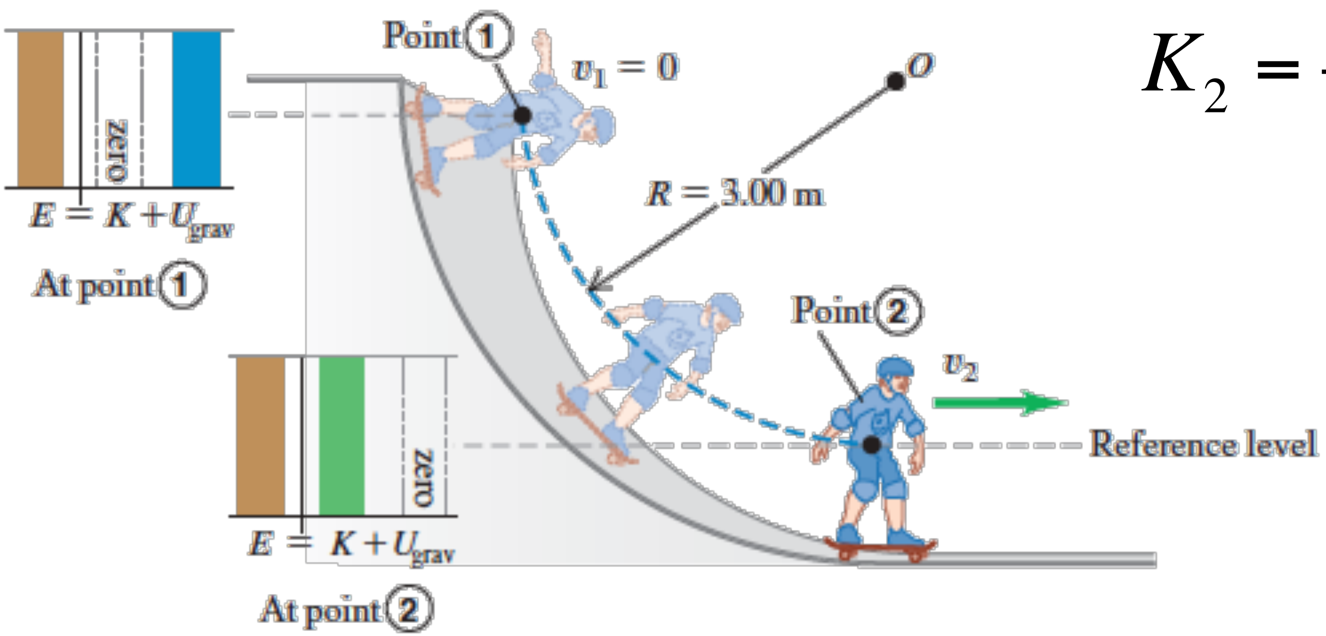
(a) Find his speed at the bottom of the ramp.

(b) Find the normal force that acts on him at the bottom of the curve.



+ 8.5. Conversation of Mechanic Energy

Answer:4

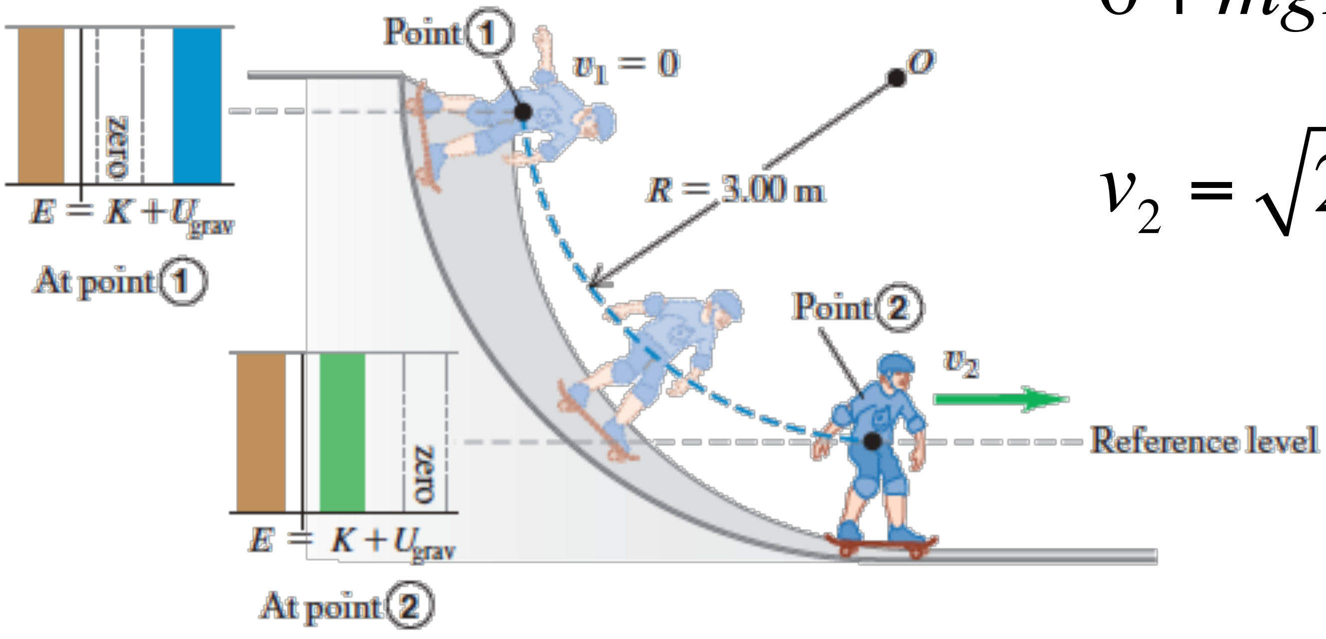


$$K_1 = 0; U_1 = mgR$$

$$K_2 = \frac{1}{2}mv_2^2; U_2 = 0$$

8.5. Conversation of Mechanic Energy

Answer:4



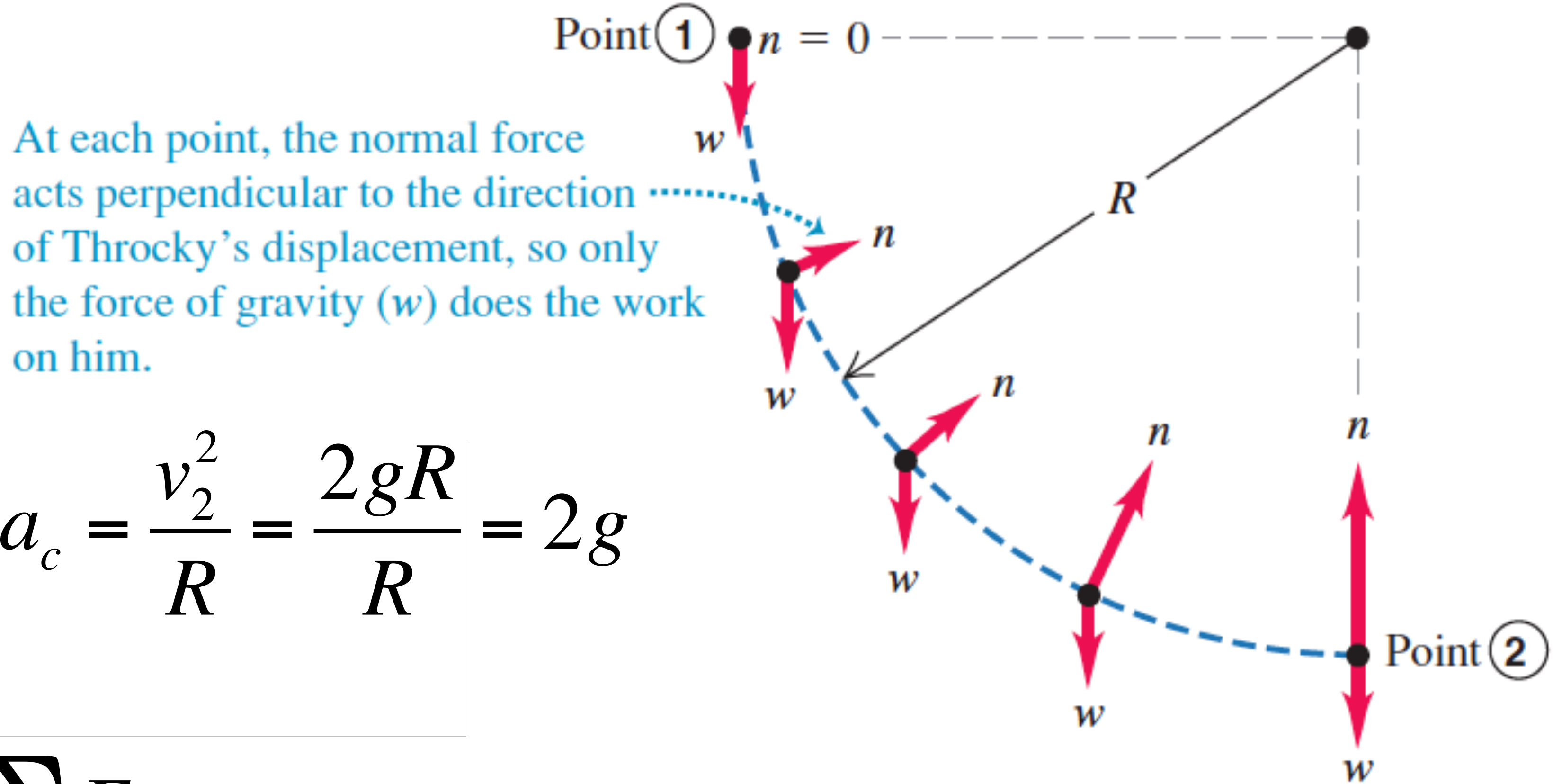
$$K_1 + U_1 = K_2 + U_2$$

$$0 + mgR = \frac{1}{2}mv_2^2 + 0$$

$$v_2 = \sqrt{2gR} = 7.67m/s$$

8.5. Conversation of Mechanic Energy

Answer:4



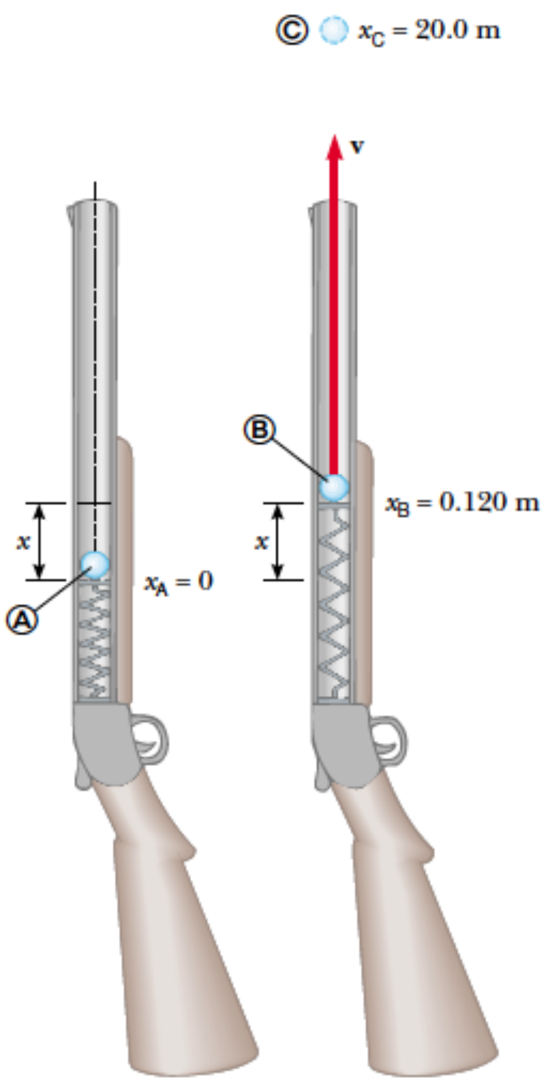
$$a_c = \frac{v_2^2}{R} = \frac{2gR}{R} = 2g$$

$$\sum F_y = n - w = ma_c$$

$$n = w + 2mg = 3mg = 755N$$

8.5. Conversation of Mechanic Energy

Example:5



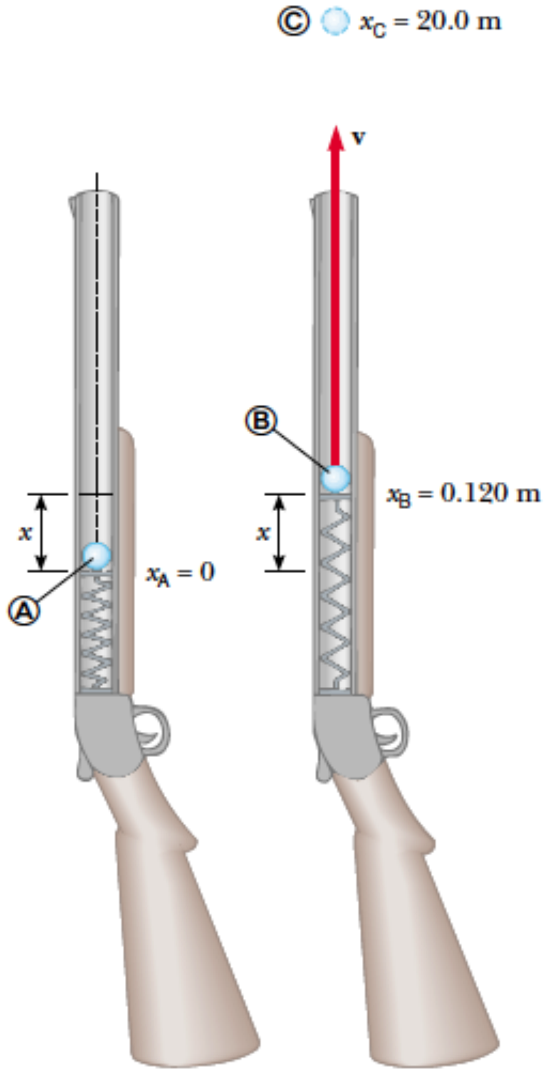
The launching mechanism of a toy gun consists of a spring of unknown spring constant. When the spring is compressed 0.120 m, the gun, when fired vertically, is able to launch a 35.0-g projectile to a maximum height of 20.0 m above the position of the projectile before firing.

a) Neglecting all resistive forces, determine the spring constant.

b) Find the speed of the projectile as it moves through the equilibrium position of the spring (where xB = 0.120m) as shown in Figure.

8.5. Conversation of Mechanic Energy

Answer:5a



$$E_C = E_A$$

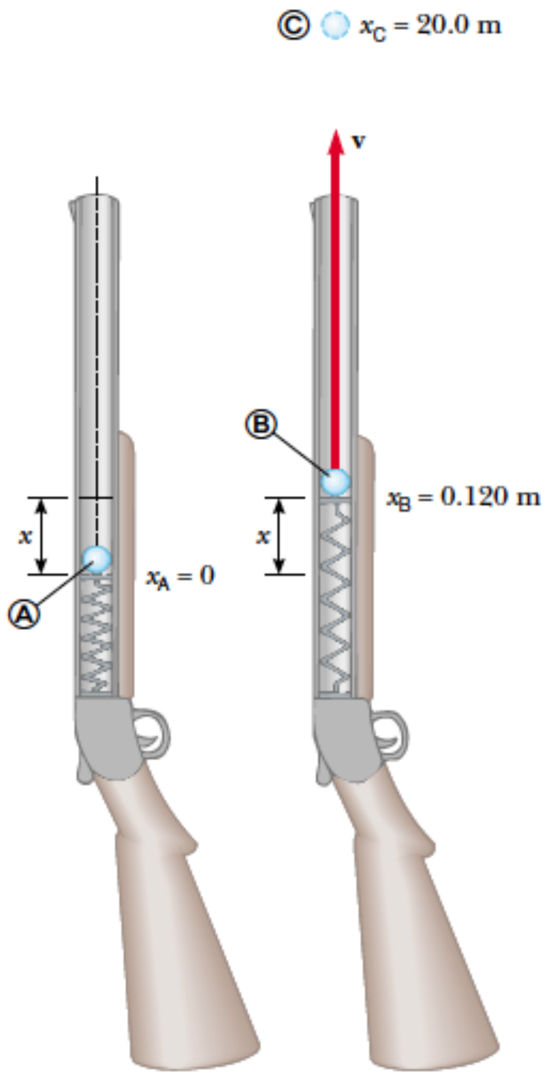
$$K_C + U_{g,C} + U_{el,C} = K_A + U_{g,A} + U_{el,A}$$

$$O + mgh + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$k = \frac{2mgh}{x^2} = 953N/m$$

8.5. Conversation of Mechanic Energy

Answer:5b



$$E_B = E_A$$

$$K_B + U_{g,B} + U_{el,B} = K_A + U_{g,A} + U_{el,A}$$

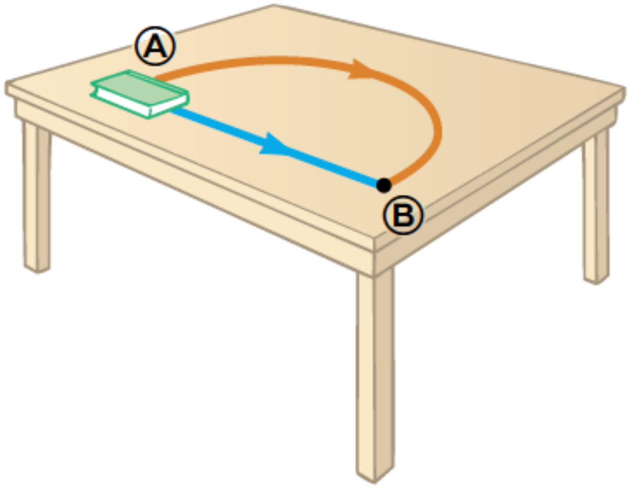
$$\frac{1}{2}mv_B^2 + mgx_B + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$v_B = \sqrt{\frac{kx^2}{m} - 2gx_B} = 19.7 m/s$$

8.6 Work Done on a System by an External Force

If the forces acting on objects within a system are conservative, then the mechanical energy of the system is conserved.

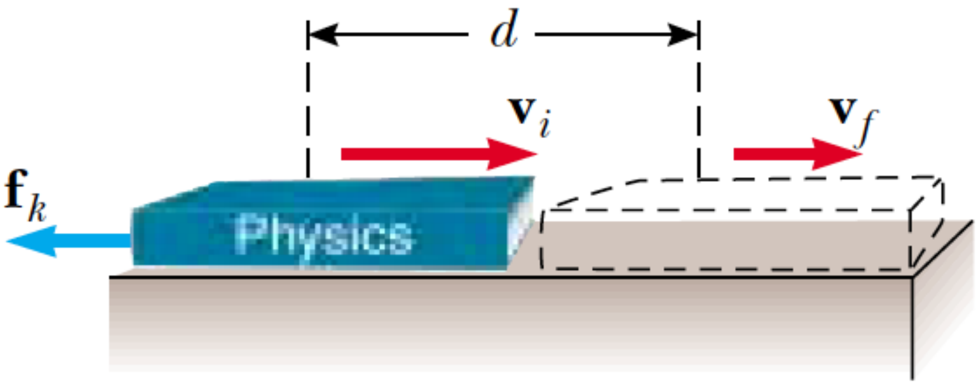
What if the forces are not conservative ?



A force is nonconservative if it does not satisfy properties 1 and 2 for conservative forces.

Nonconservative forces acting within a system cause a change in the mechanical energy E_{mech} of the system

8.6 Work Done on a System by an External Force



Applying Newton's second law to book.:

$$-f_k = ma$$

Since the friction force is constant, acceleration is constant too:

$$v_s^2 = v_i^2 + 2ad$$

Solving this equation for a , substituting the result into the first equation

8.6 Work Done on a System by an External Force

$$-f_k d = \frac{1}{2}mv_s^2 - \frac{1}{2}mv_i^2$$

$$-f_k d = \Delta K$$

In a more general situation (say, one in which the block is moving up a ramp), there can be a change in potential energy

$$\Delta K + \Delta U = -f_k d$$

If the friction do work, the change in mechanical energy is given by:

$$\boxed{\Delta E_{mek} = -f_k d}$$

$-f_k d$ is the amount by which the mechanical energy of the system changes because of the force of kinetic friction.

By experiment we know that the book and the portion of the floor along which it slides become warmer as the block slides.

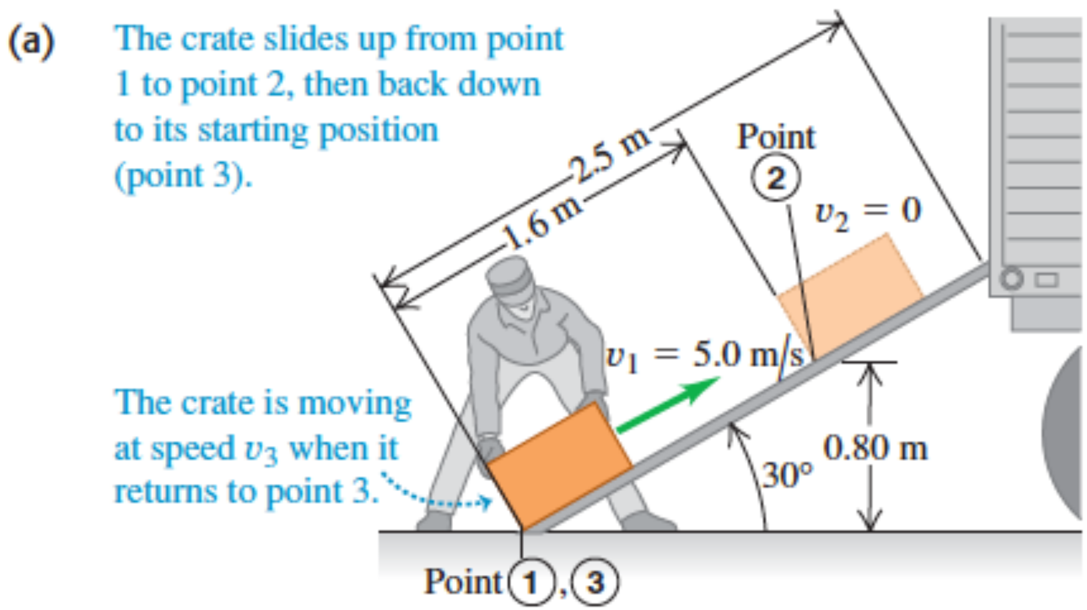
+ 8.6 Work Done on a System by an External Force

Example:6

We want to slide a 12-kg crate up a 2.5-m-long ramp inclined at 30°. A worker, ignoring friction, calculates that he can do this by giving it an initial speed of 5.0 m/s at the bottom and letting it go. But friction is not negligible; the crate slides only 1.6 m up the ramp, stops, and slides back down.

(a) Find the magnitude of the friction force acting on the crate, assuming that it is constant.

(b) How fast is the crate moving when it reaches the bottom of the ramp?



8.6 Work Done on a System by an External Force

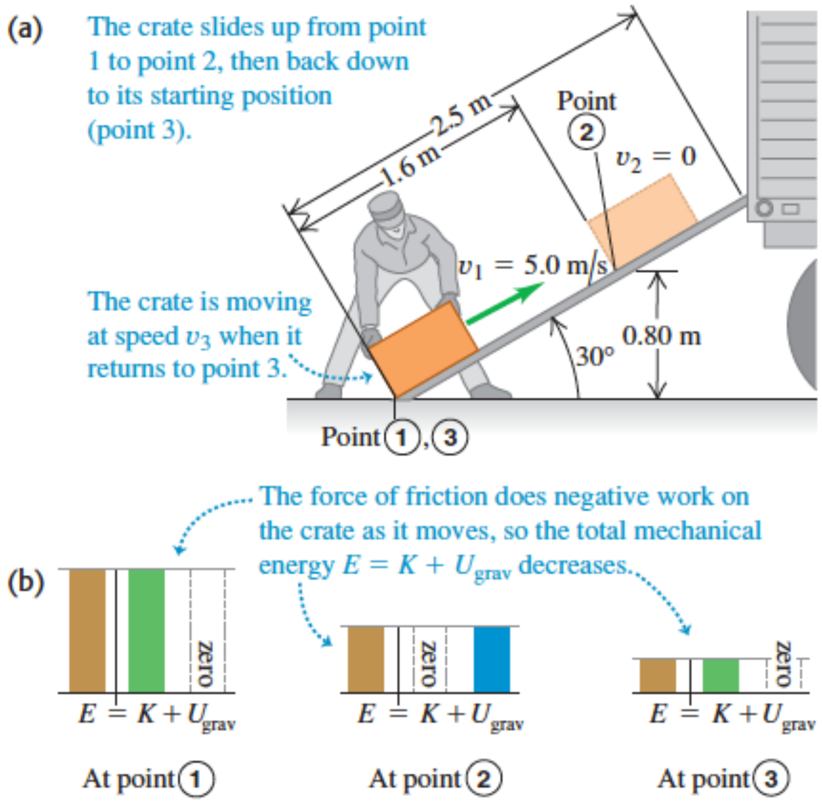
Answer:6a

$$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(12kg)(5.0m/s^2) = 150J$$

$$U_{g,1} = 0$$

$$K_2 = 0$$

$$U_{g,2} = mgy = (12kg)(9.8m/s^2)(0.80m) = 94J$$



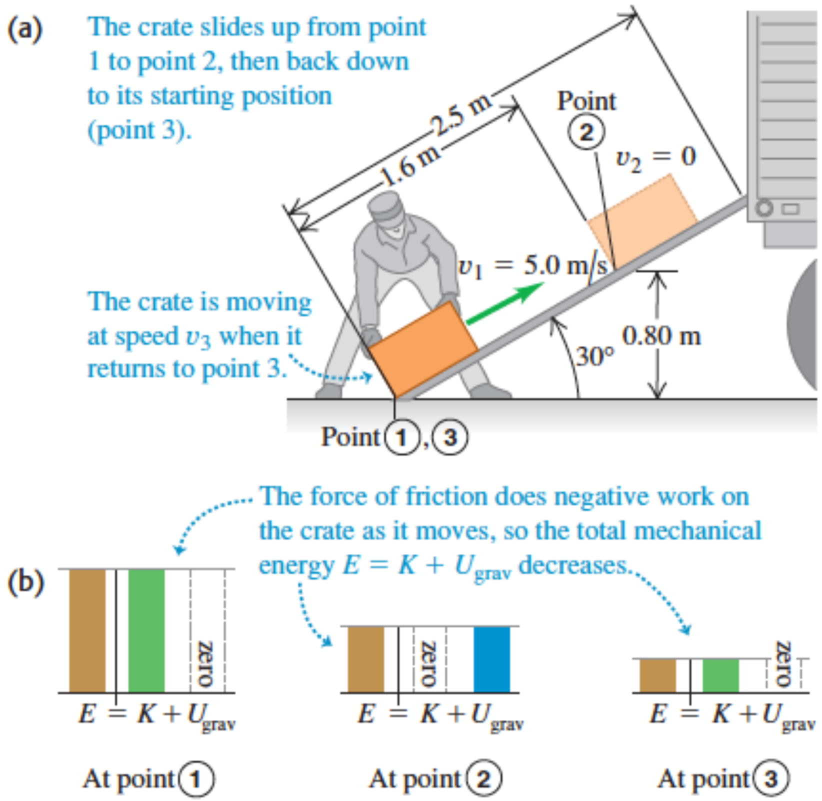
$$\Delta K + \Delta U = -f_k d$$

$$(K_2 + U_{g,2}) - (K_1 + U_{g,1}) = -f_k d$$

$$(0 + 94) - (K_1 + 0) = -(35N)(0.80m)$$

+ 8.6 Work Done on a System by an External Force

Answer:6b



As the crate moves from point 1 to point 2, the work done by friction:

$$W_{fric.,1-2} = -f_k d = -56J$$

As the crate moves from point 2 to point 3, the work done by friction:

$$W_{fric.,2-3} = -f_k d = -56J$$

$$\Delta K + \Delta U = -f_k d$$

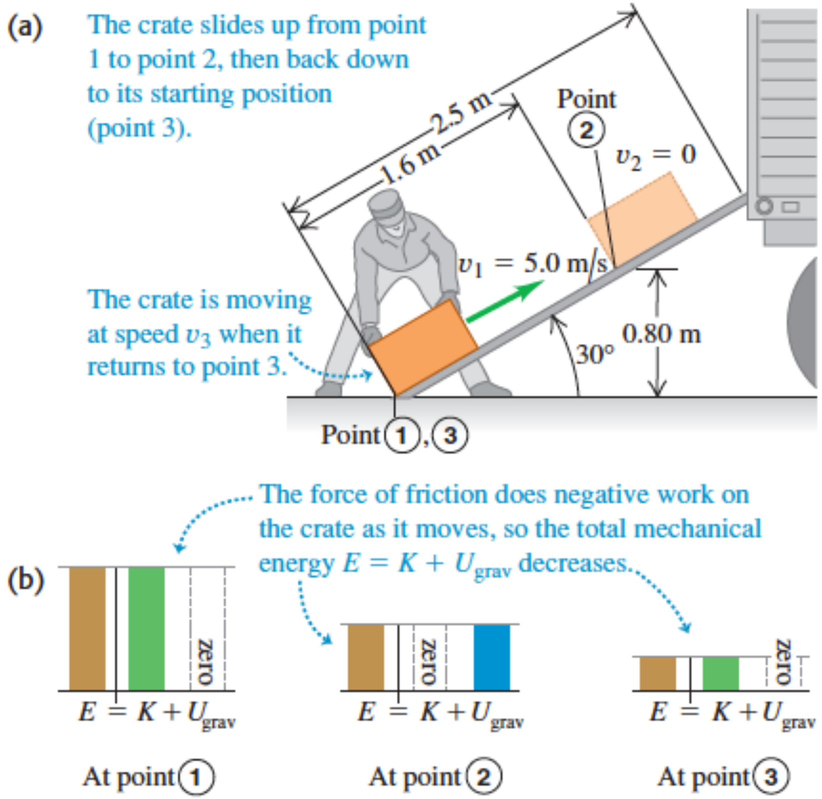
$$(K_2 + U_{g,2}) - (K_1 + U_{g,1}) = -f_k d$$

$$(0 + 94) - (150 + 0) = -f_k (0.80m)$$

$$f_k = 35N$$

+ 8.6 Work Done on a System by an External Force

Answer:6b



As the crate moves from point 2 to point 3, the work done by friction:

$$W_{fric.,2-3} = -f_k d = -56J$$

$$\Delta K + \Delta U = W_{fric.,2-3}$$

$$(K_3 + U_{g,3}) - (K_2 + U_{g,2}) = W_{fric.,2-3}$$

$$(K_3 + 0) - (0 + 94) = -56$$

$$K_3 = 38J$$

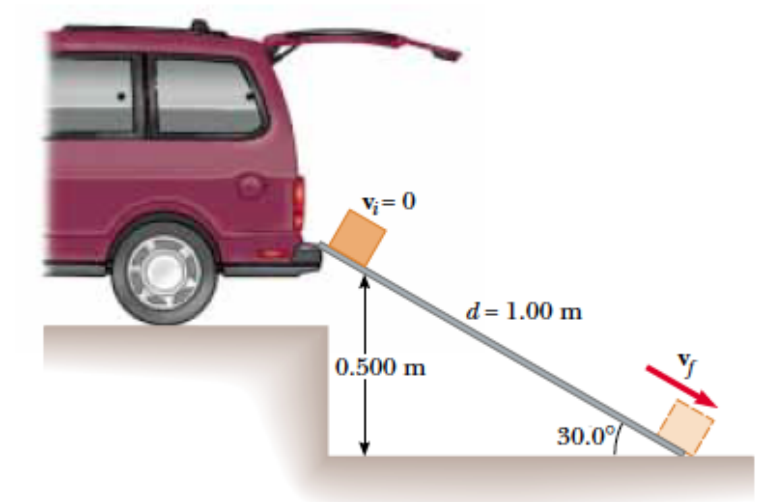
$$K_3 = \frac{1}{2} m v_3^2 = 38J$$

$$v_3 = \sqrt{\frac{2(38J)}{12kg}} = 2.5m/s$$

+ 8.6 Work Done on a System by an External Force

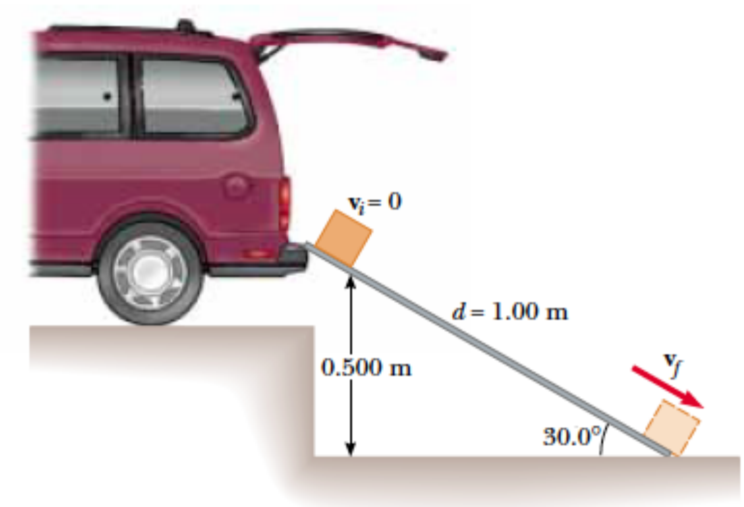
Example:7

A 3.00-kg crate slides down a ramp. The ramp is 1.00 m in length and inclined at an angle of 30.0°, as shown in Figure. The crate starts from rest at the top, experiences a constant friction force of magnitude 5.00 N, and continues to move a short distance on the horizontal floor after it leaves the ramp. Use energy methods to determine the speed of the crate at the bottom of the ramp?



8.6 Work Done on a System by an External Force

Answer:7



When the packet is top :

$$E_{top} = K_{top} + U_{top}$$

$$E_{top} = 0 + 3(9.8).0.5$$

$$E_{top} = 14.7J$$

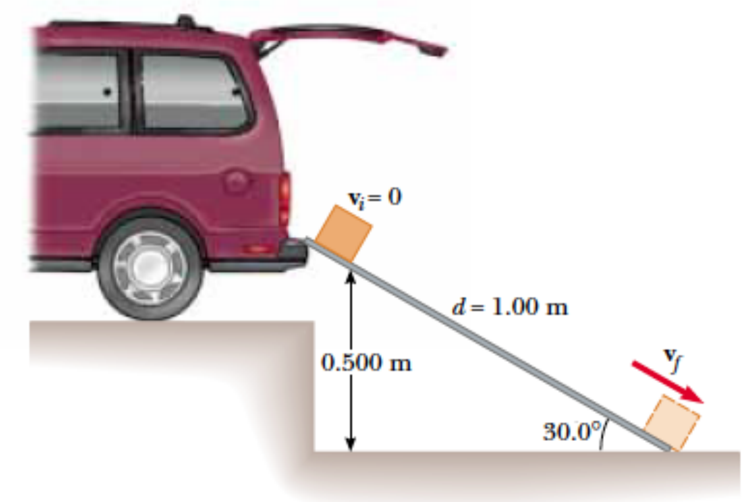
When the packet is down:

$$E_{down} = K_{down} + U_{down}$$

$$E_{down} = \frac{1}{2}mv^2 + 0 = \frac{1}{2}mv^2$$

8.6 Work Done on a System by an External Force

Answer:7



So:

$$\Delta E_{mec} = -f_k d$$

$$E_{down} - E_{top} = -f_k d$$

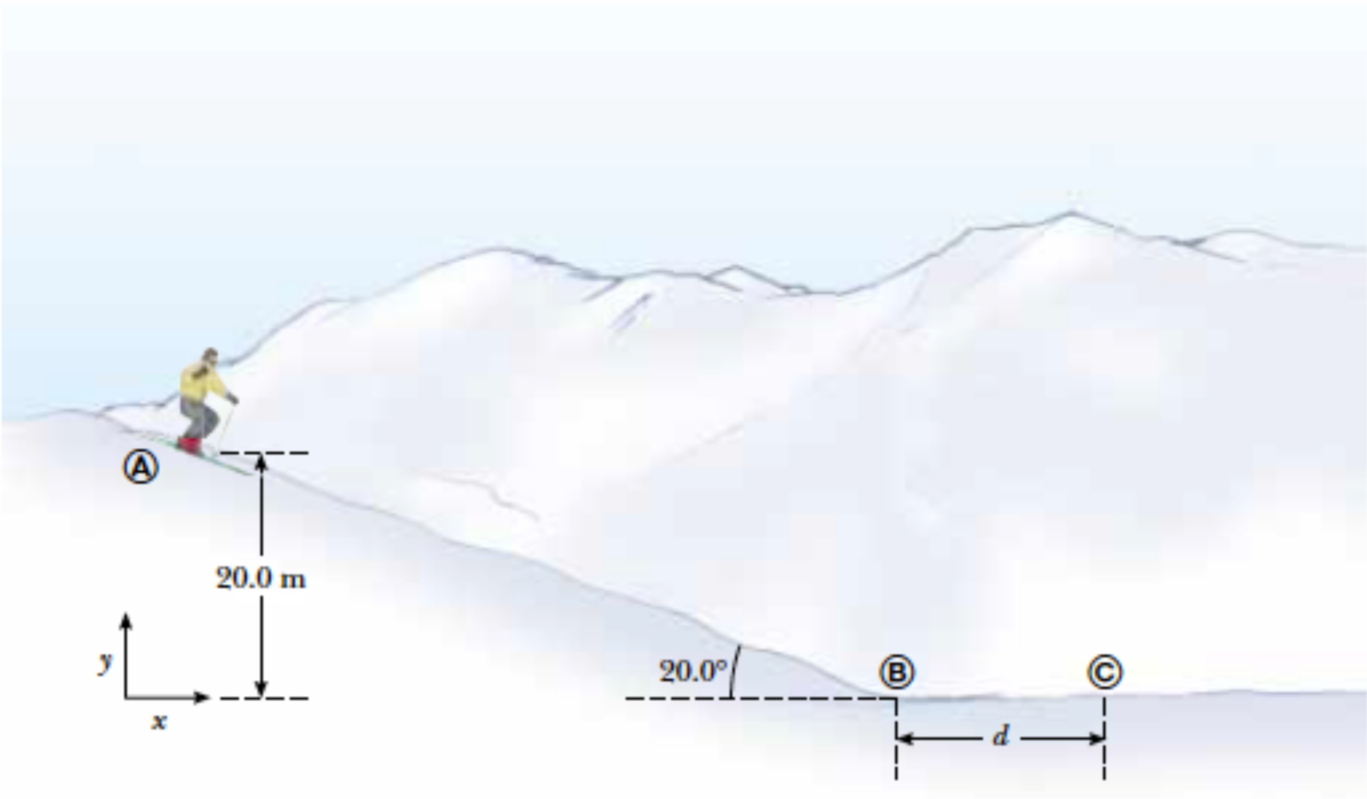
$$\frac{1}{2}mv^2 - 14.7 = -5.1$$

$$v = 2.54 m/s$$

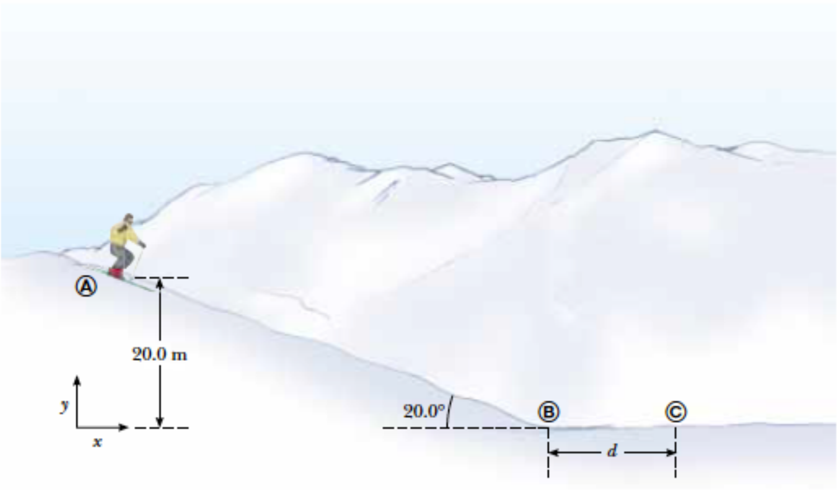
+ 8.6 Work Done on a System by an External Force

Example:8

A skier starts from rest at the top of a frictionless incline of height 20.0 m, as shown in Figure. At the bottom of the incline, she encounters a horizontal surface where the coefficient of kinetic friction between the skis and the snow is 0.210. How far does she travel on the horizontal surface before coming to rest, if she simply coasts to a stop?



Answer:8



From A to B:

$$K_A + U_A = K_B + U_B$$

$$0 + mgh = \frac{1}{2}mv_B^2 + 0$$

$$v_B = \sqrt{2gh} = 19.8m/s$$

From B to C:

$$\Delta K + \Delta U = -f_k d$$

$$(\frac{1}{2}mv_B^2 + 0) - (0 + 0) = -\mu_k mgd$$

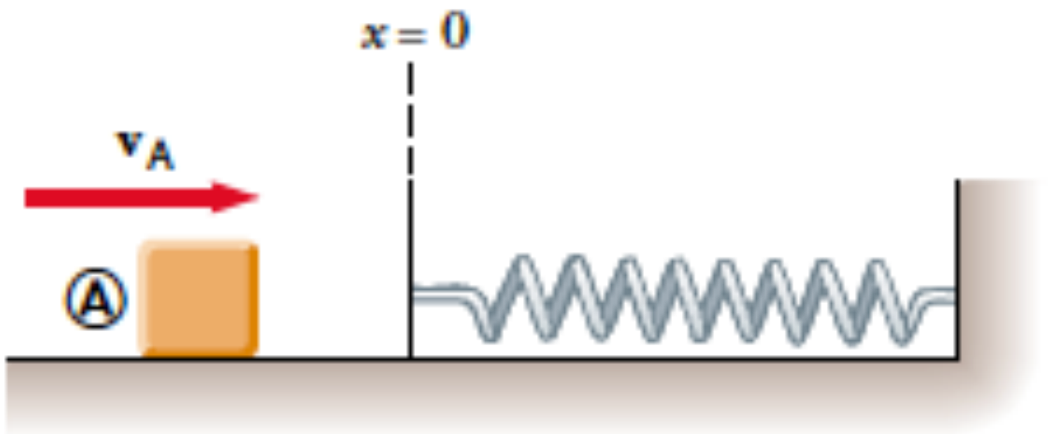
$$d = 95.2m$$

8.6 Work Done on a System by an External Force

Example:9

A block having a mass of 0.80 kg is given an initial velocity v_A = 1.2 m/s to the right and collides with a spring of negligible mass and force constant k = 50 N/m, as shown in Figure

a) Assuming the surface to be frictionless, calculate the maximum compression of the spring after the collision.
b) Suppose a constant force of kinetic friction acts between the block and the surface, with) μ_k = 0.50. If the speed of the block at the moment it collides with the spring is v_A = 1.2 m/s, what is the maximum compression x_C in the spring?



+ 8.6 Work Done on a System by an External Force

Answer:9a

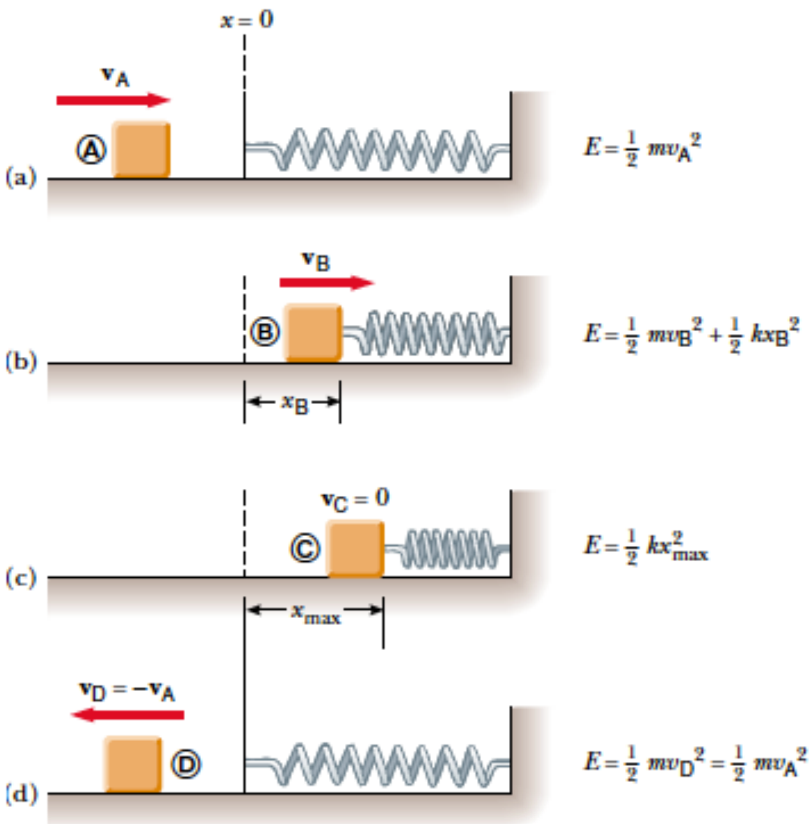


Figure 8.14 (Example 8.9) A block sliding on a smooth, horizontal surface collides with a light spring. (a) Initially the mechanical energy is all kinetic energy. (b) The mechanical energy is the sum of the kinetic energy of the block and the elastic potential energy in the spring. (c) The energy is entirely potential energy. (d) The energy is transformed back to the kinetic energy of the block. The total energy of the system remains constant throughout the motion.

$$E_{mec,A} = E_{mec,C}$$

$$K_A + U_A = K_c + U_c$$

$$\frac{1}{2}mv_A^2 + 0 = 0 + \frac{1}{2}kx_C^2$$

$$x_C = 0.15m$$

Answer:9b

$$\Delta E_{mec} = -f_k x_C$$

$$(K_c + U_c) - K_A + U_A = -f_k x_C$$

$$(0 + \frac{1}{2}kx_C^2) - (\frac{1}{2}mv_A^2 + 0) = -\mu_k m g x_c$$

$$25x_C^2 + 3.92x_C - 0.576 = 0$$

$$x_C = -0.25m \qquad \boxed{x_C = 0.092m}$$

8.7. Relationship Between Conservative Forces and Potential Energy

The work done by a conservative force F as a particle moves along the x axis is:

$$W = \int_{x_i}^{x_s} F(x)\,dx$$

where Fx is the component of F in the direction of the displacement.

Yapılan bu iş potansiyel enerjideki değişim negatifine eşit. Böylece

$$\Delta U = -\int_{x_i}^{x_s} F(x)\,dx$$

equals the negative of the change in the potential energy associated with that force when the configuration of the system changes,

8.7. Relationship Between Conservative Forces and Potential Energy

Gravitational potential energy:

$$\Delta U = -\int_{y_i}^{y_s} (-mg)\,dy$$

$$U_s - U_i = mgy_s - mgy_i$$

$$U = mgy$$

Elastic potential energy:

$$\Delta U = -\int_{x_i}^{x_s} (-kx)\,dx$$

$$U_s - U_i = \frac{1}{2}kx_s^2 - \frac{1}{2}kx_i^2$$

$$U = \frac{1}{2}kx^2$$

8.8 Reading a Potential Energy Curve

$$\Delta U = -\int_{x_i}^{x_s} F(x)\,dx$$

This equation tells us how to find the change ΔU in potential energy between two points in a one-dimensional situation if we know the force F (x). But if we know the potential energy function U (x), we can find the force associated with it::

$$W = -\Delta U$$

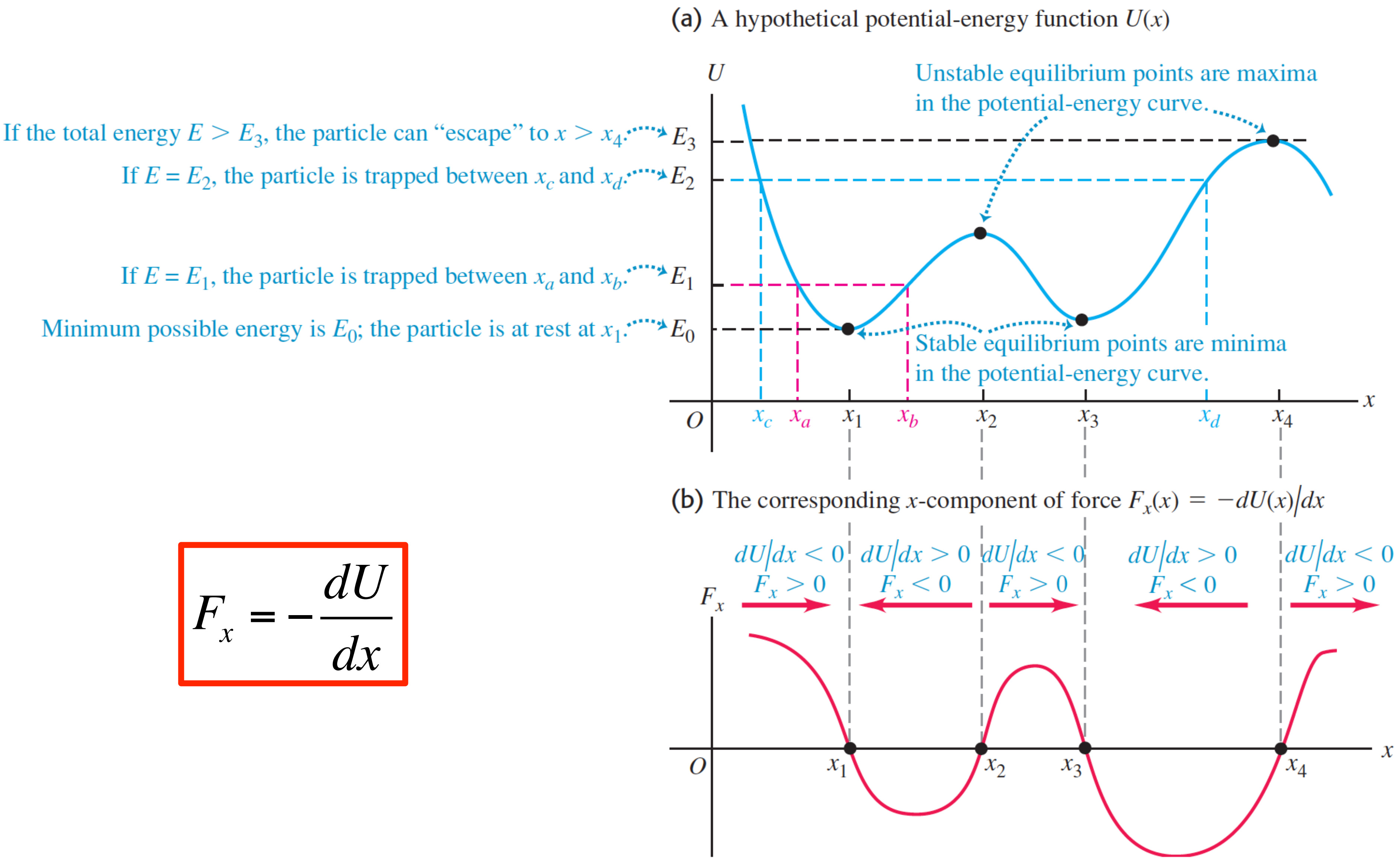
$$\Delta F_x \Delta x = -\Delta U$$

$$\Delta F_x = -\frac{\Delta U}{\Delta x}$$

Solving for F (x) and passing to the differential limit yield

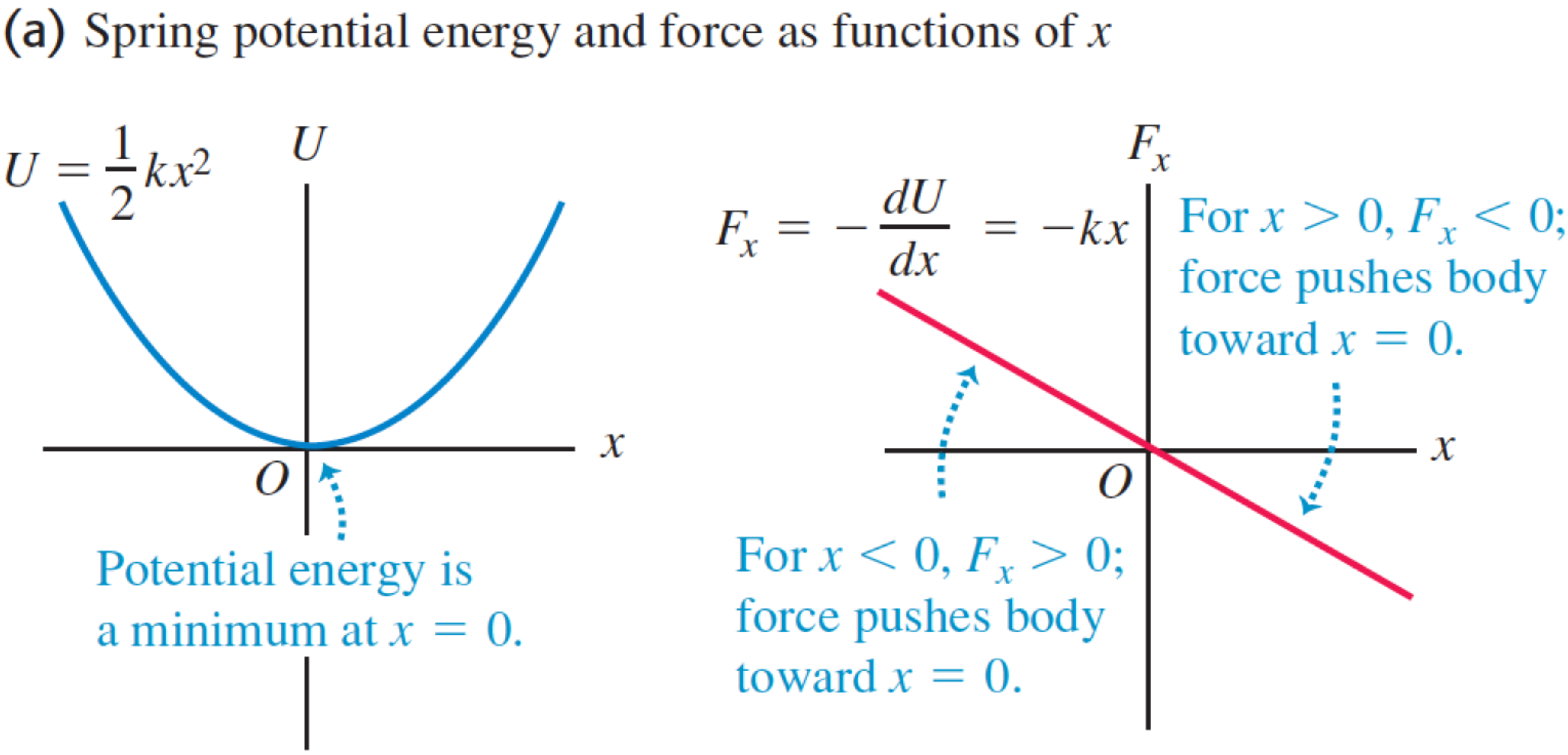
$$F_x = -\frac{dU}{dx}$$

8.8 Reading a Potential Energy Curve



$$F_x = -\frac{dU}{dx}$$

8.8 Reading a Potential Energy Curve



8.8 Reading a Potential Energy Curve

(b) Gravitational potential energy and force as functions of y