Chapter 2 Electric Fields

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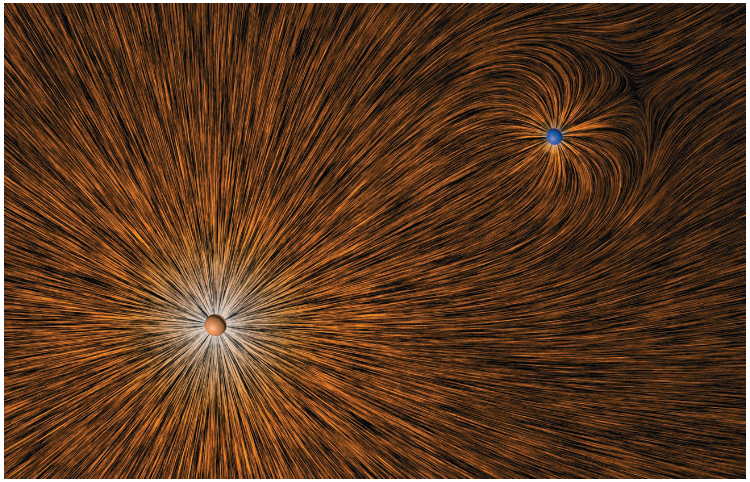
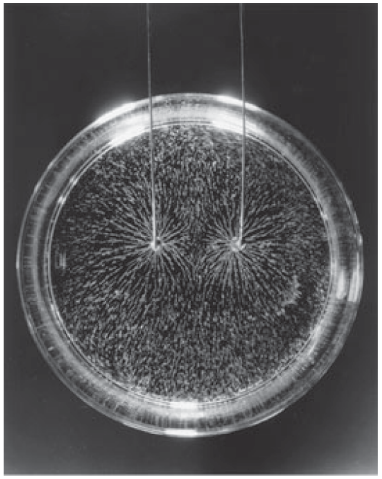
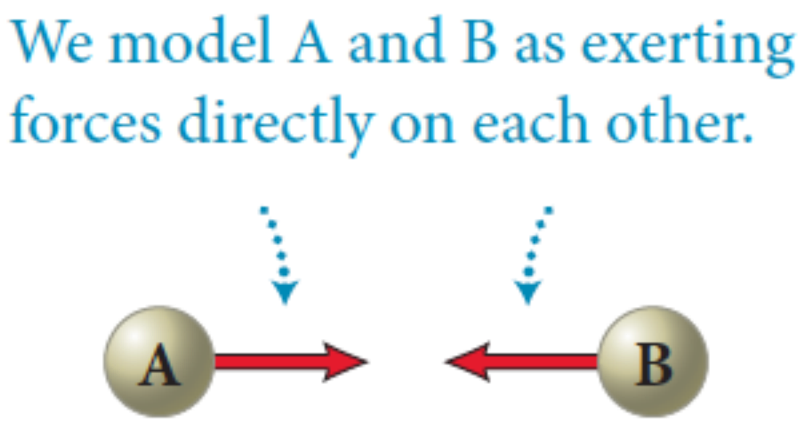




2.1 Electric Field

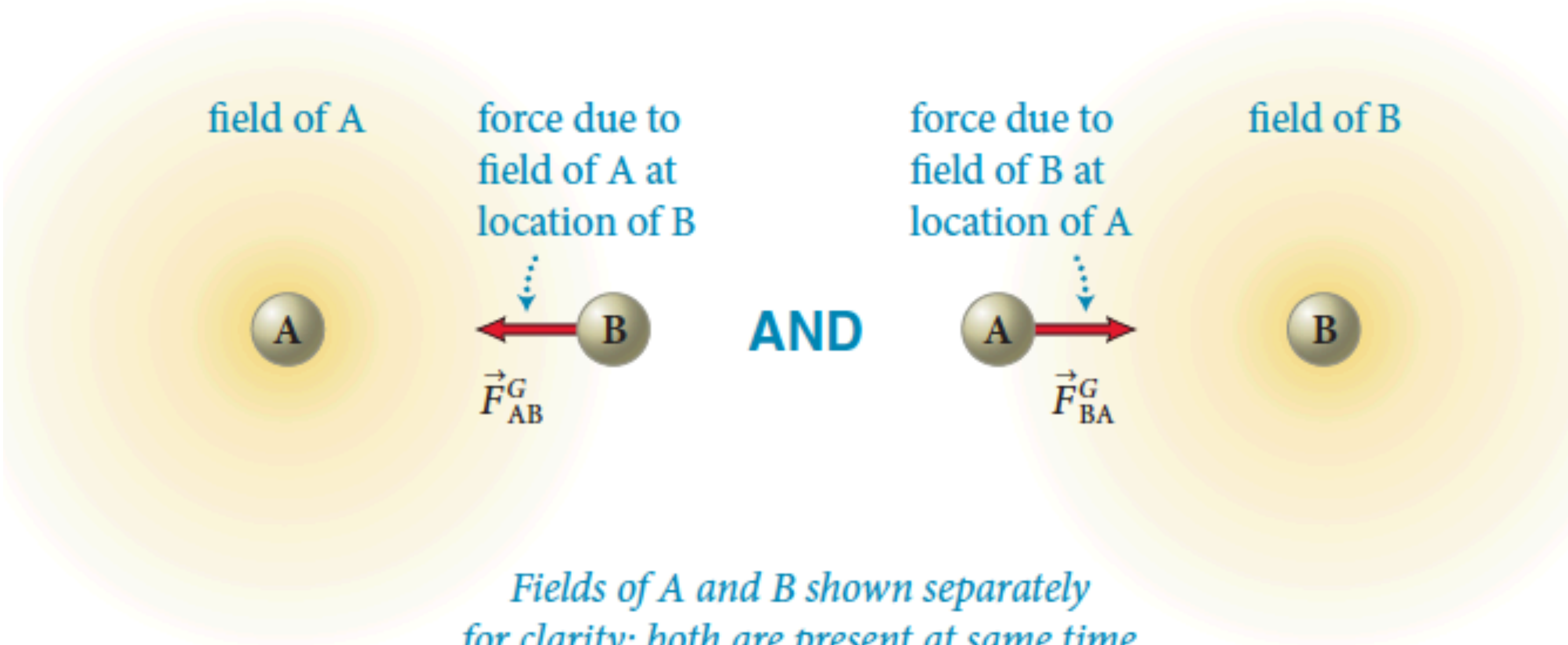


How does particle 1 "know" of the presence of particle 2?

We can adopt a model of long-range interactions, a model in which interactions take place through the intermediary of an interaction field (or simply a field).

2.1 Electric Field

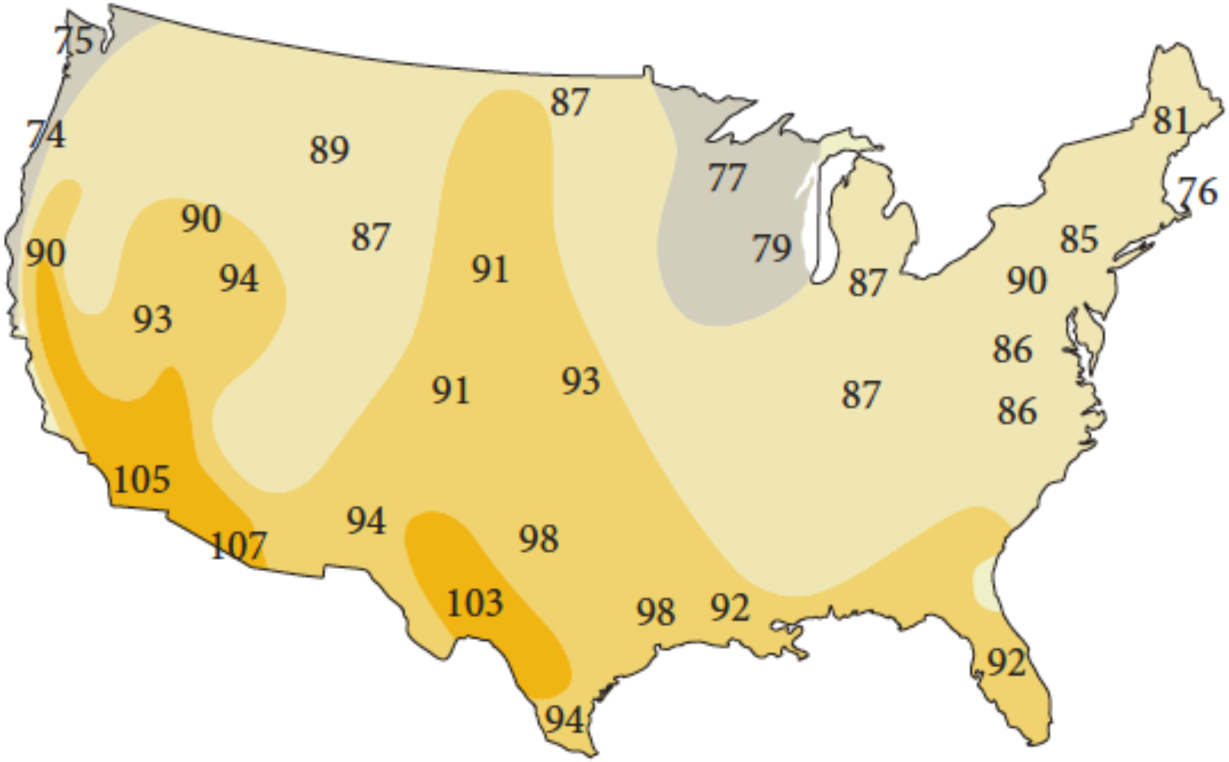
We can adopt a model of long-range interactions, a model in which interactions take place through the intermediary of an interaction field (or simply a field).



Fields of A and B shown separately for clarity; both are present at same time.

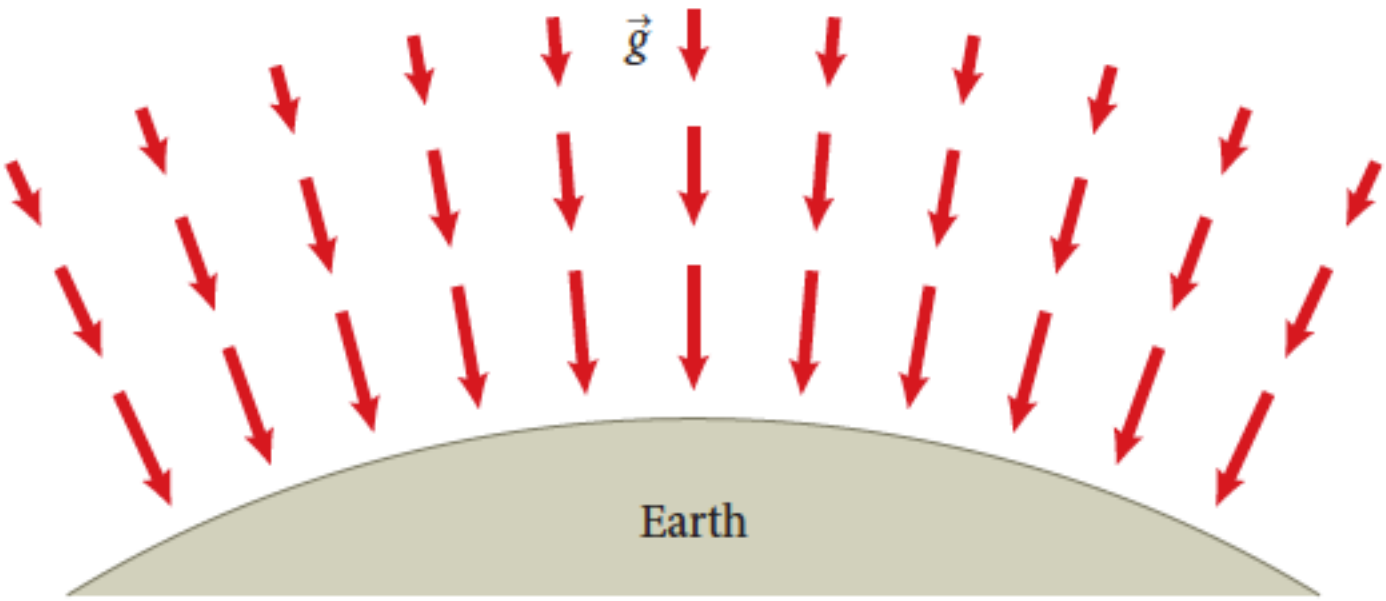
Examples of fields : Temperature (or Pressure) field (Scalar !)

Figure 23.3 The temperature across a region is specified by a set of values, with a specific temperature value for every position in that region. Such a set of values is called a *field*.



Examples of fields : Gravitational field (Vector!)

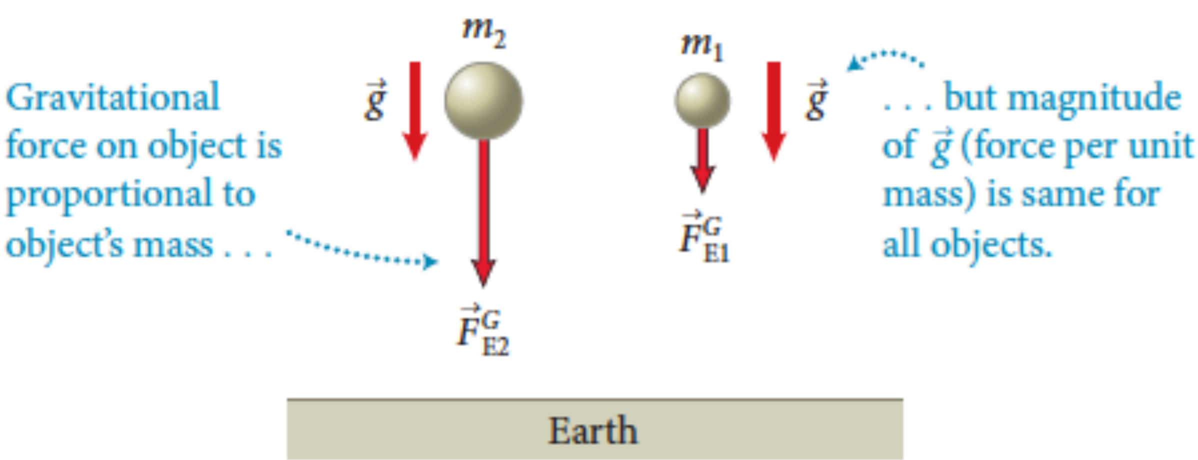
Figure 23.5 Vector field diagram for the gravitational field in a region near Earth.



2.1 Electric Field

Examples of fields : Gravitational field (Vector!)

Figure 23.4 Comparison between gravitational force and gravitational acceleration on objects of different mass at the same distance from Earth.

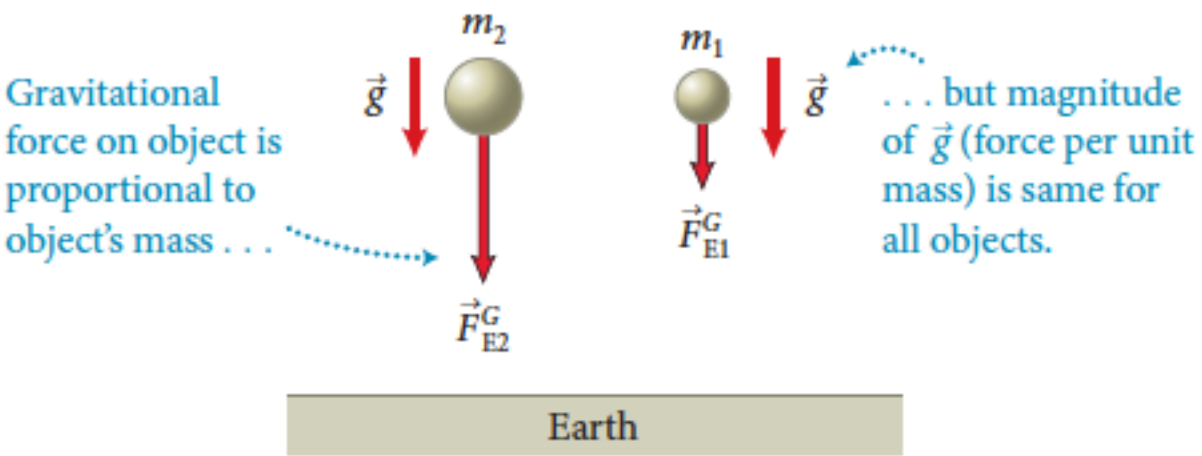


$$\boxed{F_g^1 = m_1 g} \qquad \boxed{F_g^2 = m_2 g}$$

The gravitational force is not a good quantity for describing Earth's field.

Examples of fields : Gravitational field (Vector!)

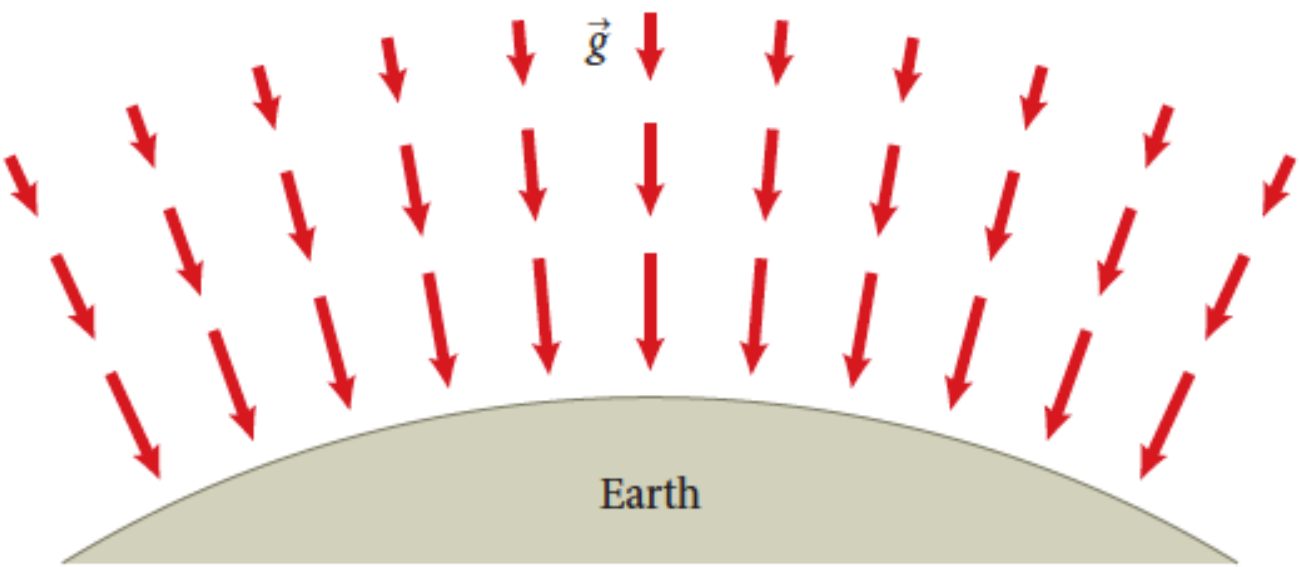
Figure 23.4 Comparison between gravitational force and gravitational acceleration on objects of different mass at the same distance from Earth.



$$\boxed{g = F_g^1 / m_1 = F_g^2 / m_2}$$

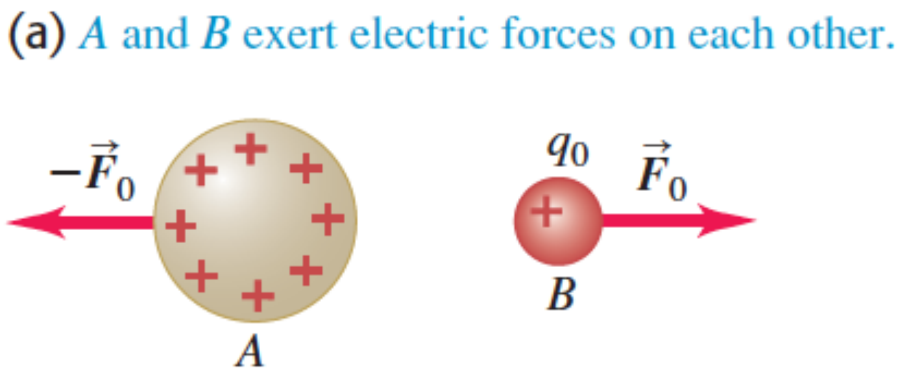
Examples of fields : Gravitational field (Vector!)

Figure 23.5 Vector field diagram for the gravitational field in a region near Earth.



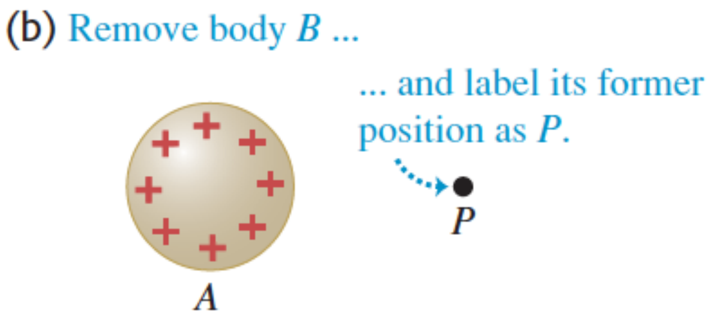
At any given location in the space surrounding a source object S, the magnitude of the gravitational field created by S is the magnitude of the gravitational force exerted on an object B placed at that location divided by the mass of B.

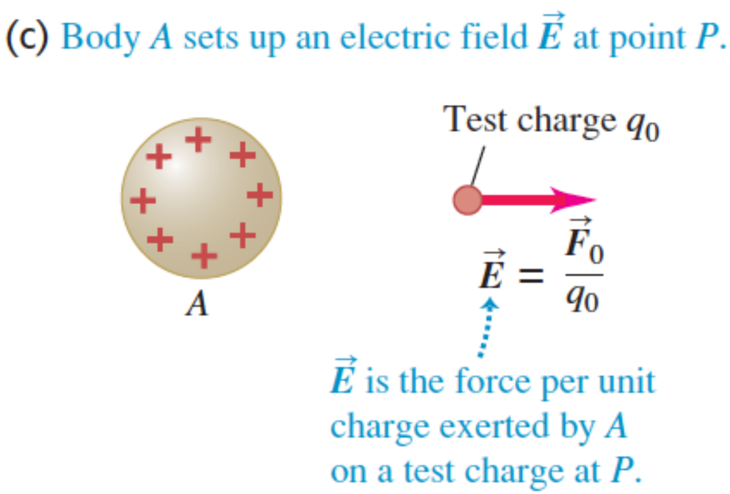
2.1 Electric Field



In this approach, an electric field is said to exist in the region of space around a charged object—**the source charge**. When another charged object—the test charge—enters this electric field, an electric force acts on it.

2.1 Electric Field

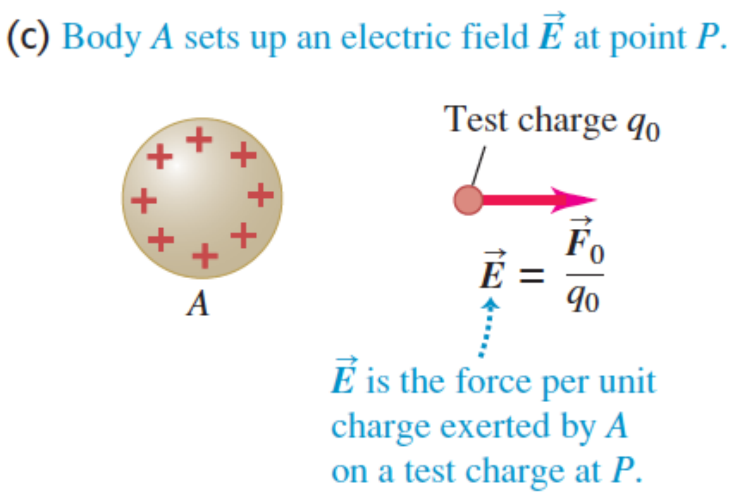




The electric field vector **E** at a point in space is defined as the electric force F_o acting on a positive test charge q_0 placed at that point divided by the test charge:

$$\boxed{\vec{E} = \frac{\vec{F}_o}{q_o}}$$

2.1 Electric Field



$$\boxed{\vec{E} = \frac{\vec{F}_o}{q_o}}$$

$$\boxed{\vec{F}_o = q_o\vec{E}}$$

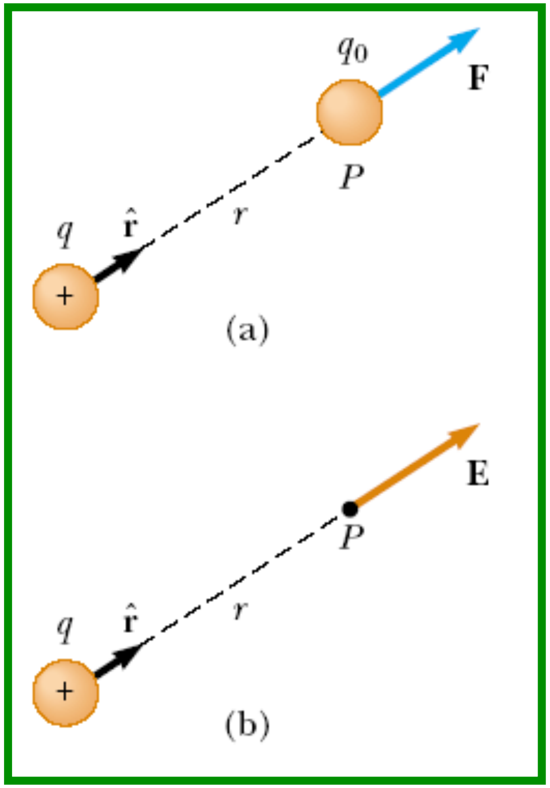
- **E is the field produced by some charge or charge distribution separate from the test charge.**
- **The existence of an electric field is a property of its source, the presence of the test charge is not necessary for the field to exist.**
- **The test charge serves as a detector of the electric field.**

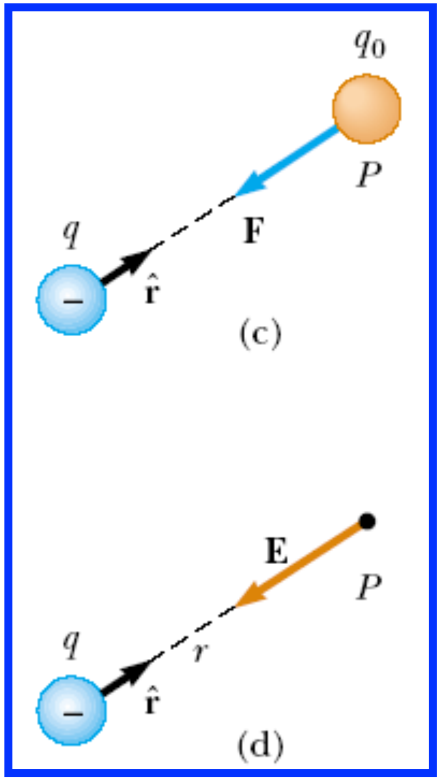
2.1 Electric Field

$$\boxed{\vec{E} = \frac{\vec{F}_o}{q_o}}$$

$$\boxed{\vec{F}_o = q_o\vec{E}}$$

The direction of E at a given location is the same as the direction of the electric force exerted on a positive test charge at that location.



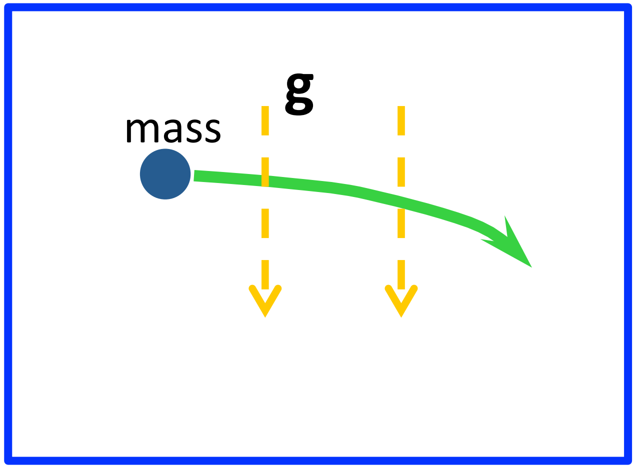


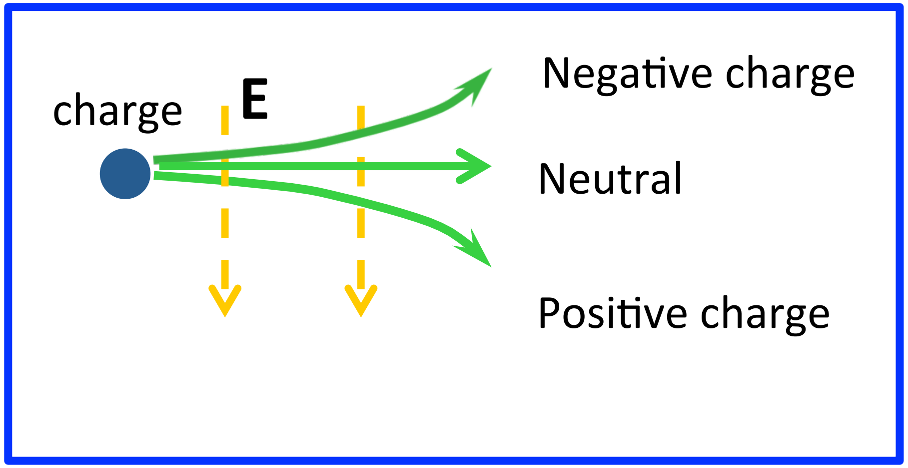
2.1 Electric Field

$$\boxed{\vec{F}_o = q_o \vec{E}}$$

$$\boxed{\vec{E} = \frac{\vec{F}_o}{q_o}}$$

If q positive, the direction of force exerted on q is same as the direction of electric field.
If q negative, the direction of force exerted on q is opposite of the direction of electric field.



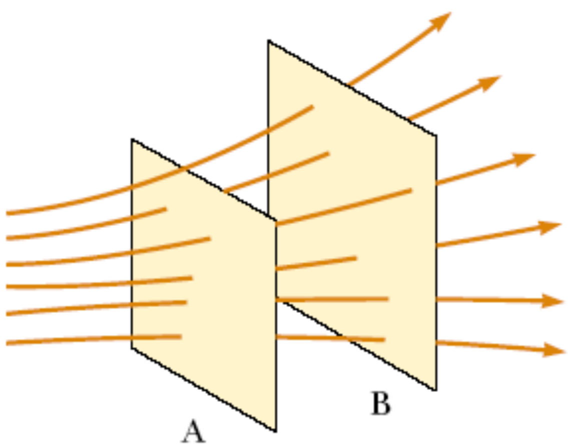


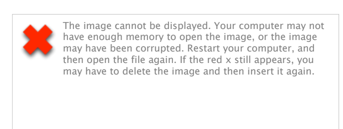
2.2 Electric Field Lines

A nice way to visualize electric field concept is to draw curved lines that are parallel to the electric field vector at any point in space. These lines, called electric field lines and first introduced by Faraday, are related to the electric field in a region of space in the following manner:

- The electric field vector E is tangent to the electric field line at each point. The line has a direction, indicated by an arrowhead, that is the same as that of the electric field vector.

- The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region. Thus, the field lines are close together where the electric field is strong and far apart where the field is weak.





2.2 Electric Field Lines

The rules for drawing electric field lines are as follows:

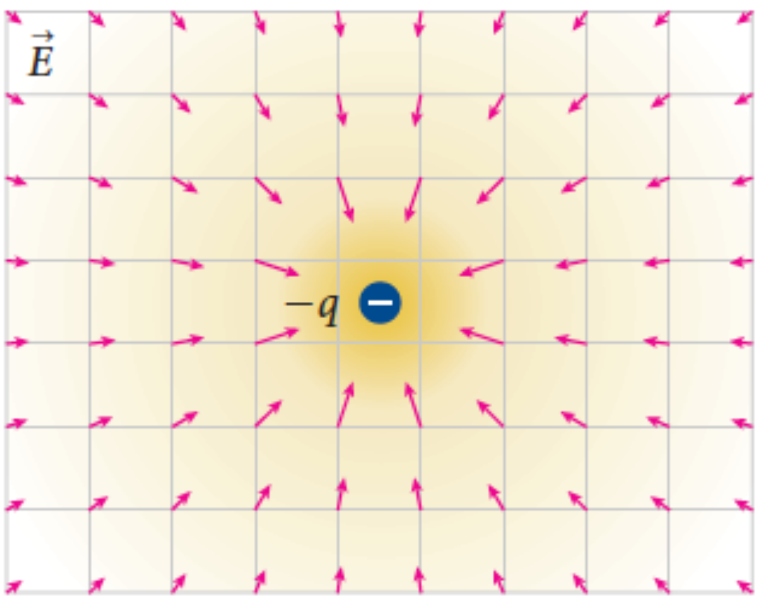
- **The lines must begin on a positive charge and terminate on a negative charge. In the case of an excess of one type of charge, some lines will begin or end infinitely far away.**

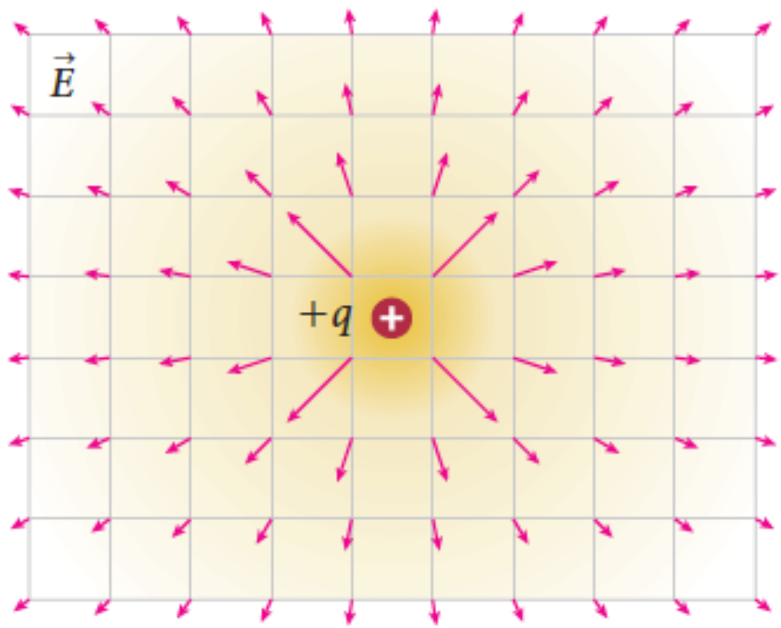
- **The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.**

- **No two field lines can cross.**

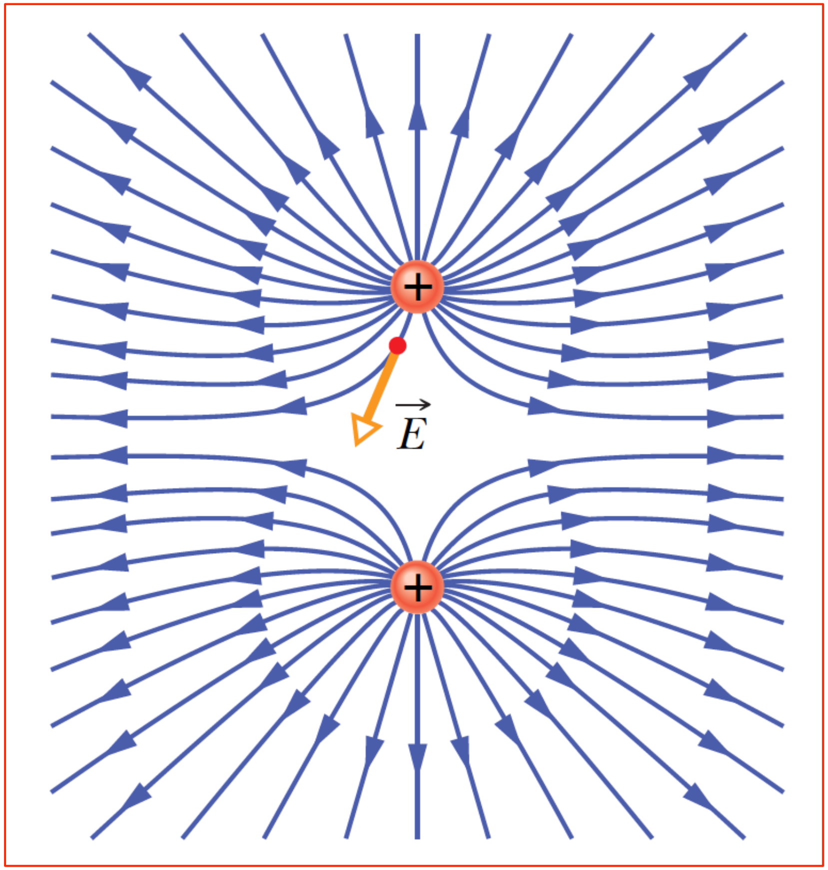
2.2 Electric Field Lines

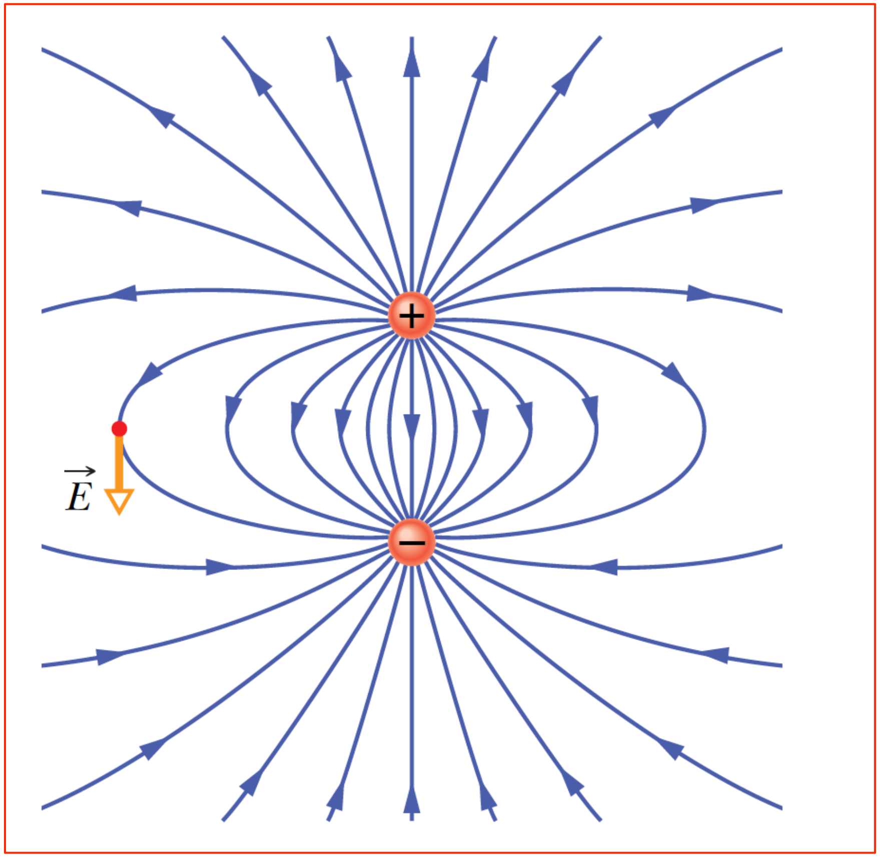
Vector field diagrams for the electric fields of particles that carry positive and negative charges.





2.2 Electric Field Lines





2.3 Electric Field Due to a Point Charge

From Coulomb's law the electrostatic force vector acting on q_0 test charge is

$$\vec{F}_o = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r^2}\hat{r}$$

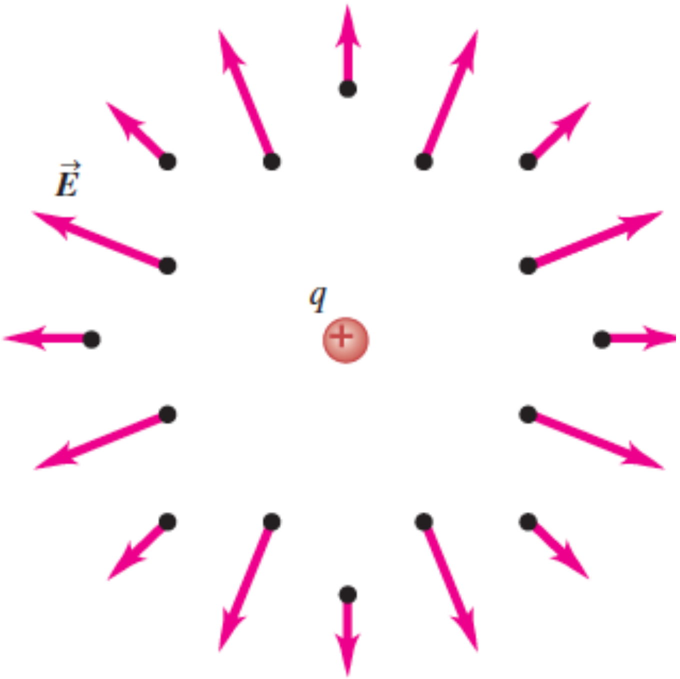
From the definition of electric field, the electric field vector created by q is

$$\vec{E} = \frac{\vec{F}_o}{q_o} \Rightarrow \qquad \boxed{\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r}}$$

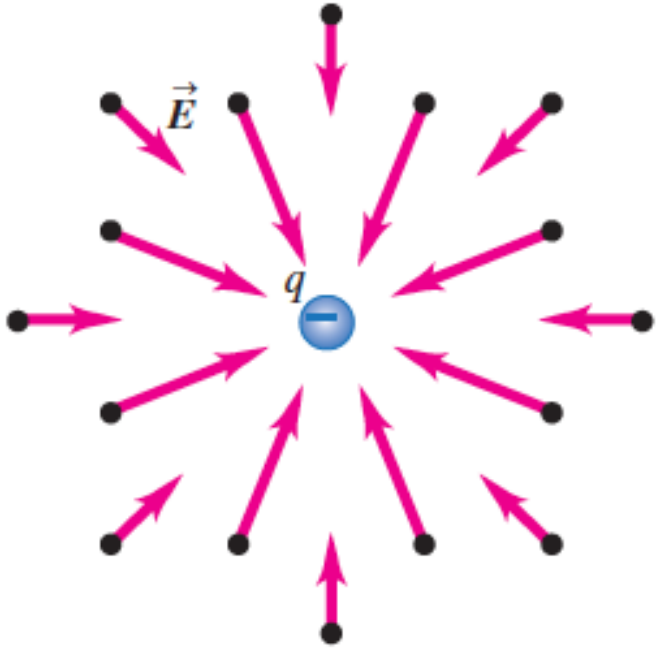
2.3 Electric Field Due to a Point Charge

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r}}$$

(a) The field produced by a positive point charge points *away from* the charge.



(b) The field produced by a negative point charge points *toward* the charge.



2.3 Electric Field Due to a Point Charge

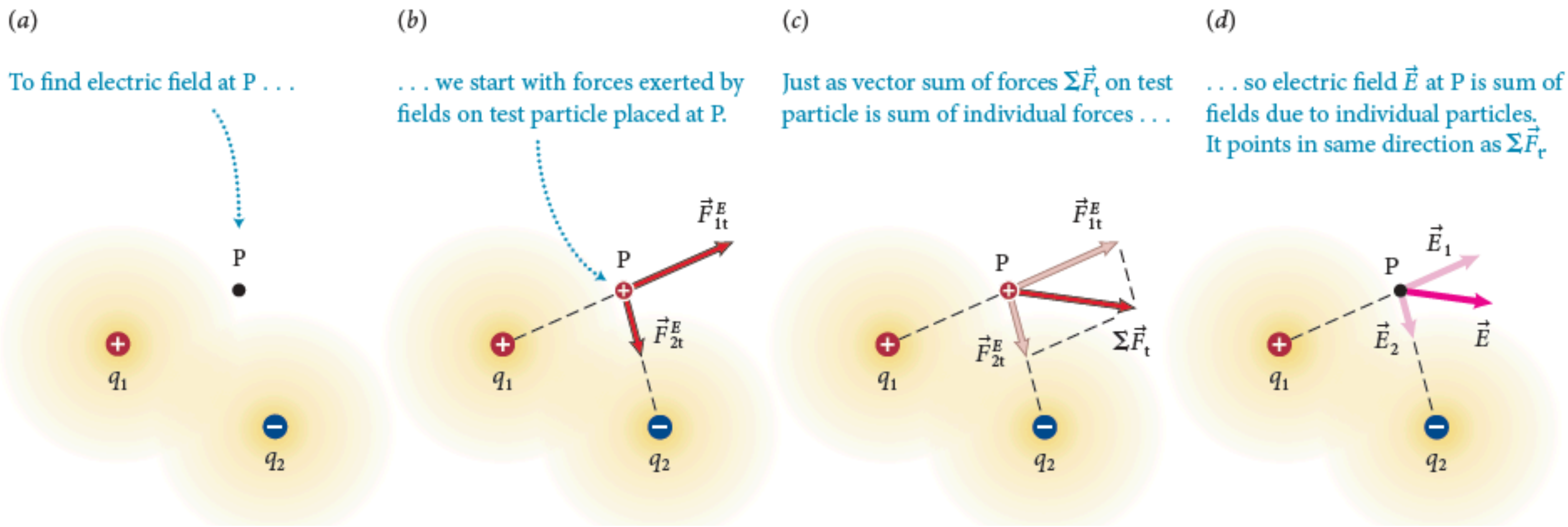
The combined electric field created by a collection of charged objects is equal to the vector sum of the electric fields created by the individual objects.

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_o}\sum_i \frac{q_i}{r_i^2}\hat{r}_i}$$

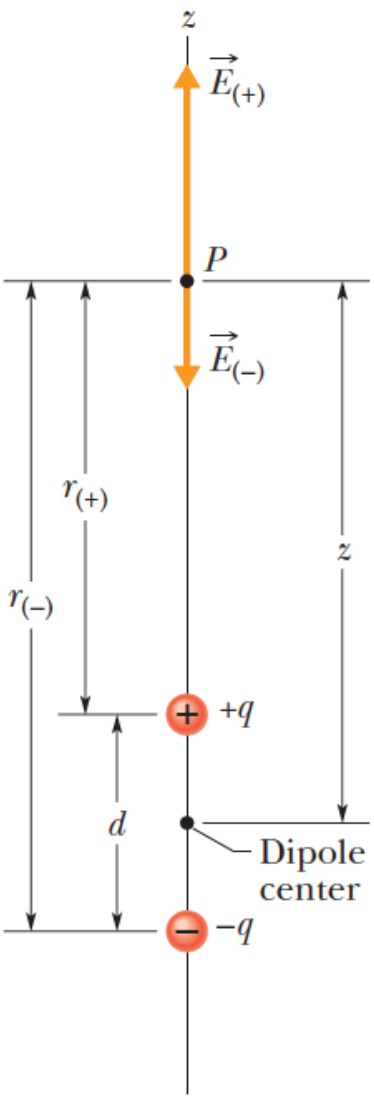
r_i is the distance from the I th source charge qi to the point P

$\hat{r}_i$: is a unit vector directed from qi toward P

2.3 Electric Field Due to a Point Charge



2.4 The Electric Field Due to an Electric Dipole



Applying the superposition principle for electric fields, we find that the magnitude E of the electric field at P:

$$E = E_{(+)} - E_{(-)}$$

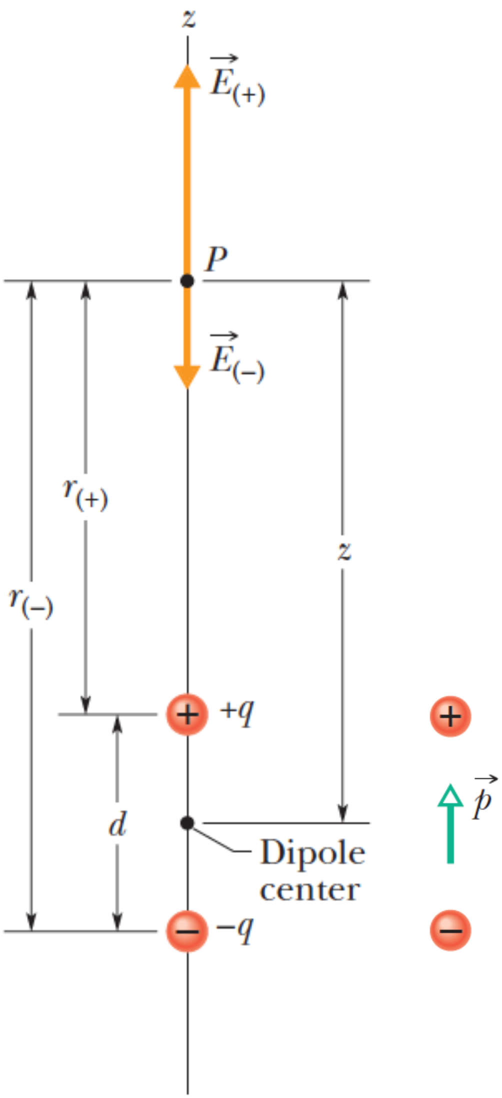
$$= \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{r_{(+)}^2} - \frac{q}{r_{(-)}^2}\right)$$

$$= \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{(z-\frac{1}{2}d)^2} - \frac{1}{(z+\frac{1}{2}d)^2}\right)$$

$$= \frac{q}{4\pi\varepsilon_o z^3}\left(\frac{d}{(1-(\frac{d}{2z})^2)^2}\right)$$

for z>> d

2.4 The Electric Field Due to an Electric Dipole



$$E = \frac{q}{2\pi\varepsilon_o z^3}\left(\frac{d}{\left(1-\left(\frac{d}{2z}\right)^2\right)^2}\right)$$

for z>> d is d/2z << 1 and therefore we obtain

$$\boxed{E = \frac{qd}{2\pi\varepsilon_o z^3}} \qquad \boxed{E = \frac{p}{2\pi\varepsilon_o z^3}}$$

p= qd : the magnitute of electric dipol moment (Vector !!!)

The direction of electric dipol moment vector **P** is taken to be from the negative to the positive end of the dipole

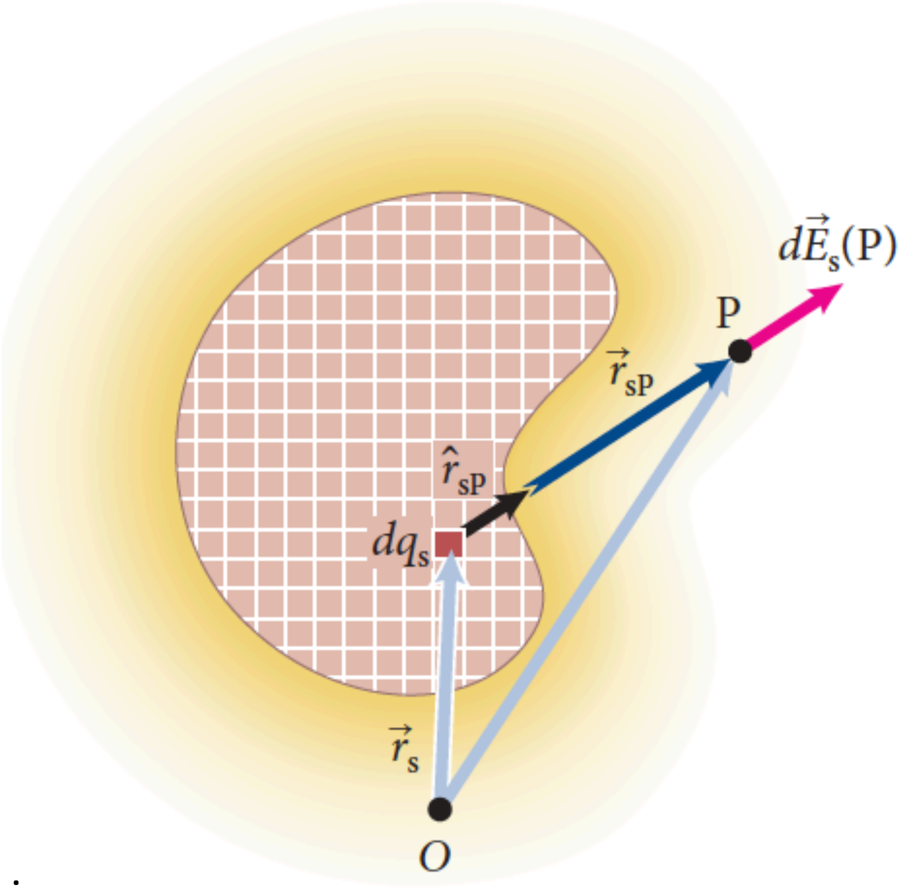
Homework 1:

A charge q_1= 7.0 μC is located at the origin, and a second charge q_2=5.0 μC is located on the x axis, 0.30 m from the origin. Find the electric field at the point P, which has coordinates (0, 0.40) m.

2.5 Electric Field of Due to Continues Charge Distribution

We can use Coulomb's law to obtain the infinitesimal portion of the electric field at point P contributed by a segment:

$$d\vec{E} = k\frac{dq_s}{r_{sP}^2}\hat{r}_{sP}$$

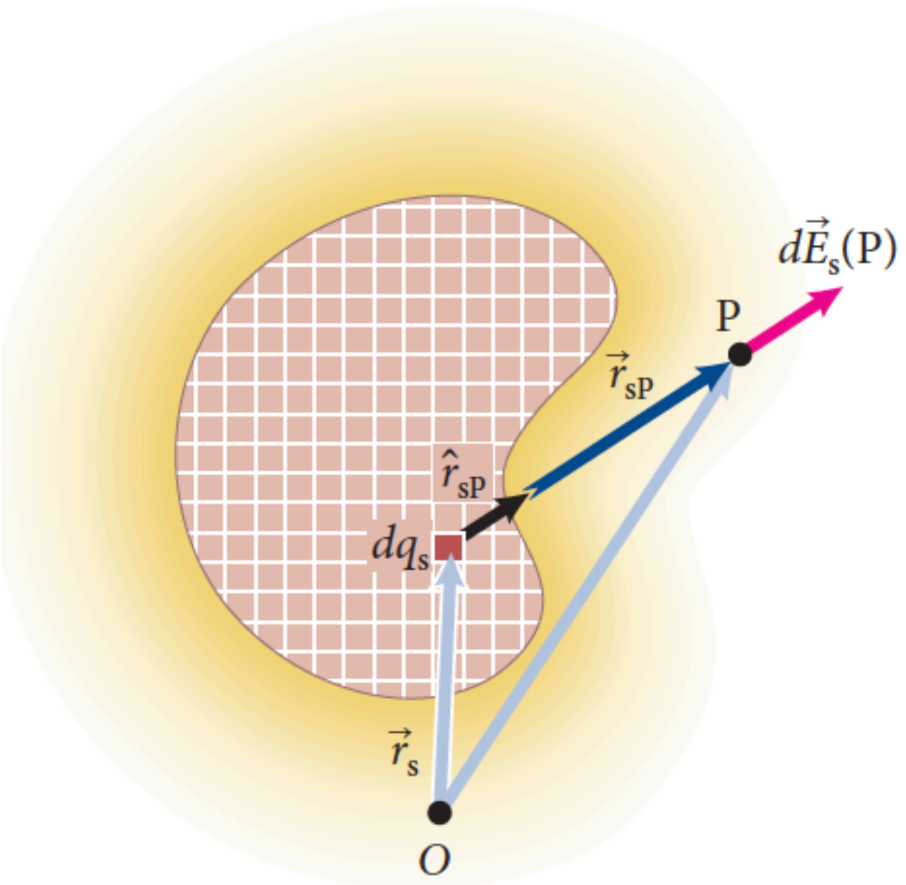


where r_{sP} is the distance from the charge element to point P and $\hat{r}_{SP}$ is a unit vector directed from the element toward P.

Here the index s refers to the s'th element in the distribution.

Using the principle of superposition, we can then sum the contributions of all the segments that make up the object.

$$\vec{E} = k\sum_{s}\frac{\Delta q_s}{r_{sP}^2}\hat{r}_{sP}$$



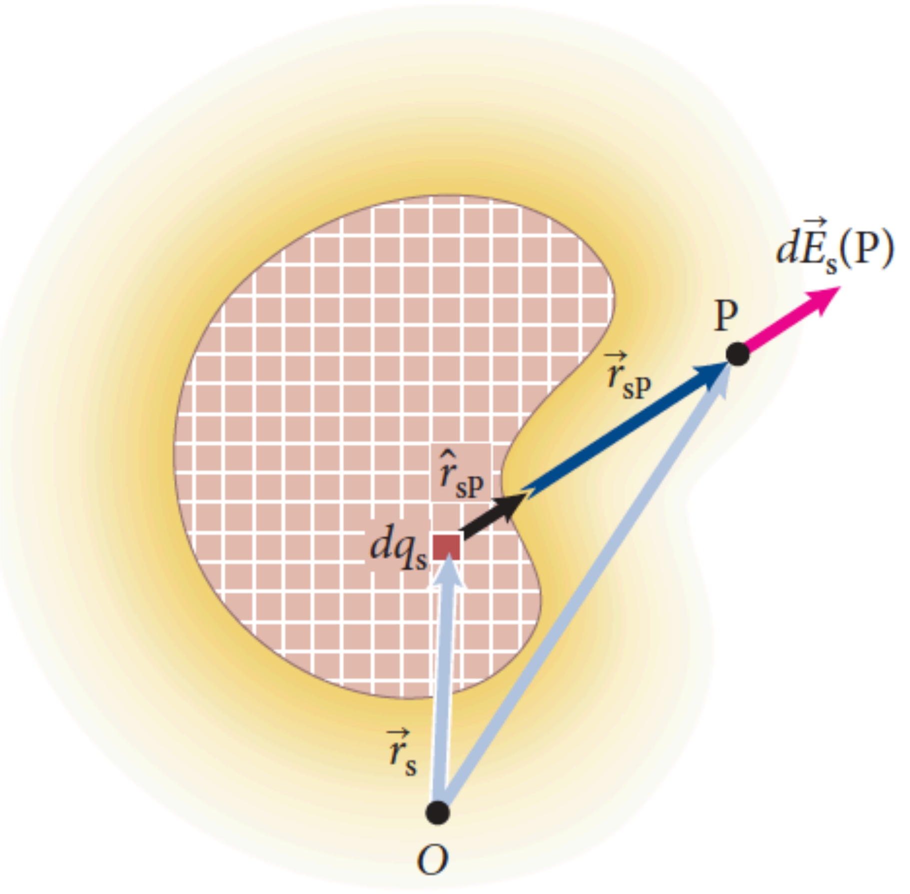
2.5 Electric Field of Due to Continues Charge Distribution

Because the charge distribution is modeled as continuous, the total field at P in the limit

$$\Delta q_s \to 0$$

$$\vec{E} = k \lim_{\Delta q_s \to 0} \sum_s \frac{\Delta q_s}{r_{sP}^2} \hat{r}_{sP}$$

$$\boxed{\vec{E} = k \int \frac{dq}{r^2} \hat{r}}$$



dq : charge density !

2.5 Electric Field of Due to Continues Charge Distribution

If a charge Q is uniformly distributed throughout a volume V, the volume charge density ρ is defined by:

$$\rho = \frac{Q}{V} \qquad (\mathrm{C/m^3})$$

If a charge Q is uniformly distributed on a surface of area A, the surface charge density σ (lowercase Greek sigma) is defined by:

$$\sigma = \frac{Q}{A} \qquad (\mathrm{C/m^2})$$

If a charge Q is uniformly distributed along a line of length ℓ, the linear charge density λ is defined by

$$\lambda = \frac{Q}{l} \qquad (\mathrm{C/m})$$

$$\boxed{dq = \rho dV} \qquad \boxed{dq = \sigma dA} \qquad \boxed{dq = \lambda dl}$$

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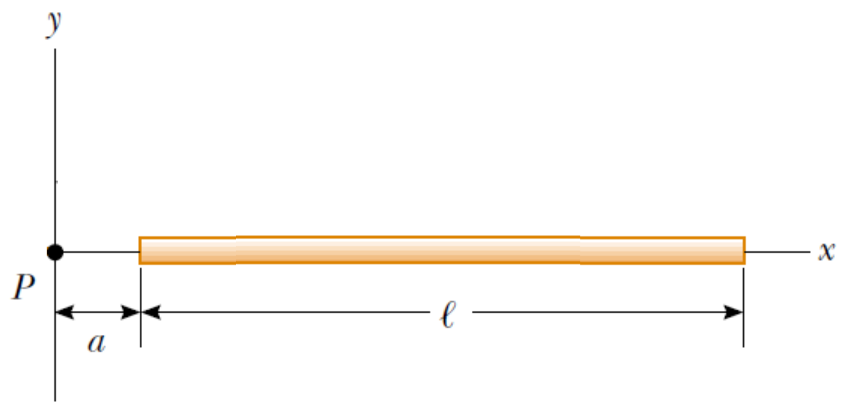
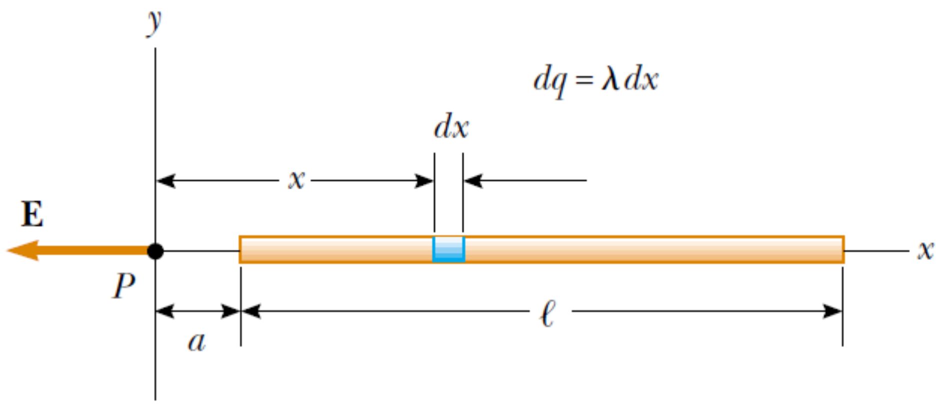
$$\boxed{dq = \rho dV} \qquad \boxed{dq = \sigma dA} \qquad \boxed{dq = \lambda dl}$$

Example: The Electric Field Due to a Charged Rod

A rod of length l has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance a from one end.



Answer: The Electric Field Due to a Charged Rod



Charge per unit length λ:

$$\lambda = \frac{Q}{l}$$

the charge dq on the small segment is:

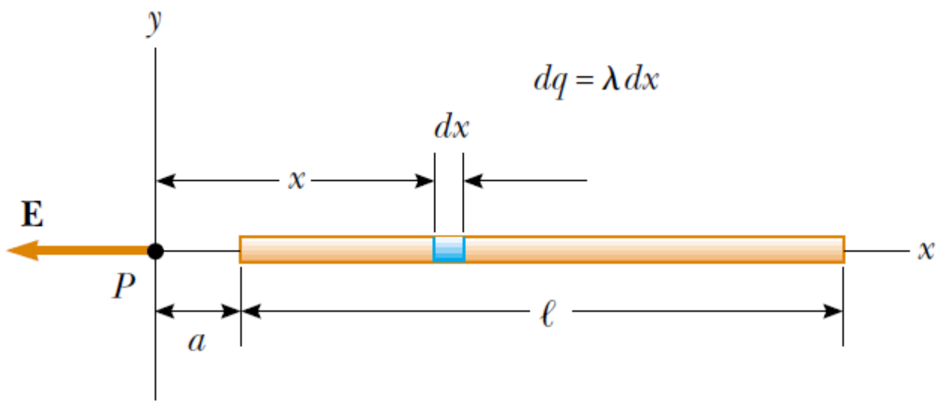
$$dq = \lambda dx$$

The field dE at P due to this segment is in the negative x direction and it's magnitute:

$$dE = k\frac{dq}{x^2} = k\frac{\lambda dx}{x^2}$$

2.5 Electric Field of Due to Continues Charge Distribution

Answer: The Electric Field Due to a Charged Rod



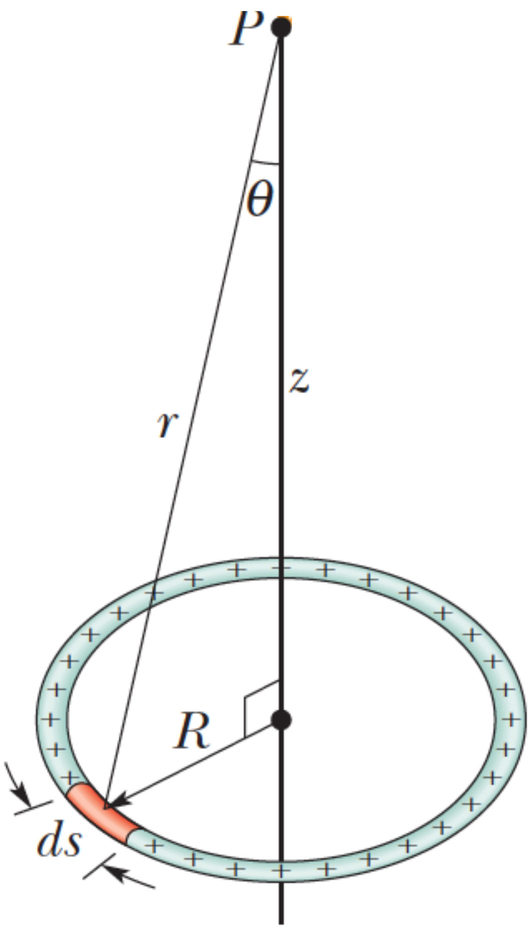
The total field at P due to all segments of the rod:

$$E = \int_{a}^{l+a} k\frac{\lambda dx}{x^2}$$

$$E = k\lambda \int_{a}^{l+a} \frac{dx}{x^2}$$

$$E = k\lambda \left[-\frac{1}{x}\right]_{a}^{l+a} = k\lambda\left(\frac{1}{a} - \frac{1}{l+a}\right) = \frac{kQ}{a(l+a)}$$

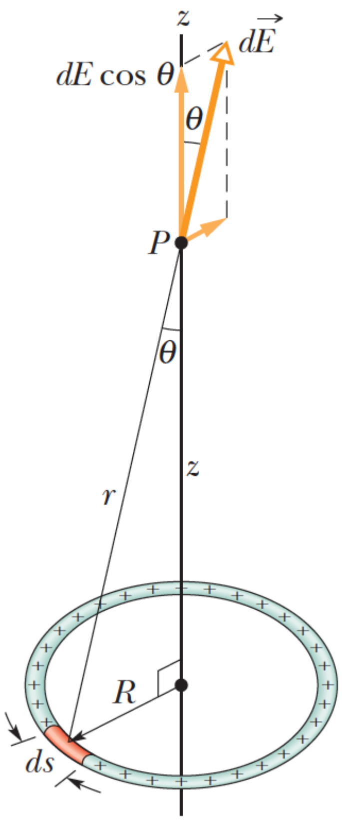
Example: The Electric Field of a Uniform Ring of Charge



A ring of radius R carries a uniformly distributed positive total charge Q. Calculate the electric field due to the ring at a point P lying a distance z from its center along the central axis perpendicular to the plane of the ring

2.5 Electric Field of Due to Continues Charge Distribution

Answer: The Electric Field of a Uniform Ring of Charge



The element ds has a charge of magnitude

$$dq = \lambda ds$$

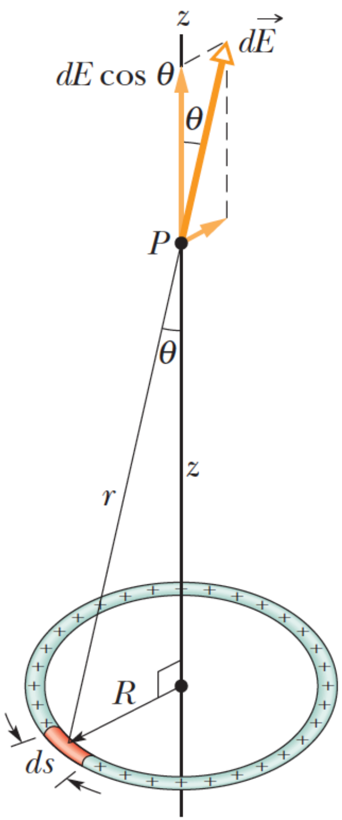
Electric filed at point P due to dq:

$$dE = k\frac{dq}{r^2} = \frac{\lambda ds}{r^2}$$

From figure we can write $r^2=(z^2+R^2)$. So we can rewrite the electric field at point P:

$$dE = k\frac{dq}{r^2} = k\frac{\lambda ds}{(z^2+R^2)}$$

Answer: The Electric Field of a Uniform Ring of Charge



The parallel component of electric field has magnitude dEcosθ and Cosθ is:

$$\cos\theta = \frac{z}{(z^2 + R^2)^{1/2}}$$

Thus

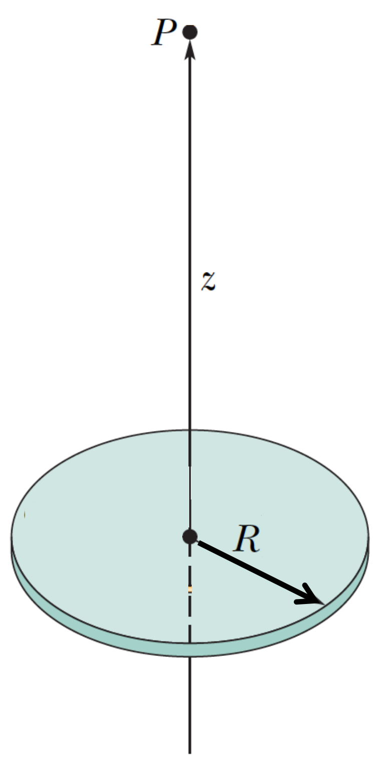
$$dE\cos\theta = \frac{z\lambda}{(z^2 + R^2)^{3/2}} ds$$

To add the parallel components dEcosθ produced by all the elements, we integrate:

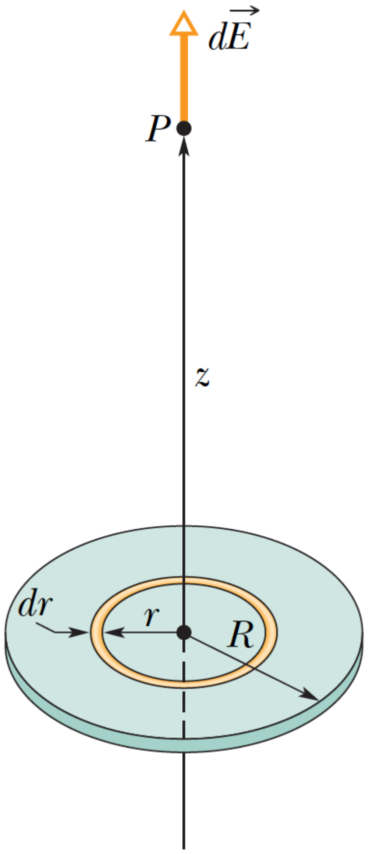
$$E = \int_{o}^{2\pi R} dE\cos\theta = \frac{z\lambda}{(z^2 + R^2)^{3/2}} \int_{o}^{2\pi R} ds = \frac{z\lambda(2\pi R)}{(z^2 + R^2)^{3/2}} = \frac{zQ}{(z^2 + R^2)^{3/2}}$$

Example: The Electric Field Due to a Charged Disk

A circular plastic disk of radius R has a positive surface charge of uniform density σ on its upper surface. What is the electric field at point P , a distance z from the disk along its central axis?



Answer: The Electric Field Due to a Charged Disk



Since σ is the charge per unit area, the charge on the ring is

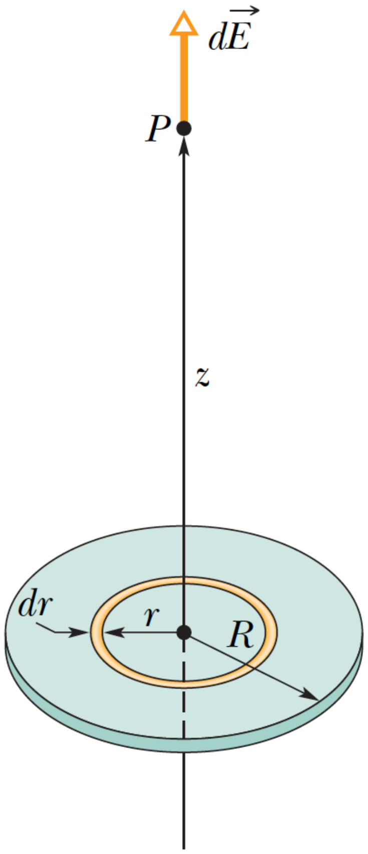
$$dq = \sigma dA = \sigma(2\pi r)dr$$

We have already solved the problem of the electric field due to a ring of charge:

$$dE = k\frac{z\sigma 2\pi r}{(z^2 + R^2)^{3/2}}dr$$

We can obtain the electric field of disk by by integrating with respect to the variable r from r= 0 to r R .

Answer: The Electric Field Due to a Charged Disk



We get:

$$E = k\pi z\sigma \int_0^R (z^2 + R^2)^{-3/2}(2r)dr$$

From the integral tables:

$$\int X^m \, dX = \frac{X^{m+1}}{m+1}$$

If we choose

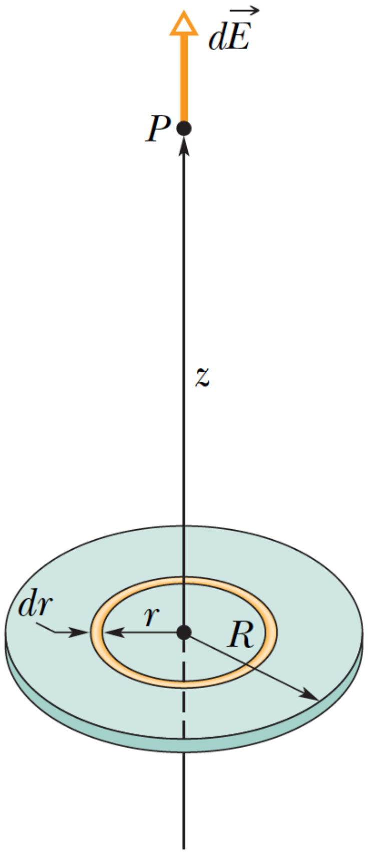
$$X = (z^2 + R^2) \qquad m = 3/2$$

then we get:

$$E = k\pi z\sigma \left[\frac{(z^2 + R^2)^{-3/2}}{-\frac{1}{2}} \right]_0^R = 2k\pi\sigma\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

2.5 Electric Field of Due to Continues Charge Distribution

Answer: The Electric Field Due to a Charged Disk



$$E = 2k\pi\sigma\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

for R>>z the electric field reduces to:

$$E = \frac{\sigma}{2\varepsilon_o}$$

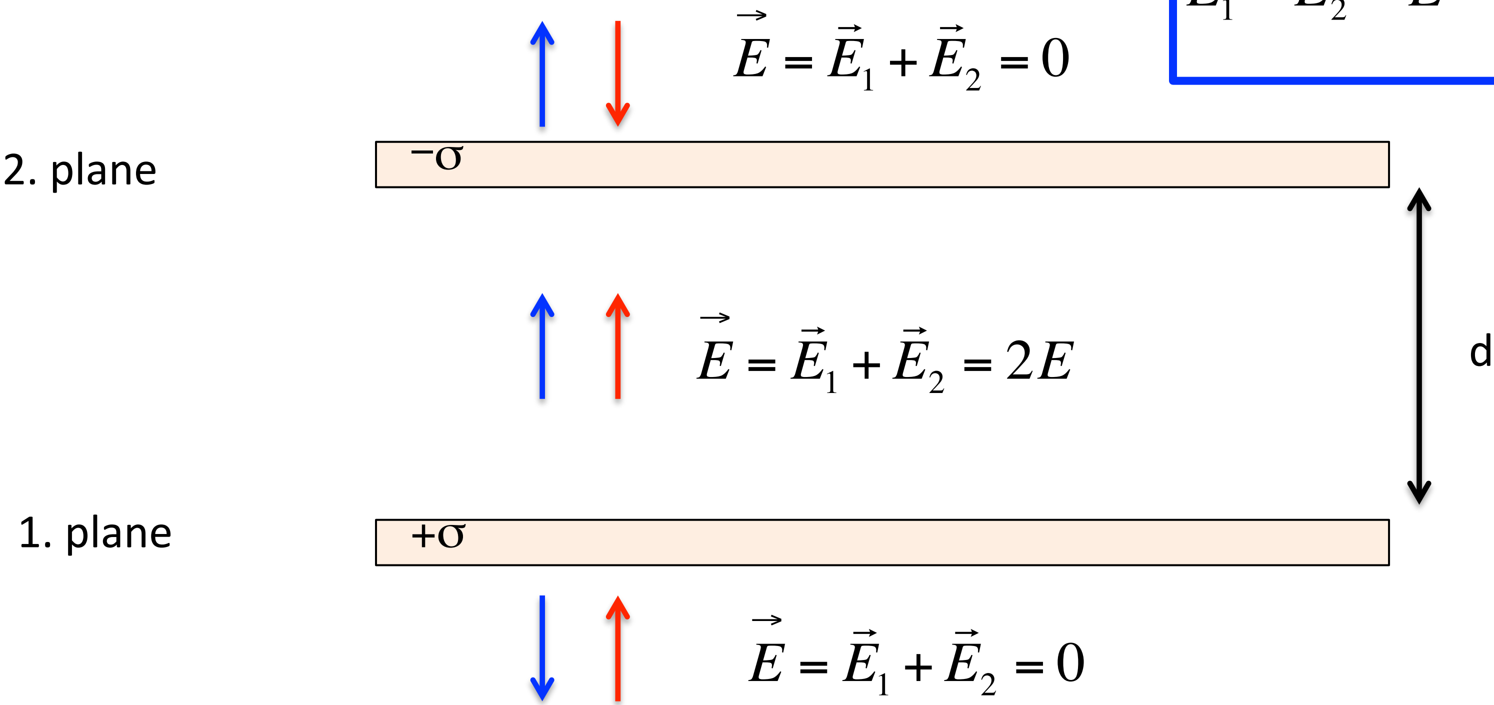
This is the electric field produced by an infinite sheet of uniform charge located on one side of a nonconductor such as plastic

Example: Electric Fid Between Two Uniformly Charged Planes

Two planes, oppositely charged but with same magnitude of charge density are placed parallel to each other. What is the electric field outside and inside the planes ?

Answer: Electric Fid Between Two Uniformly Charged Planes

$$\boxed{E_1 = E_2 = E = \frac{\sigma}{2\varepsilon_0}}$$



Example

We know that, there is an electric field near Earth surface of about 150 N/C and that points vertically down. Assume this filed is constant around Earth and that is due to charge evently spread on Earth's surface.Waht is the total charge on the Earth?

Answer:

$$\sigma = 2\varepsilon_o E = 2(8.85x10^{-12}C^2/N.m^2)(150N/C)$$

$$\sigma = 2.7x10^{-9}C/m^2$$

$$Q = \sigma 4\pi R^2 = (2.7x10^{-9}C/m^2)4\pi(6.37x10^6 m)^2$$

$$Q = 1.4x10^6 C$$

2.6 A Point Charge in an Electric Field

From the definition of electric field, an electrostatic force acts on the particle is given by

$$\vec{F}_e = q\vec{E}$$

If this is the only force exerted on the particle, it must be the net force and causes the particle to accelerate according to Newton's second law.

$$\vec{F}_e = q\vec{E} = m\vec{a}$$

Thus

$$\boxed{\vec{a} = \frac{q\vec{E}}{m}}$$

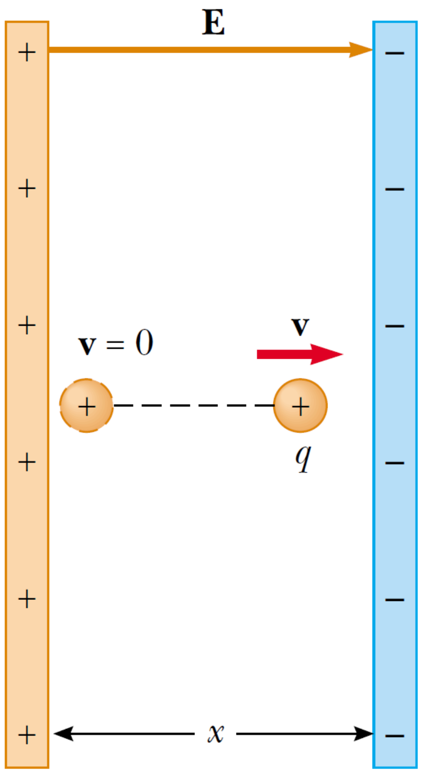
If electric field is uniform then the acceleration is constant. In this case:

if q is **positive** → Accelaration is in **same** direction of electric field

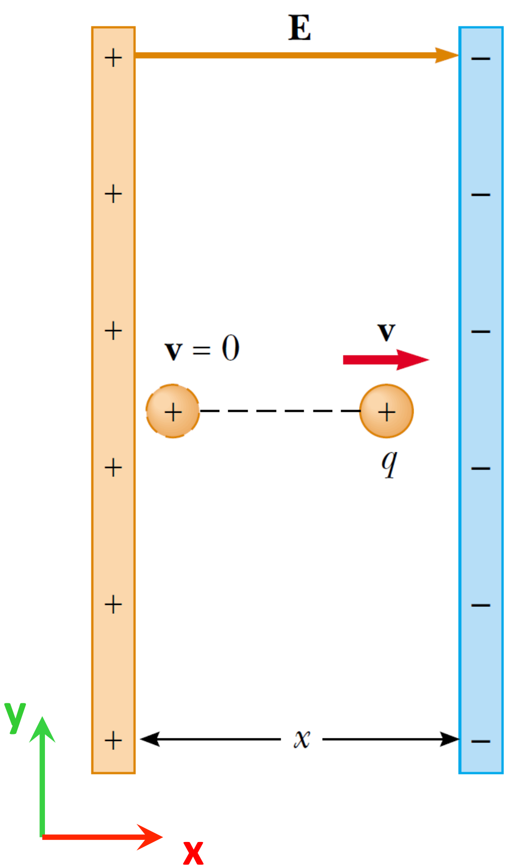
if q is **negative** → It's acceleration is in the direction **opposite** the electric field.

Example: An Accelerating Positive Charge

A positive point charge q of mass m is released from rest in a uniform electric field E directed along the x axis. Describe the motion of the particles.



Answer: An Accelerating Positive Charge



The acceleration is constant and is given by

$$\vec{a} = \frac{q\vec{E}}{m}$$

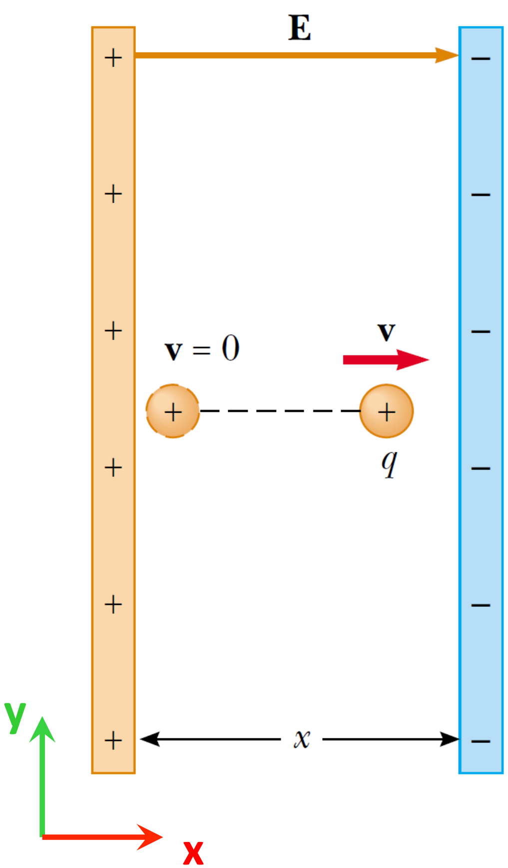
The motion is simple linear motion along the x axis. Therefore, we can apply the equations of kinematics in one dimension

$$x_s = x_i + v_{xi}t + \frac{1}{2}at^2$$

$$v_{xs} = v_{xi} + at$$

$$v_{xs}^2 = v_{xi}^2 + 2a(x_s - x_i)$$

Answer: An Accelerating Positive Charge



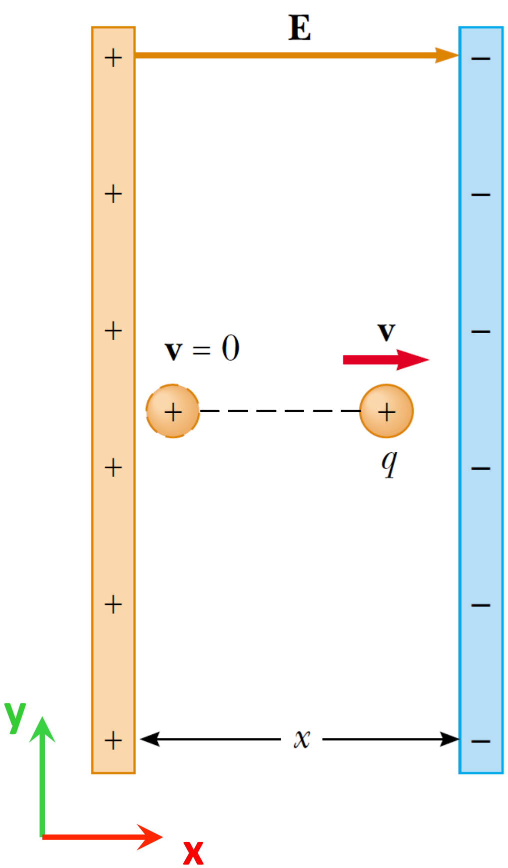
Choosing the initial position of the charge as $x_i = 0$ and assigning $v_i = 0$ because the particle starts from rest, the position of the particle as a function of time is

$$x_s = \frac{1}{2}at^2 = \frac{qE}{2m}t^2$$

And the speed of the particle is given by

$$v_{xs} = at = \frac{qE}{m}t$$

Answer: An Accelerating Positive Charge



The third kinematic equation gives us

$$v_{xs}^2 = 2ax_s = 2\frac{qE}{m}x_s$$

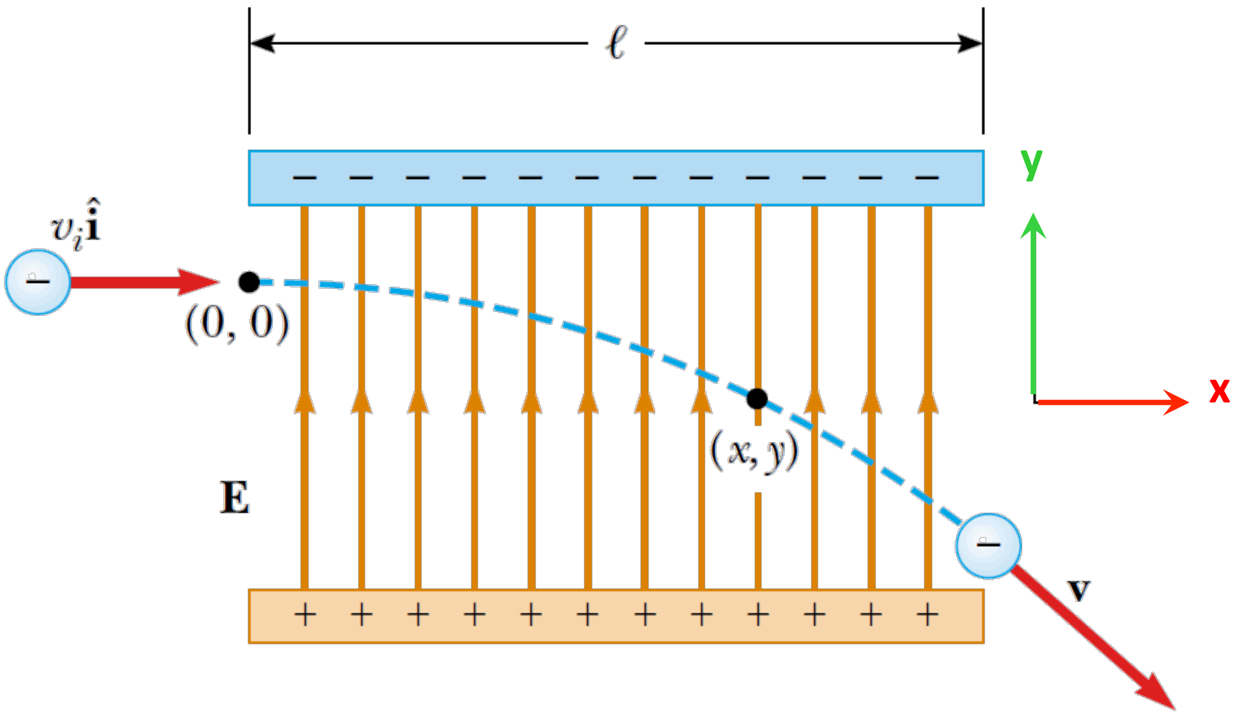
we can find the kinetic energy of the charge after it has moved a distance $\Delta x = x_s - x_i$

$$K = \frac{1}{2}mv_{xs}^2 = \frac{1}{2}m(\frac{2qE}{m}x_s) = qEx_s$$

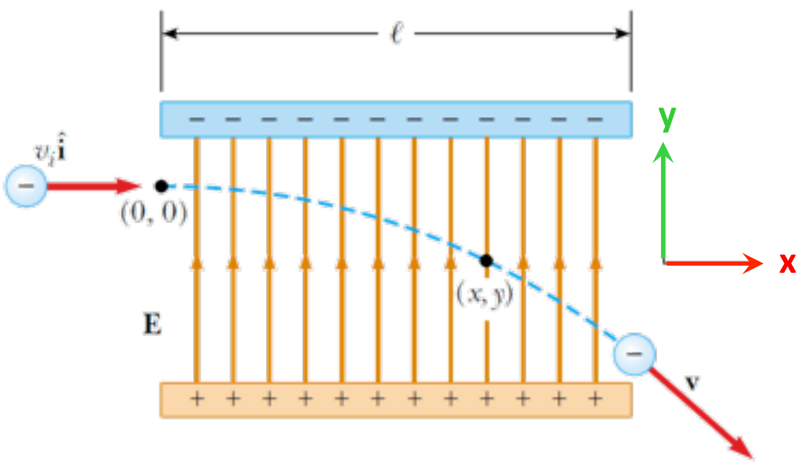
We can also obtain this result from the work–kinetic energy theorem. HOMEWORK !

Example: A Negative Charged Particle in an Electric Field

Suppose an electron of charge -e is projected horizontally into a uniform electric field between opposite charged planes from the origin with an initial velocity v_i at time t=0.
Describe the motion.



Answer: A Negative Charged Particle in an Electric Field



Elektrik alan pozitif y yönünde olduğundan yüklü parçacık elektrik alana girdikten sonra negatif y yönünde bir elektrik kuvveti hissedecek. Bunun sonucu olarak –y yönünde

$$\vec{a} = -\frac{e\vec{E}}{m}\hat{j}$$

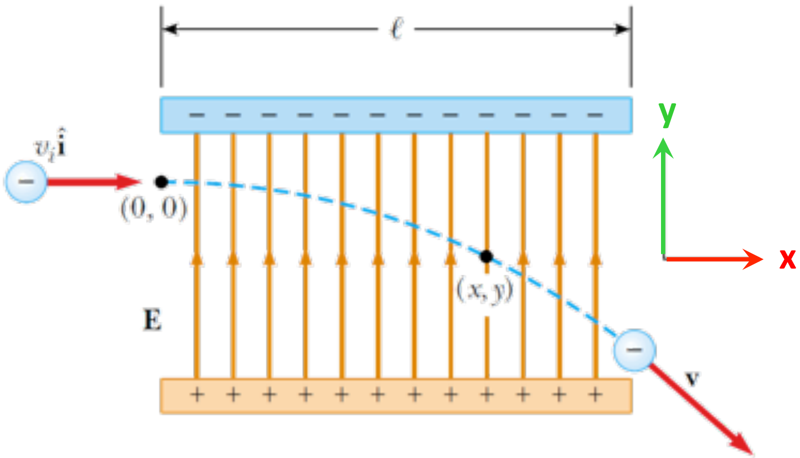
ivmesine sahip olur.

Parçacığın elektrik alana girdiği noktayı orijin olarak alırsak, bu noktada parçacağın t=0 anında x ve y yönündeki hızları

$$v_{xi} = v_i \qquad v_{yi} = 0$$

2.6 A Point Charge in an Electric Field

Answer: A Negative Charged Particle in an Electric Field



Negatif yüklü parçacık elektrik alan içerisinde t saniye boyunca kalıyorsa bu süre için parçacığın hız bileşenleri (Dördüncü bölüm)

$$v_{xi} = v_i = sabit$$

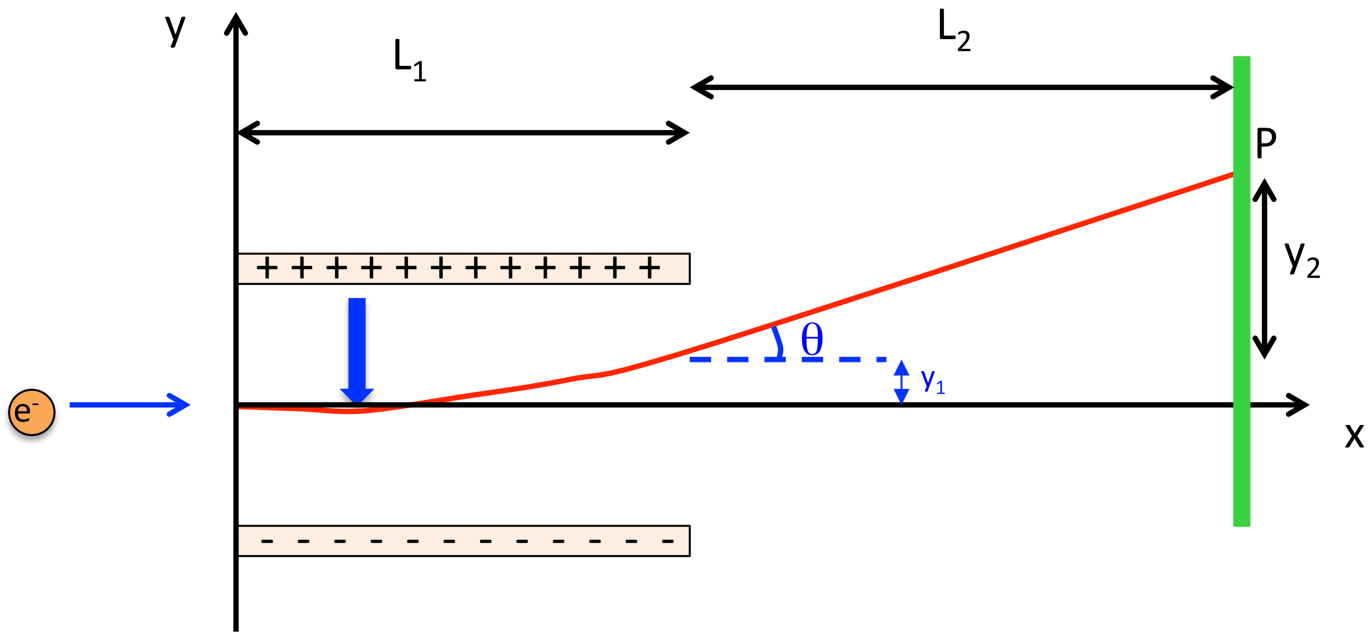
$$v_{ys} = 0 + a_y t = -\frac{eE}{m} t$$

Parçacığın bu süre için konumundaki değişim ise

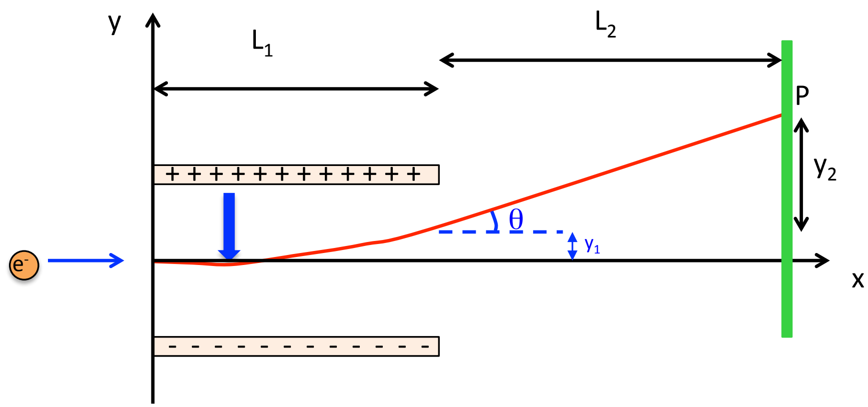
$$x_s = v_x t = v_i t \qquad y_s = \frac{1}{2} a_y t^2 = -\frac{eE}{2m} t^2$$

Homwwork: A Negative Charged Particle in an Electric Field

An electron moving horizontally at a speed V_o=3x10^6 m/s enters the region between two horizontally oriented plates of length L_1=3 cm. A fluorescent screen is located L_2=12 cm past these plates. Find the electron's total vertical deflectipn on the screen from its inital direction if the electric field between the plates points dowward with a magnitute of E=10^3 N/C.



Answer: A Negative Charged Particle in an Electric Field



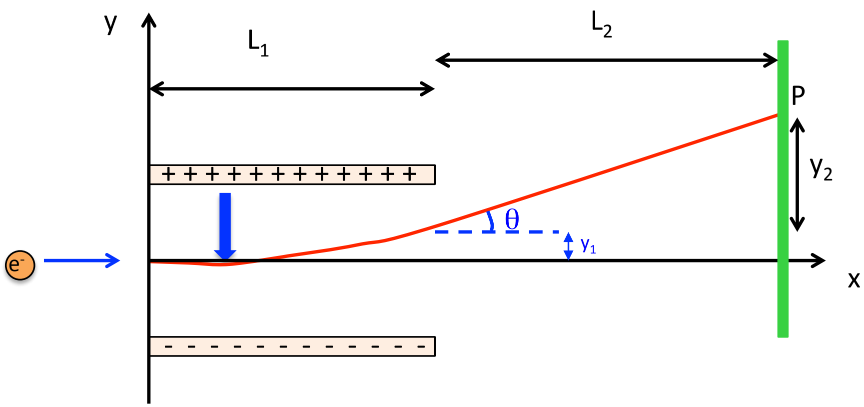
Assume that, the electron travels the length L_1 between the charged plates in time T

$$L_1 = v_o T \Rightarrow T = \frac{L_1}{v_o}$$

The electrons's deflection in the y direction:

$$y_1 = \frac{1}{2}at^2 = \frac{1}{2}\frac{eE}{m}T^2 = \frac{1}{2}\frac{eE}{m}(\frac{L_1}{v_o})^2$$

Answer: A Negative Charged Particle in an Electric Field



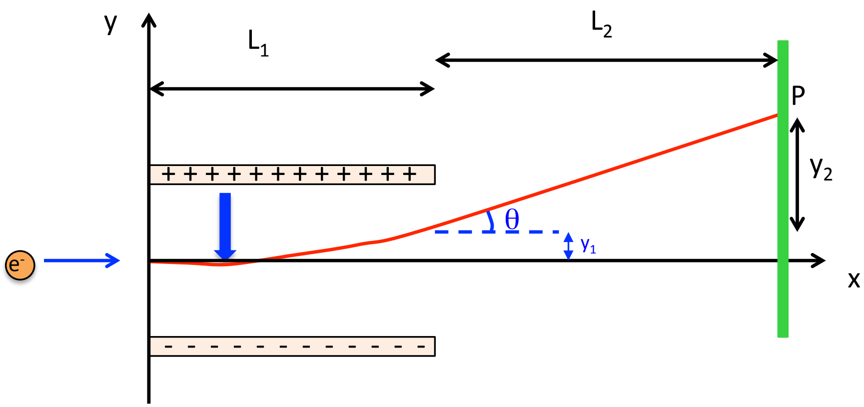
The component of the velocity vector of electron at time T

$$v_x = v_o \qquad v_y = at = \frac{eE}{m}T = \frac{eEL_1}{mv_o}$$

Therefore the tanθ

$$\tan\theta = \frac{v_y}{v_x} = \frac{(eE/m)T}{v_o} = \frac{(eE/m)(L_1/v_o)}{v_o} = \frac{eEL_1}{mv_o^2}$$

Answer: A Negative Charged Particle in an Electric Field



From figure:

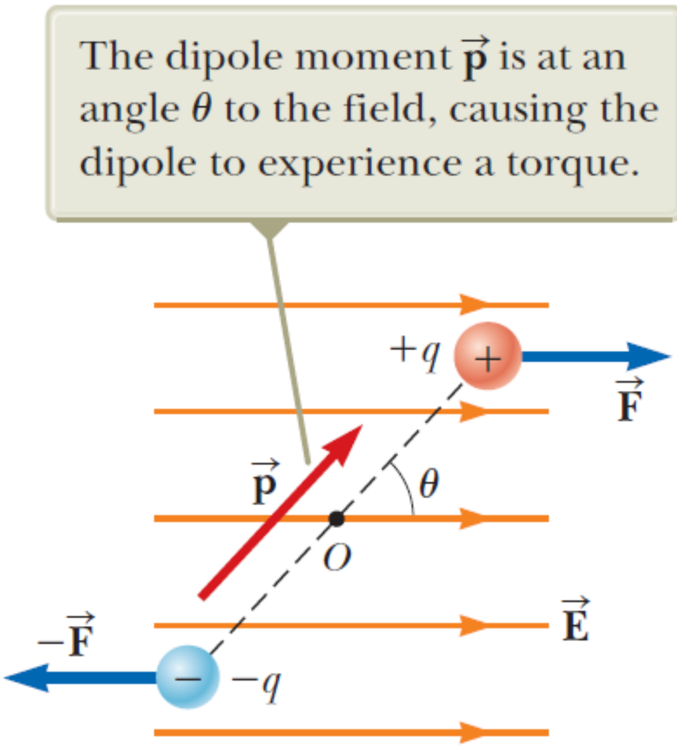
$$\tan\theta = \frac{y_2}{L_2} \Rightarrow y_2 = L_2 \tan\theta = \frac{eEL_1L_2}{mv_o^2}$$

The net deflection is then:

$$y_1 + y_2 = \frac{1}{2}\frac{eE}{m}\frac{L_1^2}{v_o^2} + \frac{eEL_1L_2}{mv_o^2} = \frac{eEL_1}{mv_o^2}\left(\frac{1}{2}L_1 + L_2\right)$$

$$y_1 + y_2 = 8x10^{-2}\,m$$

2.7 A Dipole in an Electric Field



The dipole moment **P** makes an angle θ with field **E**.

we can write the magnitude of the net torque

$$\tau = Fx\sin\theta + F(d-x)\sin\theta = Fd\sin\theta$$

$$\tau = pE\sin\theta$$

We can generalize this equation to vector form as

$$\boxed{\vec{\tau} = \vec{p}\times\vec{E}}$$