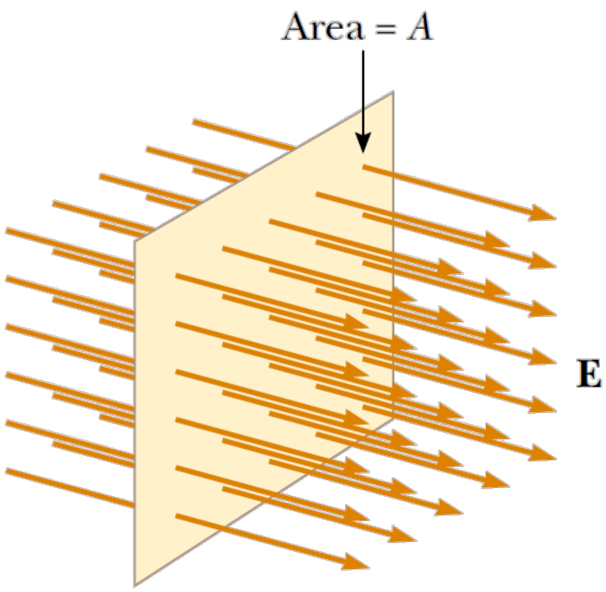
3.1 Electric Flux
3.2 Gauss Law
3.3 Gauss' Law and Coulomb's Law
3.4 Applying Gauss Law
3.5 Charged Isolated Conductor

3.1 Electric Flux

We now treat electric field lines in a more quantitative way.



The number of lines per unit area (in other words, the line density) is proportional to the magnitude of the electric field.

The total number of lines penetrating the surface is proportional to the product EA. This product of the magnitude of the electric field E and surface area A perpendicular to the field is called the electric flux

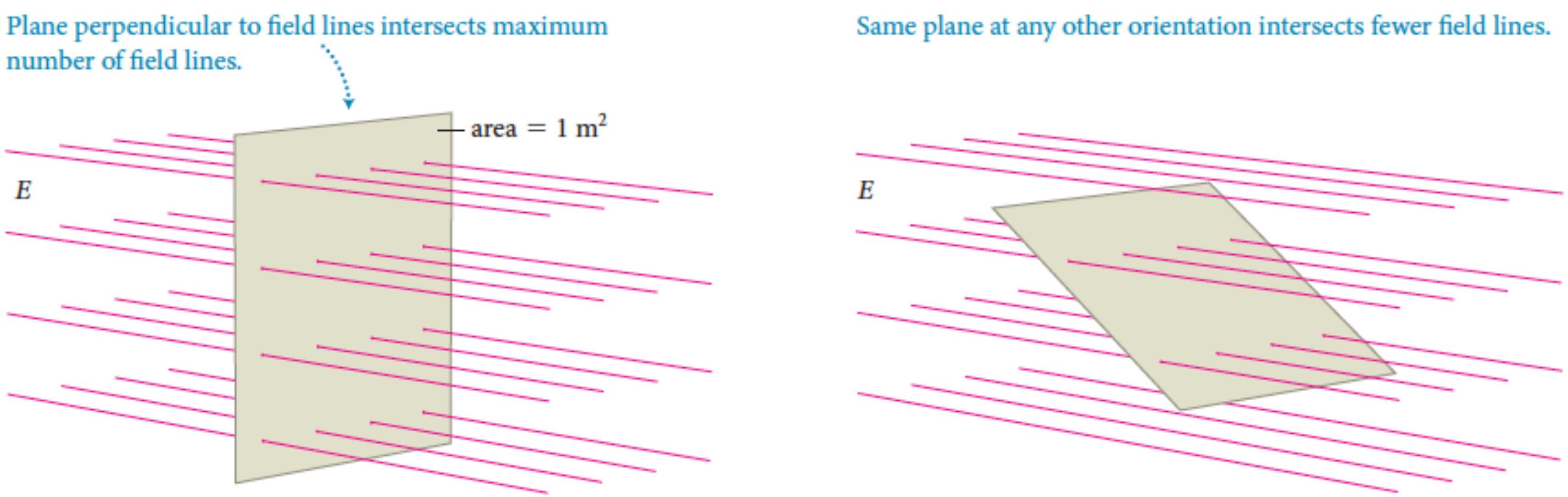
$$\Phi_E = EA$$

Unit: $N.m^2/C$

Electric flux is proportional to the number of electric field lines penetrating some surface.

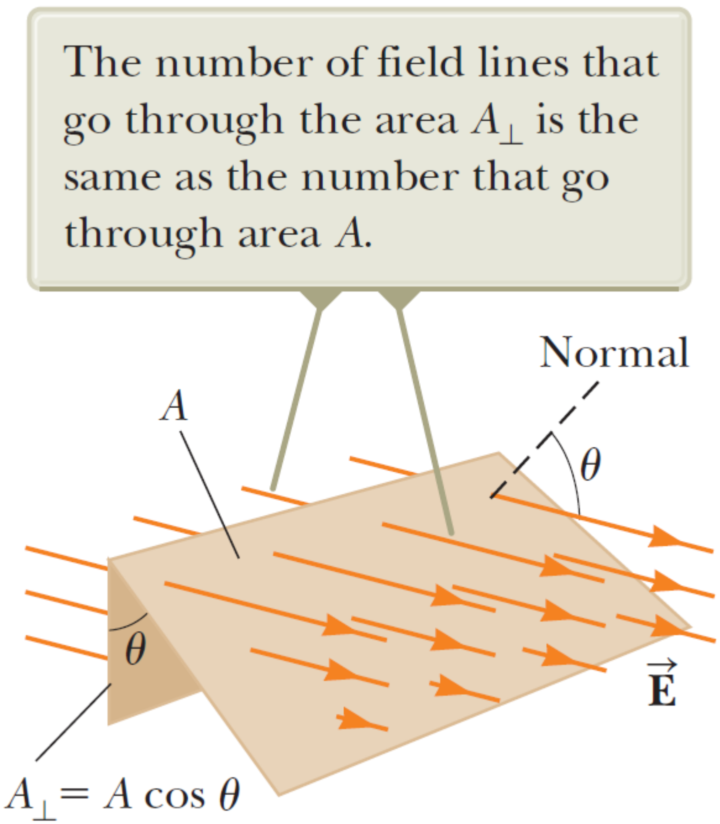
3.1 Electric Flux

The number of field lines that cross the surface depends on the orientation of the surface



3.1 Electric Flux

The number of field lines that cross the surface depends on the orientation of the surface



What is the flux thourugh surface A

The number of lines that cross this area A is equal to the number that cross the area $A_\perp$ which is a projection of area A onto a plane oriented perpendicular to the field. These two areas are related by

$$A_\perp = A\cos\theta$$

Because the flux through A equals the flux through $A_\perp$ we can write:

$$\Phi_E = EA_\perp = EA\cos\theta$$

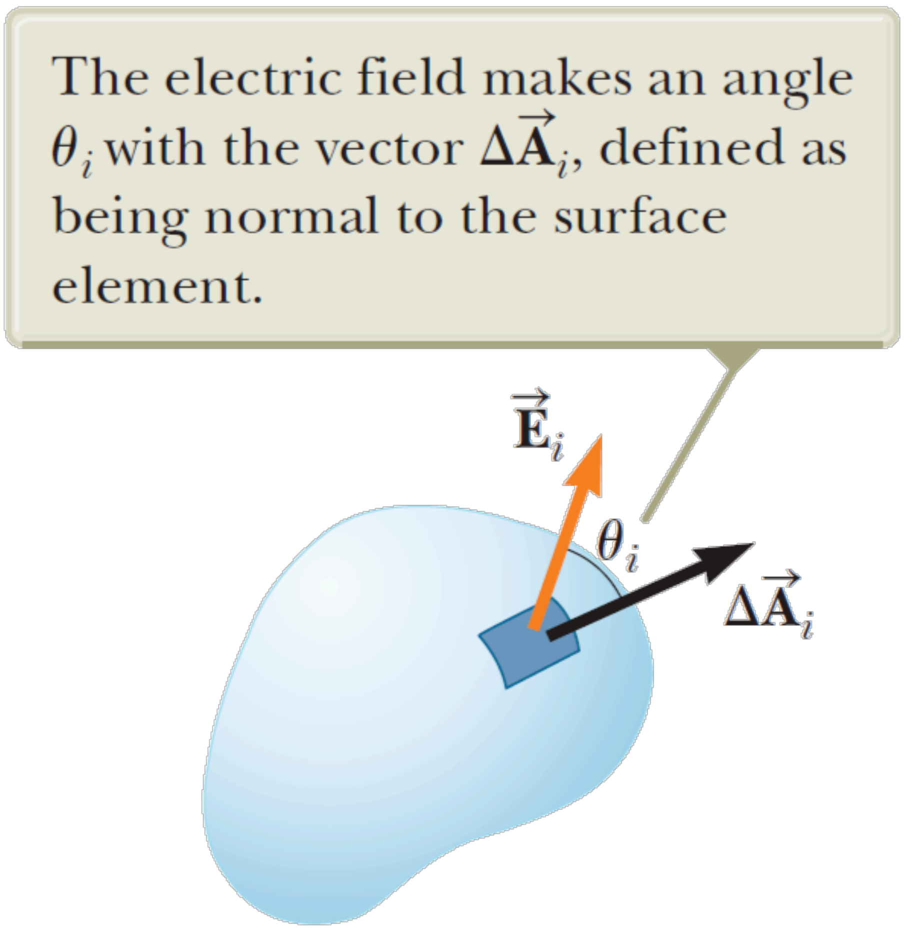
3.1 Electric Flux

$$\Phi_E = EA_{\perp} = EA\cos\theta$$

From this result, we see that the flux through a surface of fixed area A has a **maximum** value EA when the surface is perpendicular to the field (when the normal to the surface is parallel to the field, that is θ=0°);

the flux is **zero** when the surface is parallel to the field (when the normal to the surface is perpendicular to
the field, that is, θ=90°).

3.1 Electric Flux



Consider a general surface divided up into a large number of small elements, each of area $\Delta\vec{\mathbf{A}}_i$

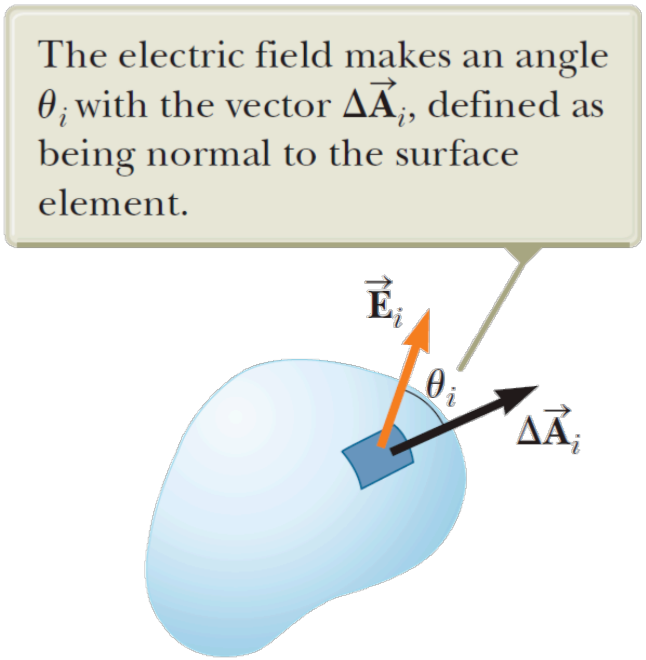
For this area, electric flux

$$\Delta\Phi_{E,i} = E_i \Delta A_i \cos\theta_i = \vec{E}_i . \Delta\vec{A}_i$$

By summing the contributions of all elements, we can obtain the total flux through the surface.

$$\Phi_E \approx \sum \vec{E}_i . \Delta\vec{A}_i$$

3.1 Electric Flux

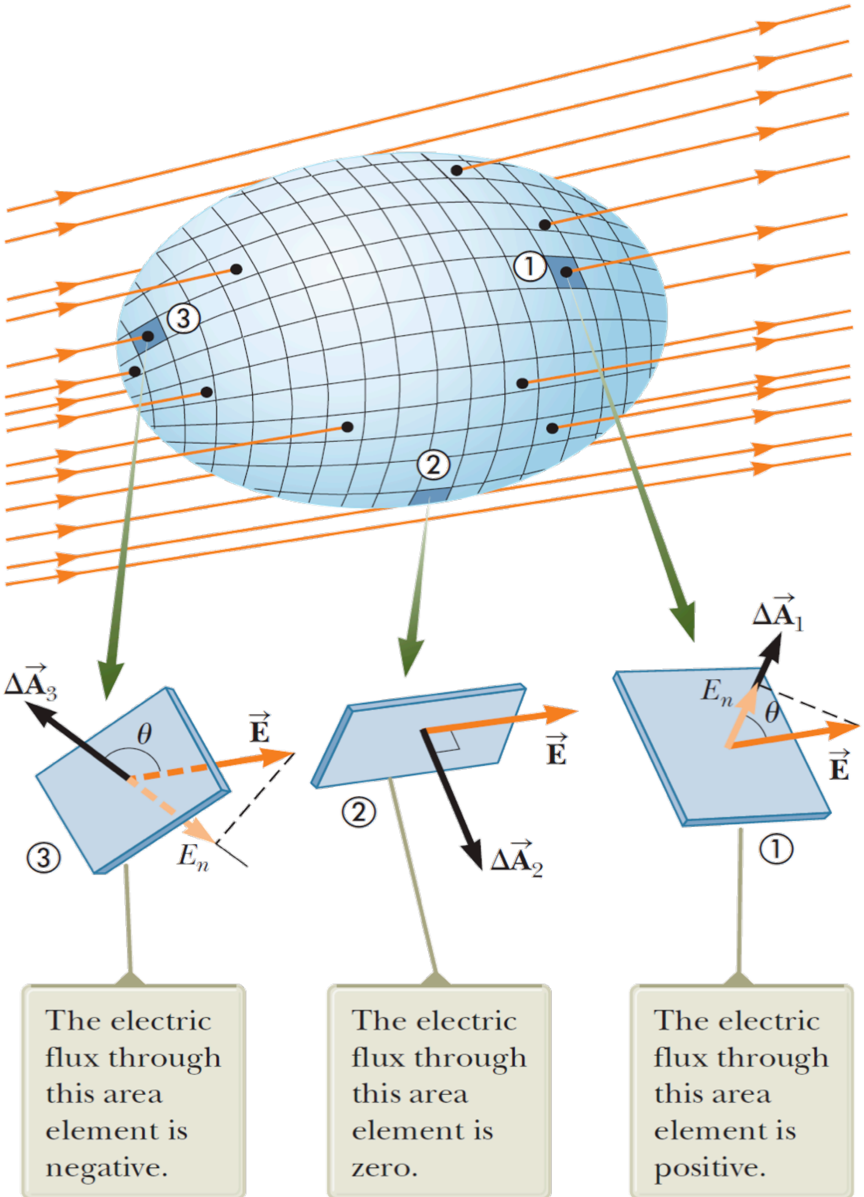


If we let the area of each element approach zero, the sum can be replaced by an integral. Therefore, the general definition of electric flux is:

$$\Phi_E \approx \lim_{\Delta\vec{A}_i \to 0} \sum \vec{E}_i . \Delta\vec{A}_i$$

$$\Phi_E = \int_{surface} \vec{E} . d\vec{A}$$

3.1 Electric Flux



The net flux through the surface is proportional to the net number of lines leaving the surface, where the net number means the number leaving the surface minus the number entering the surface.

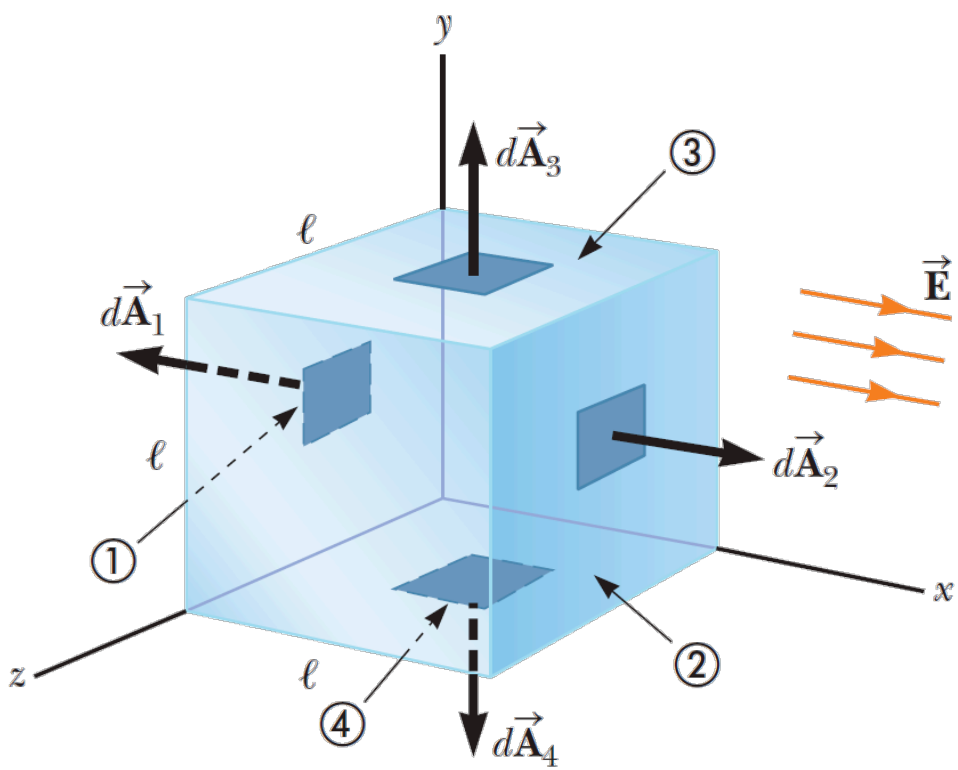
we can write the net flux through a closed surface as

$$\Phi_E = \oint \vec{E}.d\vec{A} = \oint E_n \, dA$$

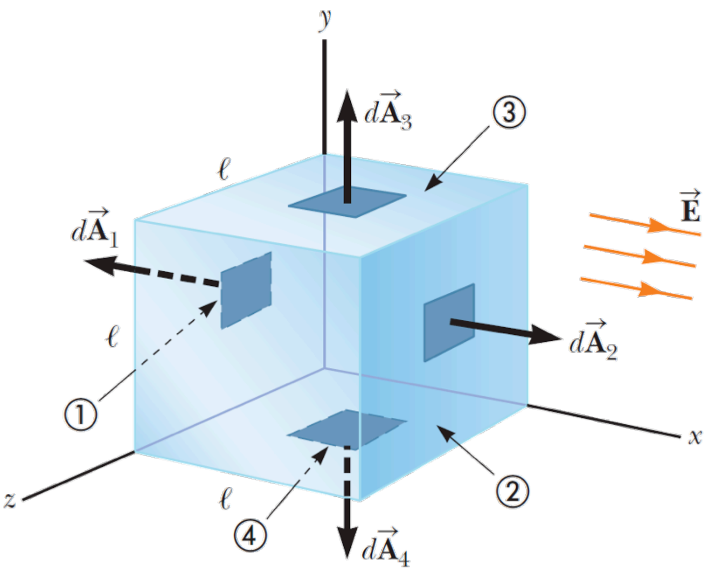
Burada E_n elektrik alanın normal bileşenini gösterir.

Example: Flux Through a Cube

Consider a uniform electric field E oriented in the x direction. Find the net electric flux through the surface of a cube of edge length !, oriented as shown in Figure.



Answer: Flux Through a Cube

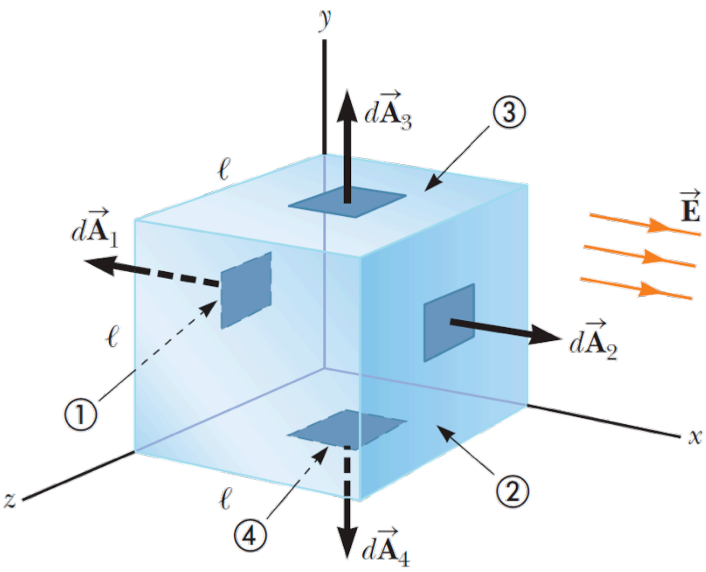


There is only net flux through faces 1 and 2. (WHY!)

The net flux through faces 1 and 2 is

$$\Phi_E = \int_1 \vec{E}.d\vec{A} + \int_2 \vec{E}.d\vec{A}$$

Answer: Flux Through a Cube



Flux though the surface 1

$$\Phi_{E,1} = \int_1 \vec{E}.d\vec{A} = \int_1 (E\cos 180)dA$$

$$\Phi_{E,1} = -E\int_1 dA = -EA = -El^2$$

Flux though the surface 2

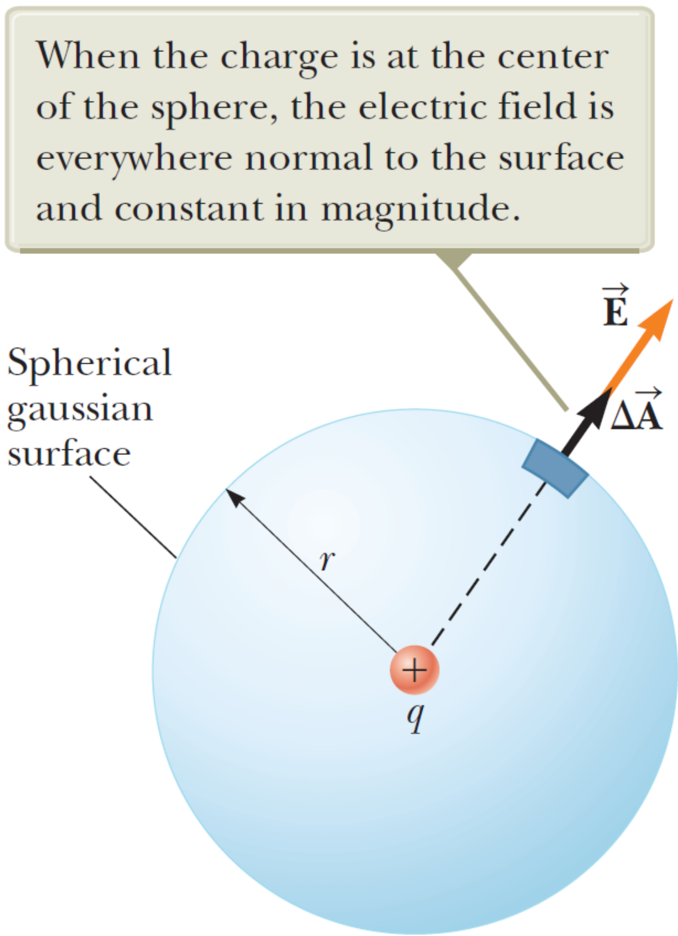
$$\Phi_{E,2} = \int_2 \vec{E}.d\vec{A} = \int_2 (E\cos 0)dA$$

$$\Phi_{E,2} = E\int_2 dA = EA = El^2$$

Therefore net flux:

$$\Phi_{E,top} = -El^2 + El^2 + 0 + 0 + 0 + 0 = 0$$

3.2 Gauss Law



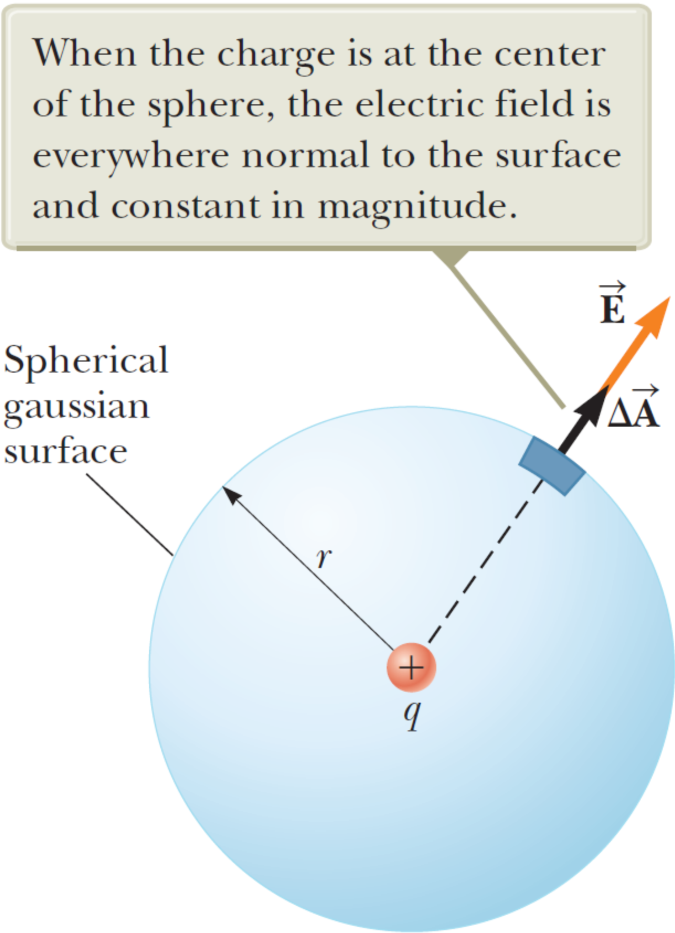
Consider a point charge q located at the center of sphere of raidus r. The electric field of point charge at the surface:

$$E = k\frac{q}{r^2}$$

The magnitude of the electric field everywhere on the surface of the sphere is same.
The field lines are directed radially outward and hence are perpendicular to the surface at every point on the surface. Therefore

$$\vec{E}.\Delta\vec{A}_i = E\Delta A_i$$

3.2 Gauss Law



The net flux through the gaussian surface is

$$\Phi_E = \oint \vec{E}.d\vec{A} = \oint E\,dA = E\oint dA$$

By symmetry, E is constant over the surface and therefore can be moved outside the integral.

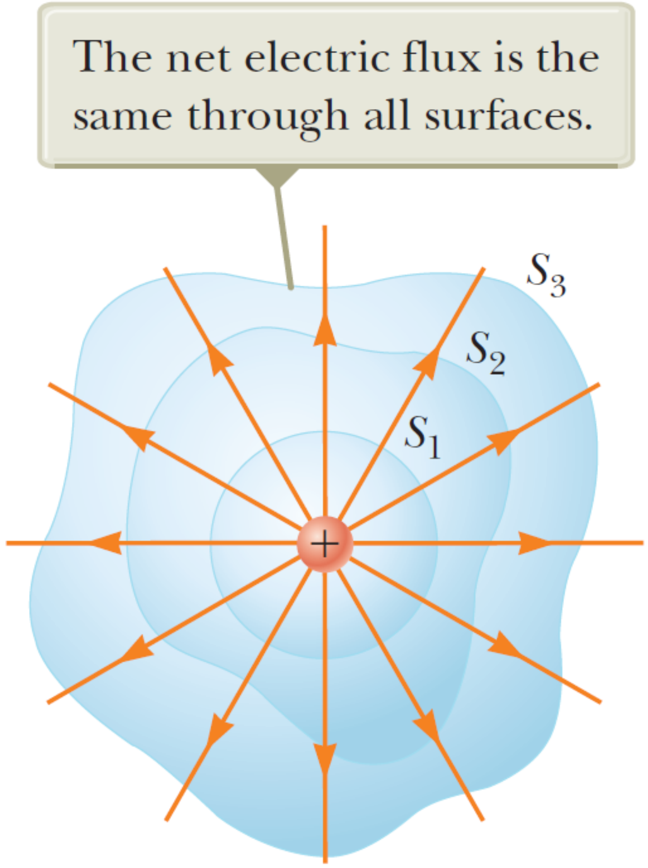
$$\Phi_E = k\frac{q}{r^2}(4\pi r^2) = 4\pi kq$$

Using $k=1/4\pi\varepsilon_0$

$$\boxed{\Phi_E = \frac{q}{\varepsilon_0}}$$

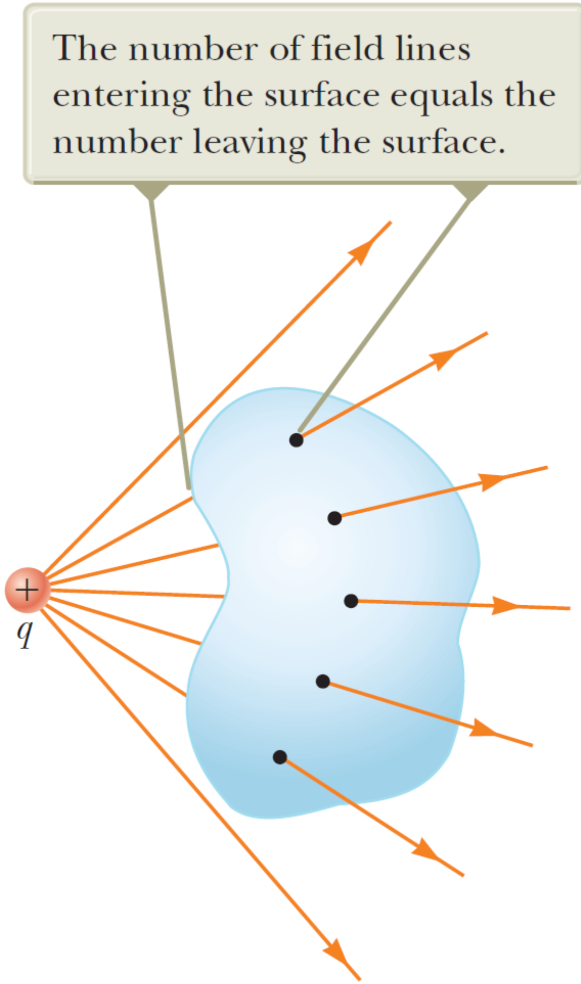
The flux is independent of the radius r and is proportional to the charge inside the sphere.

3.2 Gauss Law



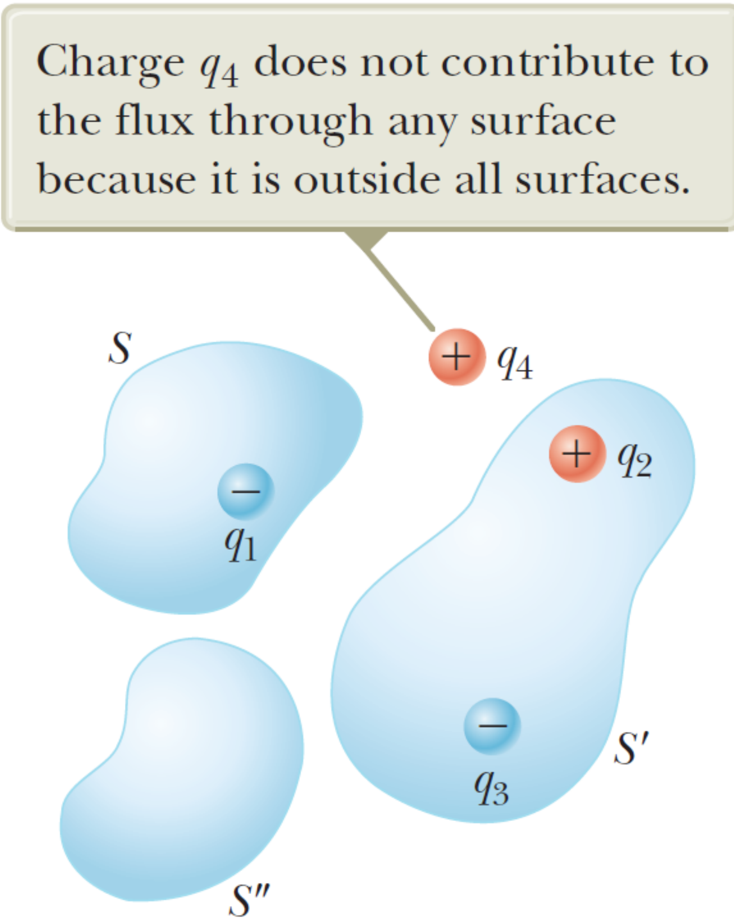
The net flux through any closed surface surrounding a point charge q is given by q/ε_0 and is independent of the shape of that surface

3.2 Gauss Law



The net electric flux through a closed surface that surrounds no charge is zero.

3.2 Gauss Law



$$\oint \vec{E}.d\vec{A} = \oint (\vec{E}_1 + \vec{E}_2 + \ldots).d\vec{A}$$

Net flux through S : q_1/ε_0

Net flux through S’ : $(q_2+q_3)/\varepsilon_0$

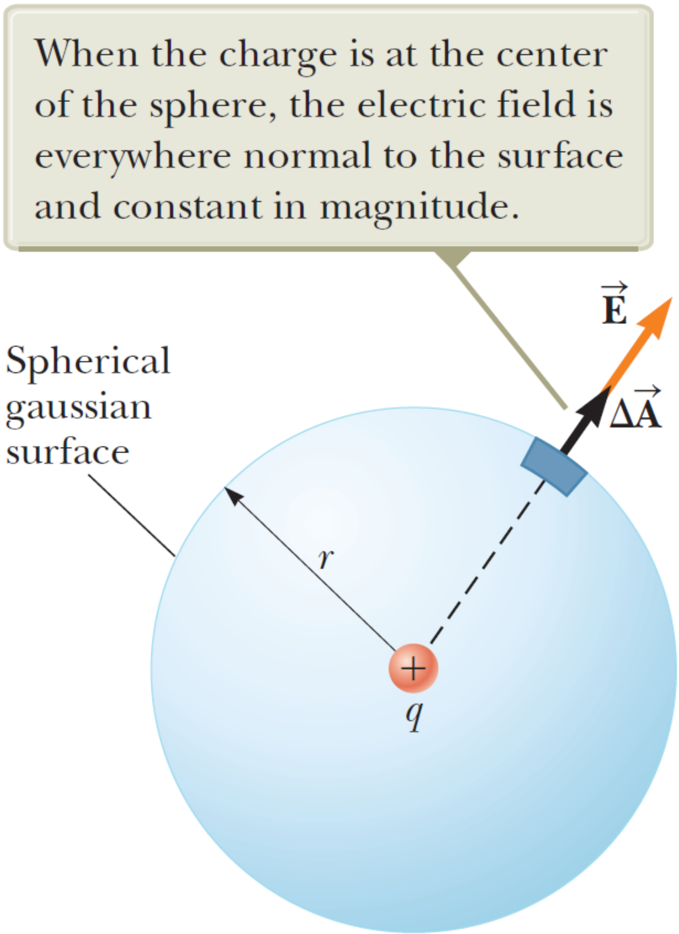
Net flux through S’’ : 0

Gauss law state that, the net flux through any closed surface is

$$\Phi_E = \oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

the net electric flux through a closed surface is proportional only the charge enclosed by the surface.

3.3 Gauss' Law and Coulomb 's Law



A positive point charge q, around which we have drawn a concentric spherical Gaussian surface of radius r. Let us divide this surface into differential areas dA. Since the angle between electric filed and surface normal vector zero we can rewrite Gauss law:

$$\Phi_E = \oint \vec{E}.d\vec{A} = \frac{q}{\varepsilon_0}$$

$$\Phi_E = \oint E\,dA = \frac{q}{\varepsilon_0}$$

Since E is a constant over the surface:

$$\Phi_E = \oint E\,dA = E\oint dA = \frac{q}{\varepsilon_0}$$

$$\Phi_E = E4\pi r^2 = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2} = k\frac{q}{r^2}$$

3.4 Application of Gauss' Law

$$\Phi_E = \oint \vec{E}.d\vec{A} = \frac{q_{iç}}{\varepsilon_0}$$

The goal in this type of calculation is to determine a surface that satisfies one or more of the following conditions:

1.) The value of the electric field can be argued by symmetry to be constant over the surface.

2.) The dot product $\vec{E}.d\vec{A}$ can be expressed as a simple algebraic product E dA if E and dA are parallel.

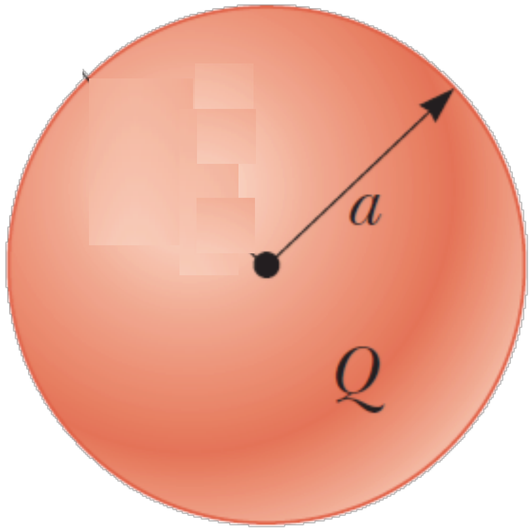
3.) The dot product $\vec{E}.d\vec{A}$ is zero if E and dA are perpendicular.

4.) The field can be argued to be zero over the same surfaces.

Example: A Spherically Symmetric Charge Distribution

An insulating solid sphere of radius a has a uniform volume charge density ρ and carries a total positive charge Q.

a) Calculate the magnitude of the electric field at a point outside the sphere.
b) Find the magnitude of the electric field at a point inside the sphere.



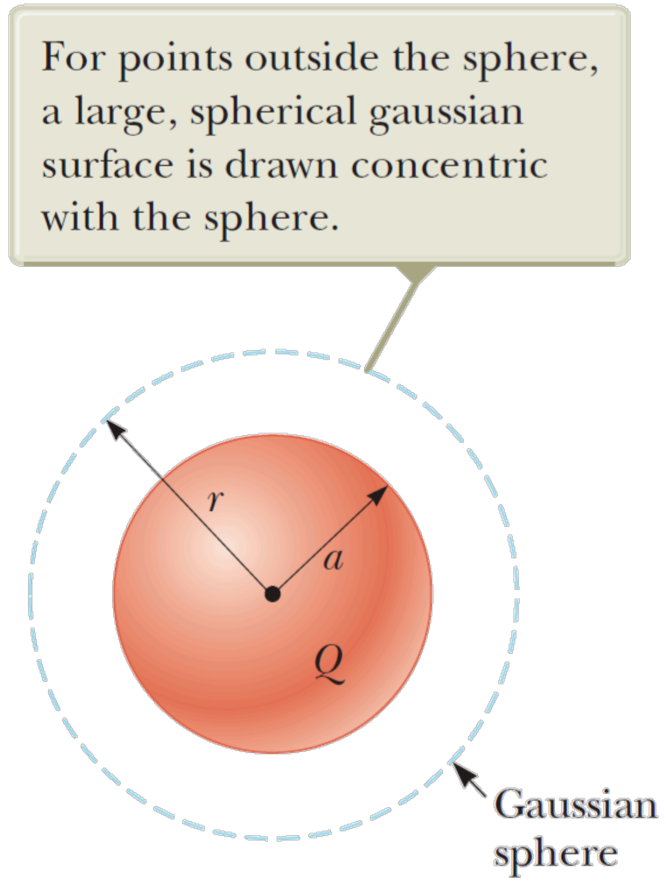
3.4 Application of Gauss' Law

Answer: A Spherically Symmetric Charge Distribution

a) Outside the sphere

$$\Phi_E = \oint \vec{E}.d\vec{A} = E\oint dA = \frac{q_{iç}}{\varepsilon_0}$$

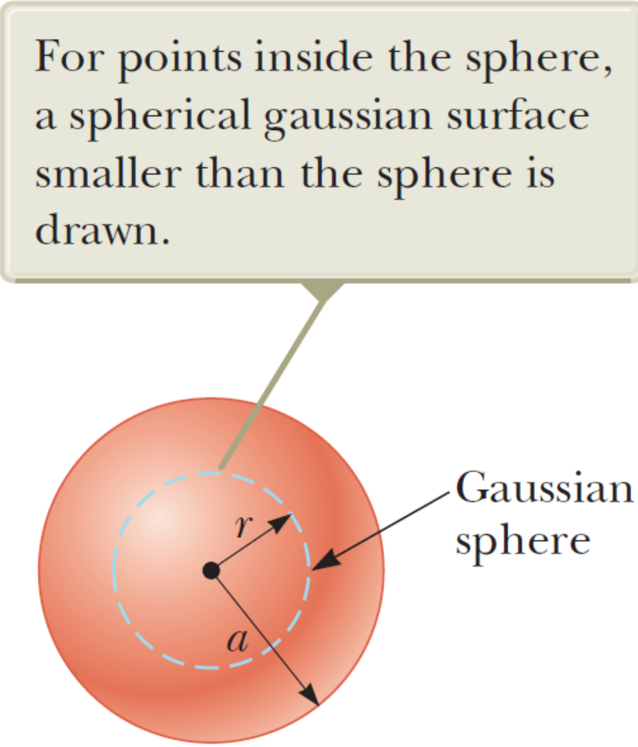
$$E(4\pi r^2) = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2} = k\frac{Q}{r^2}$$



For a uniformly charged sphere, the field in the region external to the sphere is equivalent to that of a point charge located at the center of the sphere.

Answer: A Spherically Symmetric Charge Distribution

a) Inside the sphere



To apply Gauss's law in this situation, it is important to recognize that the charge q_{in} within the gaussian surface of volume V' is less than Q. To calculate q_{in}:

$$q_{in} = \rho V' = \rho\left(\frac{4}{3}\pi r^3\right)$$

Yarıçapı r olan küre için Gauss yasasını uygulayalım!

Answer: A Spherically Symmetric Charge Distribution

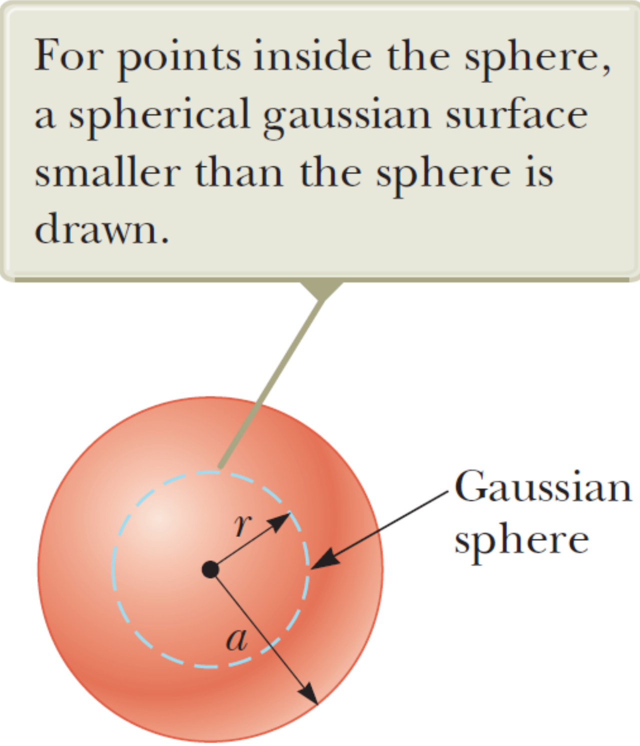
a) Inside the sphere

$$\Phi_E = \oint \vec{E}.d\vec{A} = \frac{q_{iç}}{\varepsilon_0}$$

$$E\oint dA = \frac{q_{iç}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{q_{iç}}{\varepsilon_0}$$

$$E = \frac{q_{iç}}{4\pi\varepsilon_0 r^2} = \frac{\rho(\frac{4}{3}\pi r^3)}{4\pi\varepsilon_0 r^2} = \frac{\rho}{3\varepsilon_0} r$$

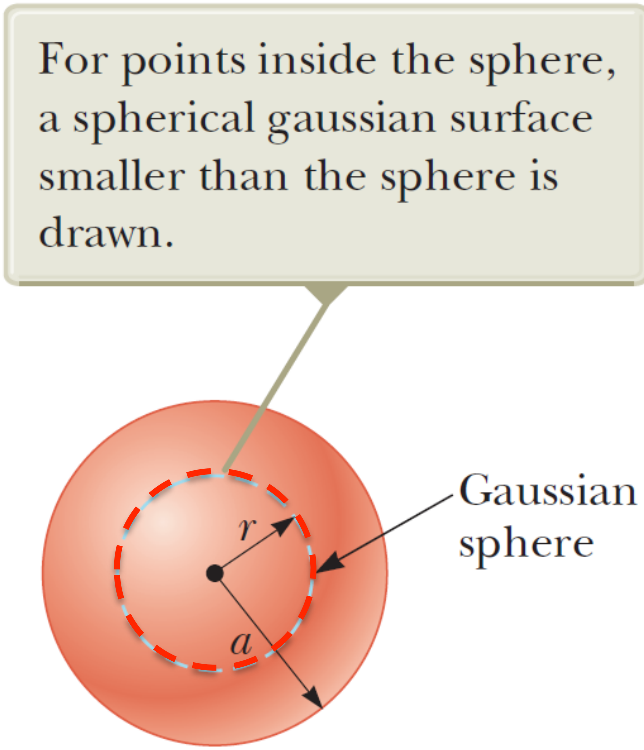


Answer: A Spherically Symmetric Charge Distribution

a) Inside the sphere

Sİnce Q distributed uniformly throughtout the volume:

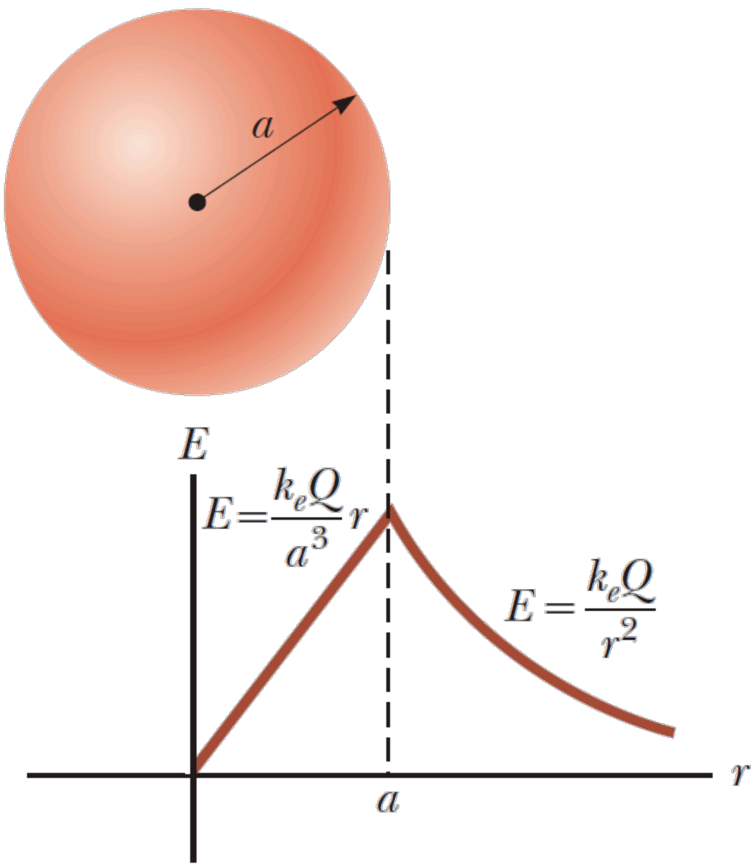
$$\rho = \frac{Q}{V} = Q / \frac{4}{3}\pi a^3$$



Using this and definition of constant k, we obtain:

$$E = \frac{Q / \frac{4}{3}\pi a^3}{3(1/4\pi k)} r = k\frac{Q}{a^3} r$$

Answer: A Spherically Symmetric Charge Distribution



Outside sphere for r>a

$$E = k\frac{Q}{r^2}$$

Inside sphere for r<a

$$E = k\frac{Q}{a^3}r$$

we approach the radial position r = a from inside the sphere and from outside

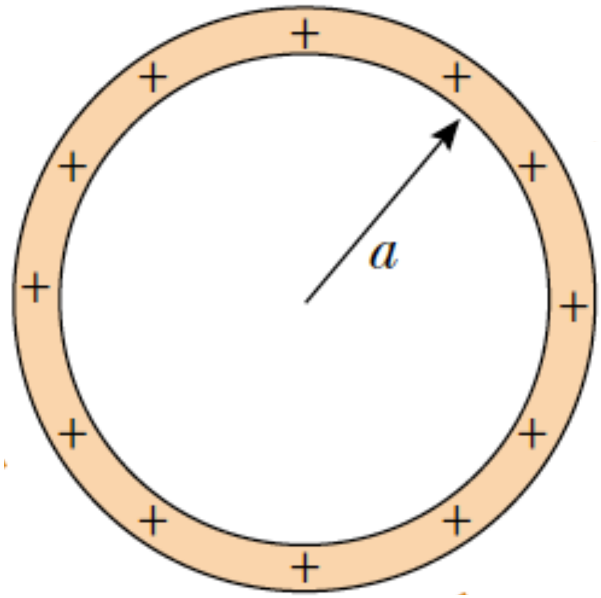
$$E = \lim_{r\to a} k\frac{Q}{r^2} = k\frac{Q}{a^2}$$

$$E = \lim_{r\to a} k\frac{Q}{a^3}r = k\frac{Q}{a^2}$$

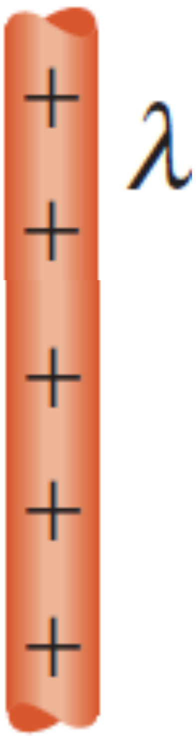
Example: The Electric Field Due to a Thin Spherical Shell

A thin spherical shell of radius a has a total charge Q distributed uniformly over its surface. Find the electric field at points

a) outside and
b) inside the shell



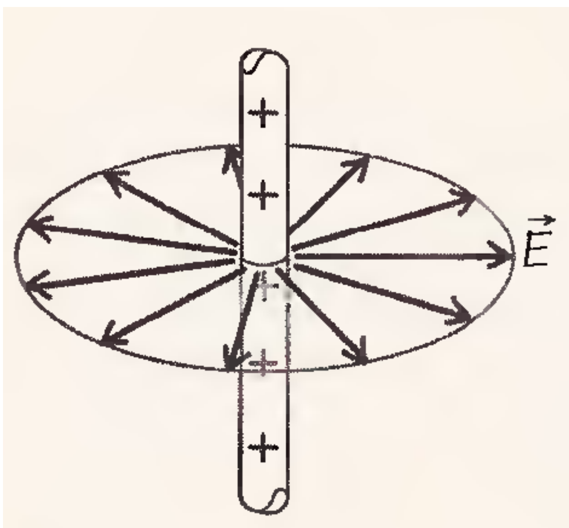
Example: A Cylindrically Symmetric Charge Distribution

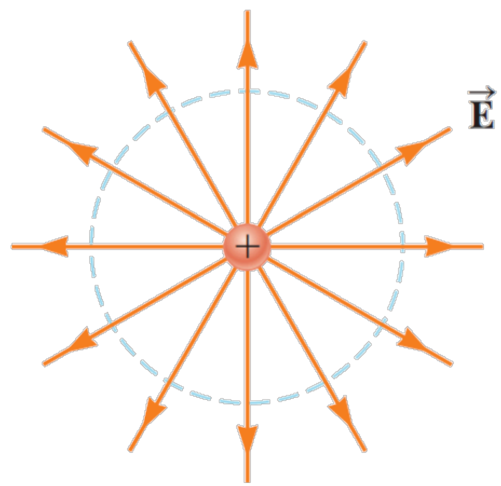


Find the electric field a distance r from a line of positive charge of infinite length and constant charge per unit length λ.

Answer: A Cylindrically Symmetric Charge Distribution

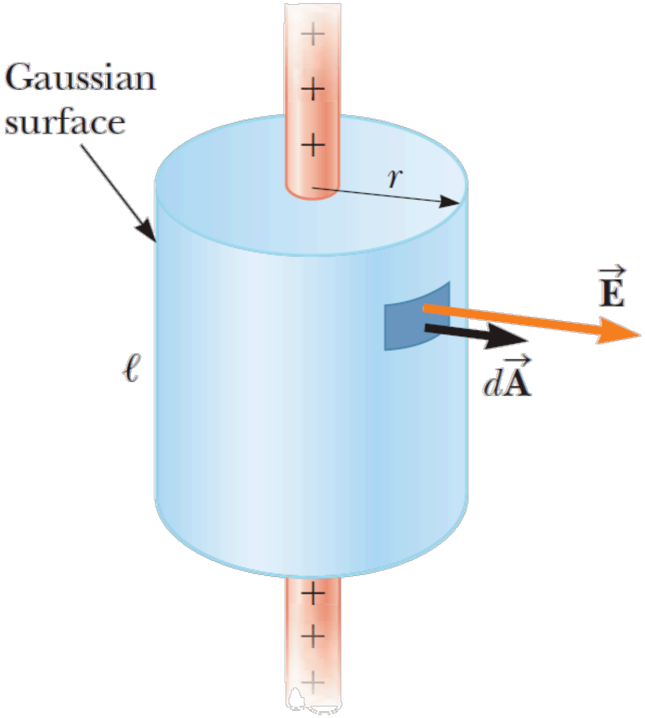
The symmetry of the charge distribution requires that E be perpendicular to the line charge and directed outward.





Answer: A Cylindrically Symmetric Charge Distribution

Our Gaussian surface should match the symmetry of the problem.



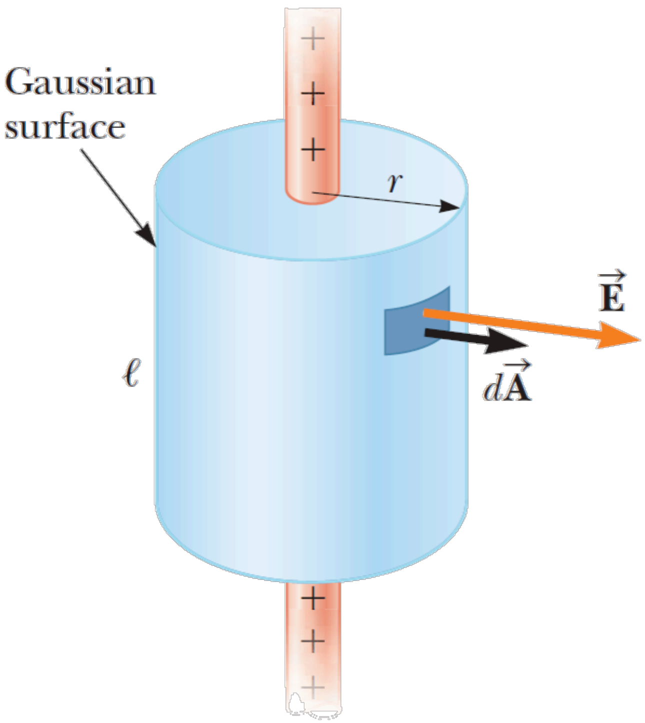
We take the surface integral in Gauss's law over the entire gaussian surface.

$$\Phi_E = \oint \vec{E}.d\vec{A} = E\oint dA = \frac{q_{in}}{\varepsilon_0}$$

We have to find the charge inside the gaussian surface:

$$q_{in} = \lambda l$$

Answer: A Cylindrically Symmetric Charge Distribution



$$\Phi_E = \oint \vec{E}.d\vec{A} = E\oint dA = \frac{q_{in}}{\varepsilon_0}$$

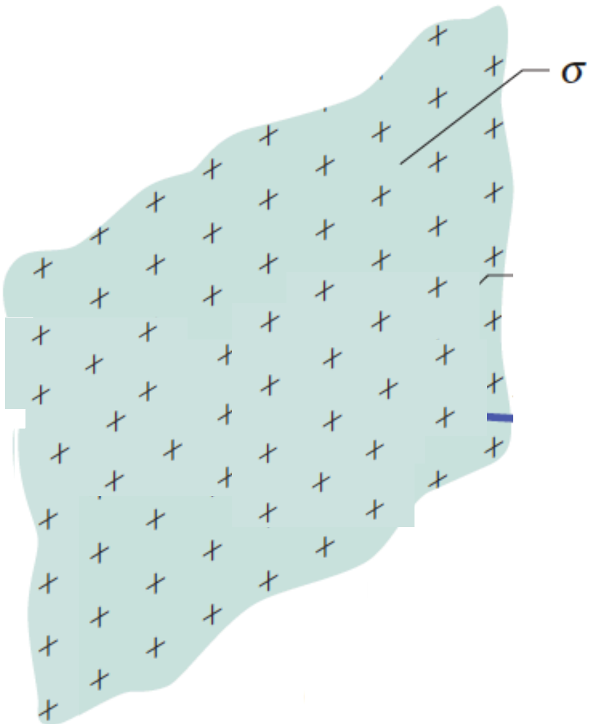
$$EA = \frac{\lambda l}{\varepsilon_o}$$

The area of the curved surface is A=2πrl

$$E(2\pi rl) = \frac{\lambda l}{\varepsilon_o}$$

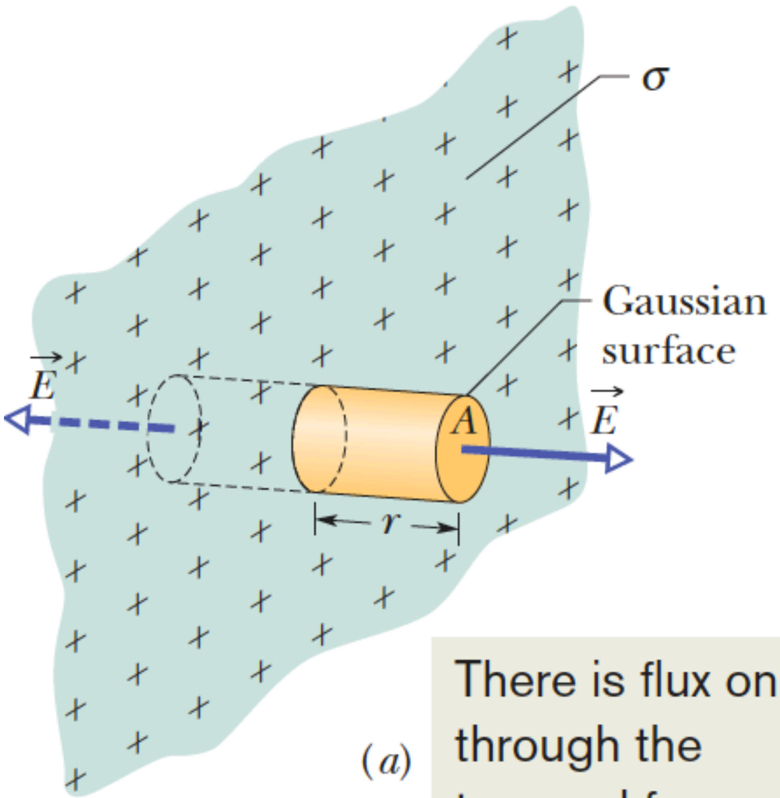
$$E = \frac{\lambda}{2\pi r\varepsilon_o} = 2k\frac{\lambda}{r}$$

Example: A Plane of Charge



Find the electric field due to an infinite plane of positive charge with uniform surface charge density σ.

Answer: A Plane of Charge

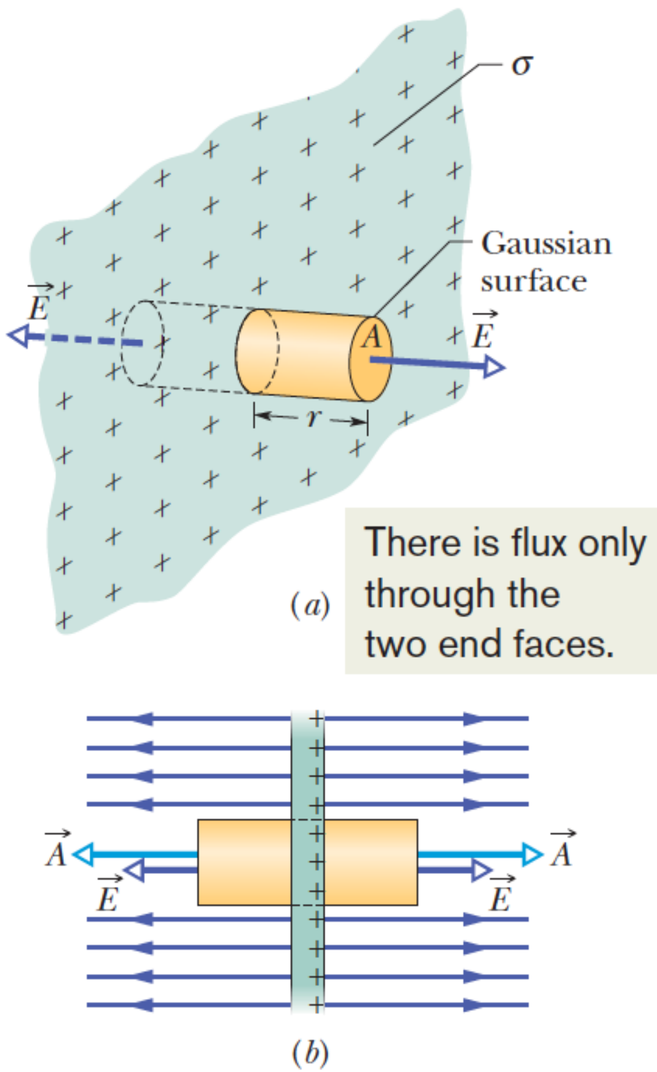


(a) There is flux only through the two end faces.

From symmetry, must be perpendicular to the sheet and furthermore, since the charge is positive, is directed away from the sheet,

A useful Gaussian surface is a closed cylinder with end caps of area A, arranged to pierce the sheet perpendicularly

Answer: A Plane of Charge



The flux through each end of the cylinder is EA; hence, the total flux through the entire gaussian surface is just that through the ends

$$\Phi_E = \oint \vec{E}.d\vec{A} = 2EA = \frac{q_{iç}}{\varepsilon_0}$$

The total charge inside the surface is

$$q_{in} = \sigma A$$

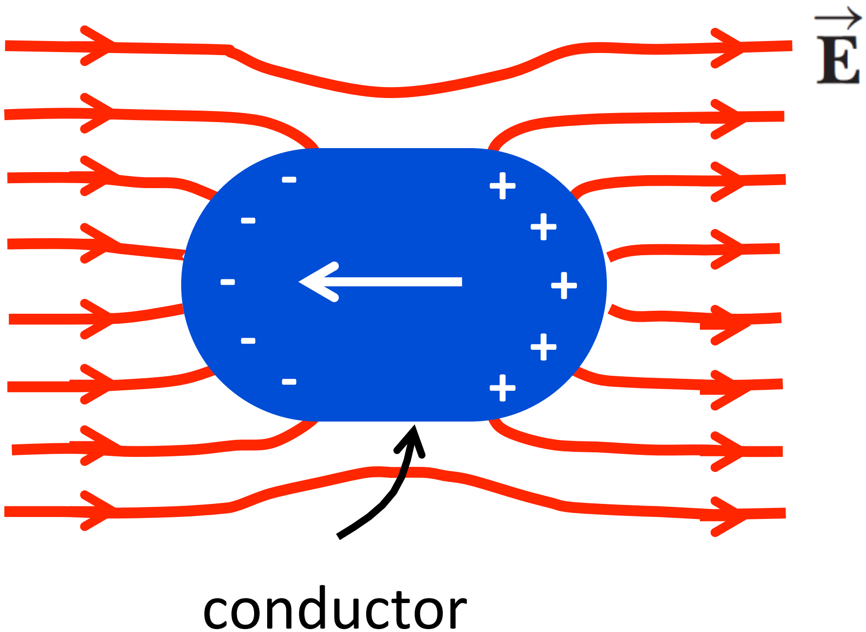
Thus the electric field of a plane of charge:

$$E = \frac{\sigma}{2\varepsilon_0}$$

3.5 Charged Isolated Conductor

A good electrical conductor contains charges (electrons) that are not bound to any atom and therefore are free to move about within the material. When there is no net motion of charge within a conductor, the conductor is in **electrostatic equilibrium**.

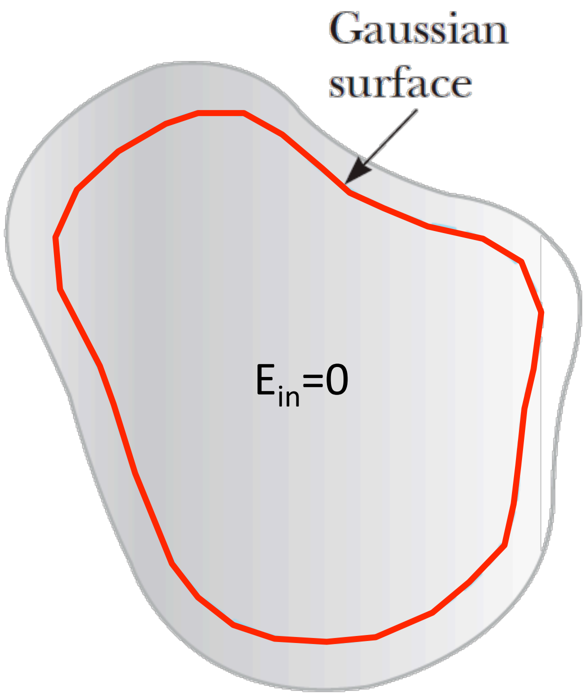
Consider a conducting slab placed in an external field E:



The electric field is zero everywhere inside the conductor if the conductor is in electrostatic equilibrium.

3.5 Charged Isolated Conductor

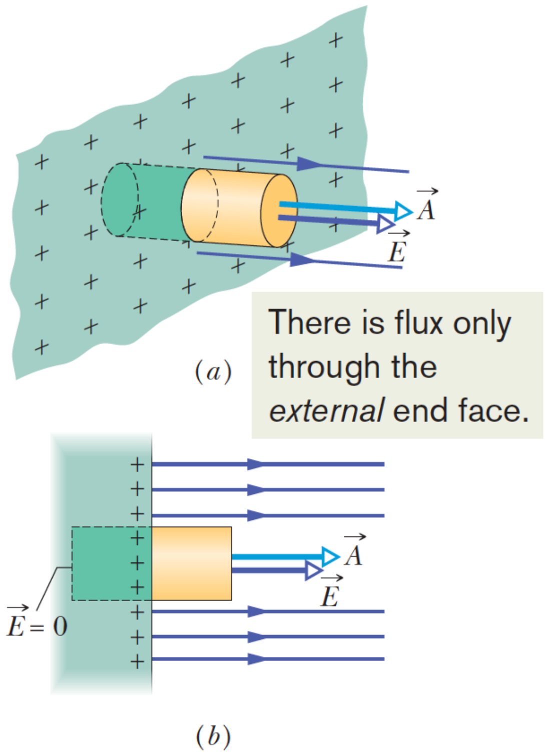
Let's apply Gauss law to a conductor of arbitrary shape which is in electrostatic equlibrium..



Any net charge on the conductor must reside on its surface

3.5 Charged Isolated Conductor

The electric field is zero everywhere inside the conductor if the conductor is in electrostatic equilibrium. Let's find the electric field outside the conductor



We can choose a cylinderical gaussian surface.

$$\Phi_E = \oint \vec{E}.d\vec{A} = EA = \frac{q_{in}}{\varepsilon_0}$$

$$EA = \frac{\sigma A}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

The electric field just outside a charged conductor is perpendicular to the surface of the conductor and has a magnitude σ/ε_0.

3.5 Charged Isolated Conductor

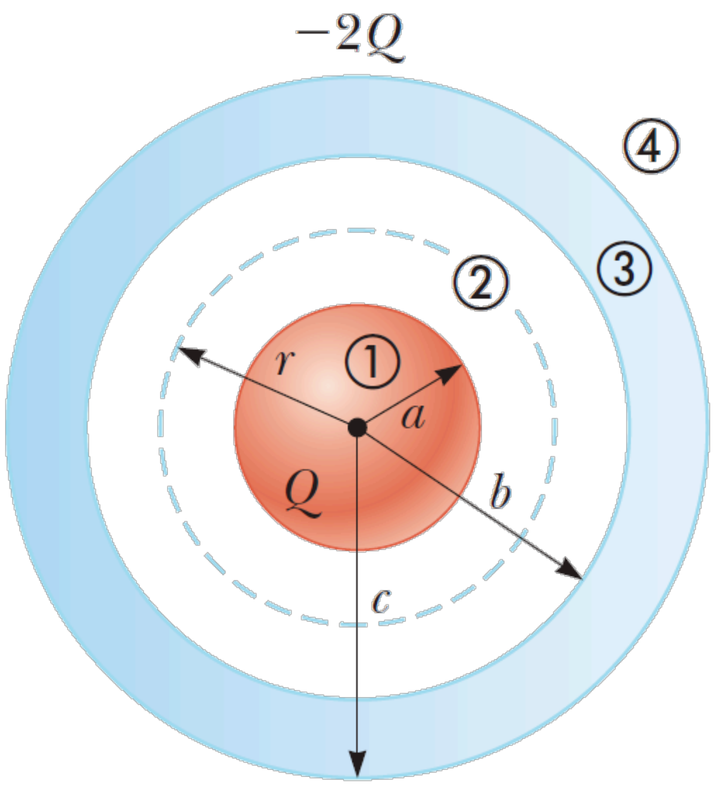
A conductor in electrostatic equilibrium has the following properties:

1. The electric field is zero everywhere inside the conductor.

2. If an isolated conductor carries a charge, the charge resides on its surface.

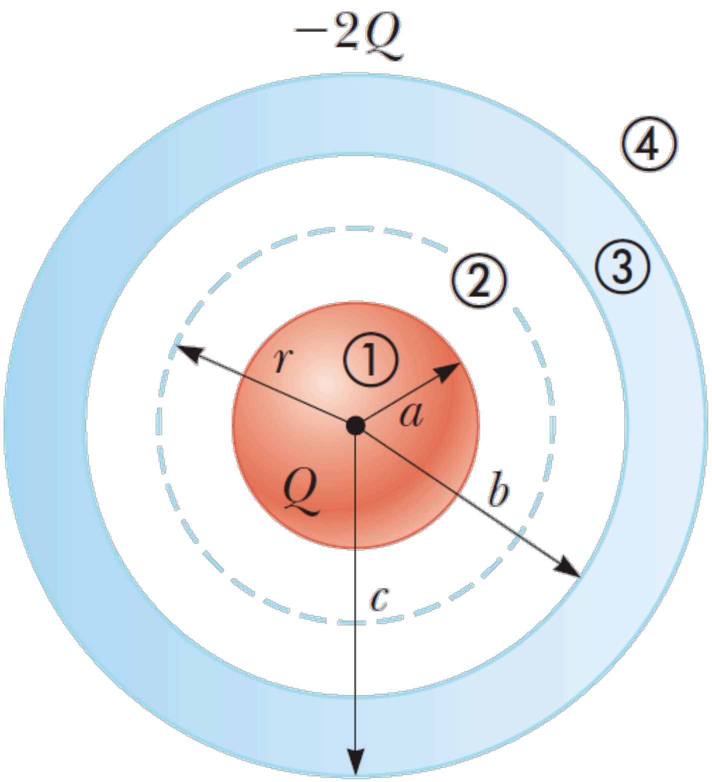
3. The electric field just outside a charged conductor is perpendicular to the surface of the conductor and has a magnitude 4//0, where 4 is the surface charge density at that point.

Example: A Sphere Inside a Spherical Shell



A solid conducting sphere of radius a carries a net positive charge 2Q. A conducting spherical shell of inner radius b and outer radius c is concentric with the solid sphere and carries a net charge Q’. Using Gauss’s law, find the electric field in the regions labeled 1, 2, 3 and 4 as in figure and the charge distribution on the shell when the entire system is in electrostatic equilibrium.

Answer: A Sphere Inside a Spherical Shell



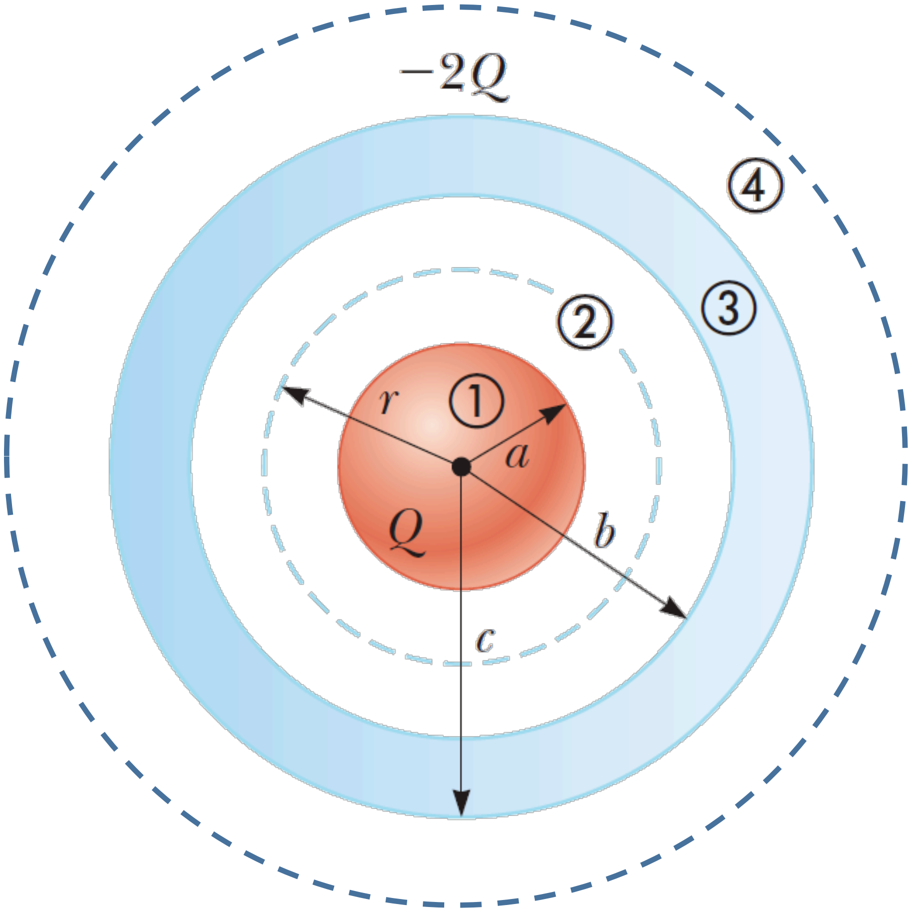
For region 1

$$E_1 = 0 \ \ (r < a)$$

For region 2

$$E_2 = k\frac{Q}{r^2} \ \ (a < r < b)$$

Answer: A Sphere Inside a Spherical Shell



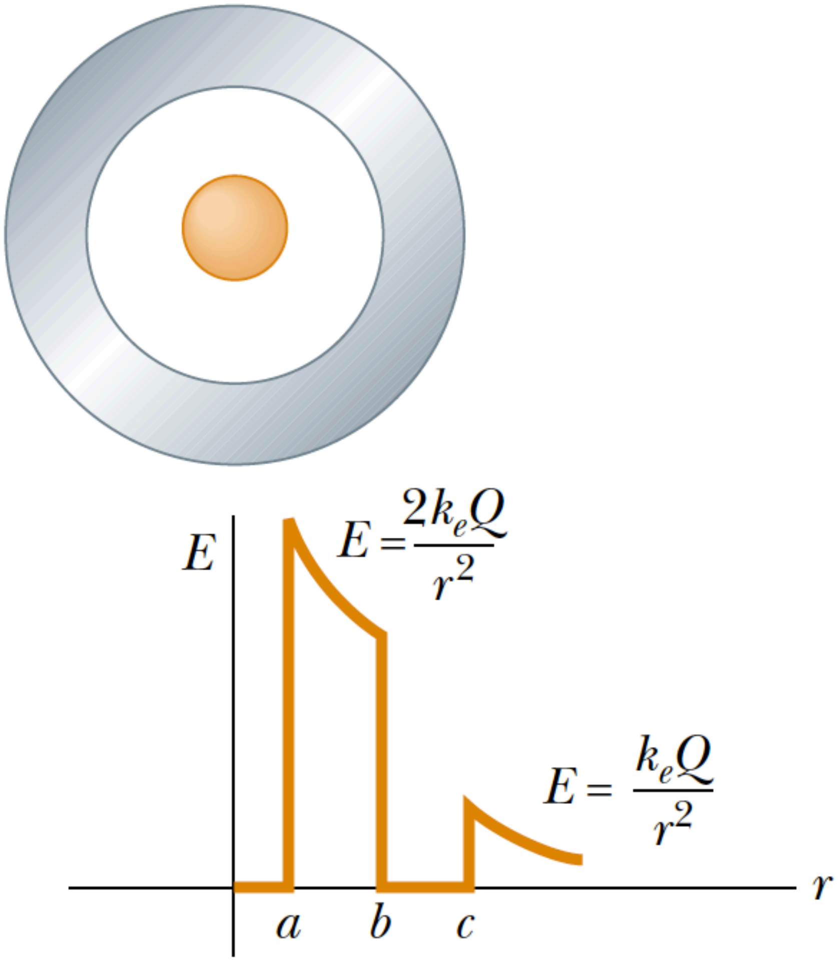
For region 3

$$E_3 = 0 \ \ (b < r < c)$$

For region 4

$$E_4 = k\frac{Q}{r^2} \ \ (r > c)$$

Answer: A Sphere Inside a Spherical Shell



3.SUMMARY

Electric flux is proportional to the number of electric field lines that penetrate a surface. If the electric field is uniform and makes an angle θ with the normal to a surface of area A, the electric flux through the surface is:

$$\Phi_E = EA\cos\theta$$

In general, the electric flux through a surface is

$$\Phi_E = \int_{surface} \vec{E}.d\vec{A}$$

Gauss's law says that the net electric flux Φ_E through any closed gaussian surface is equal to the net charge qin inside the surface divided by ε_o :

$$\Phi_E = \oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

3.SUMMARY

Typical Electric Field Calculations Using Gauss's Law

Charge Distribution	Electric Field	Location
Insulating sphere of radius R, uniform charge density, and total charge Q	$\begin{cases} k_e \dfrac{Q}{r^2} \\ k_e \dfrac{Q}{R^2} r \end{cases}$	$r > R$ $r < R$
Thin spherical shell of radius R and total charge Q	$\begin{cases} k_e \dfrac{Q}{r^2} \\ 0 \end{cases}$	$r > R$ $r < R$
Line charge of infinite length and charge per unit length λ	$2k_e \dfrac{\lambda}{r}$	Outside the line
Infinite charged plane having surface charge density σ	$\dfrac{\sigma}{2\epsilon_0}$	Everywhere outside the plane
Conductor having surface charge density σ	$\begin{cases} \dfrac{\sigma}{\epsilon_0} \\ 0 \end{cases}$	Just outside the conductor Inside the conductor

24. BÖLÜM ÖZET

A conductor in electrostatic equilibrium has the following properties:

1. The electric field is zero everywhere inside the conductor.

2. If an isolated conductor carries a charge, the charge resides on its surface.

3. The electric field just outside a charged conductor is perpendicular to the surface of the conductor and has a magnitude σ/ε_0, where σ is the surface charge density at that point.