

Capacitance

25.1 Capacitance

25.2 Calculating the capacitance

25.3 Capacitors in parallel or series

25.4 Energy stored in capacitor

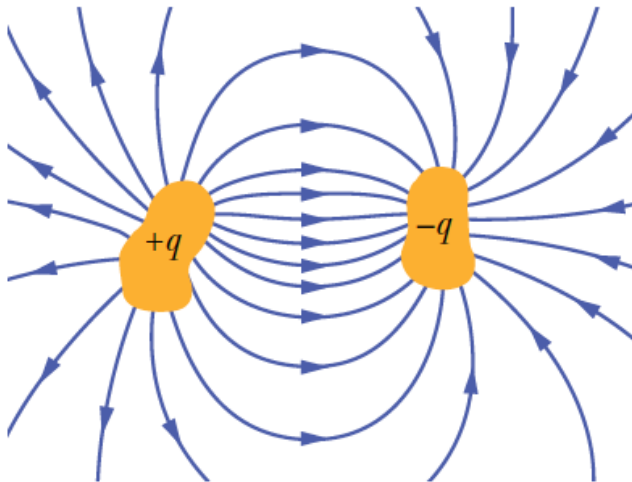
25.5 Capacitors with dielectrics



25.1 Capacitance

Such a combination of two conductors is called a capacitor.

When the capacitor is charged, the conductors carry charges of equal magnitude and opposite sign.



The conductors are called plates. A potential difference ΔV exists between the conductors due to the presence of the charges.

Experiments show that the quantity of charge Q on a capacitor¹ is linearly proportional to the potential difference between the conductors; that is,

$$q \propto \Delta V$$

We can write this relationship as

$$q = C\Delta V$$

25.1 Capacitance

The capacitance C of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors:

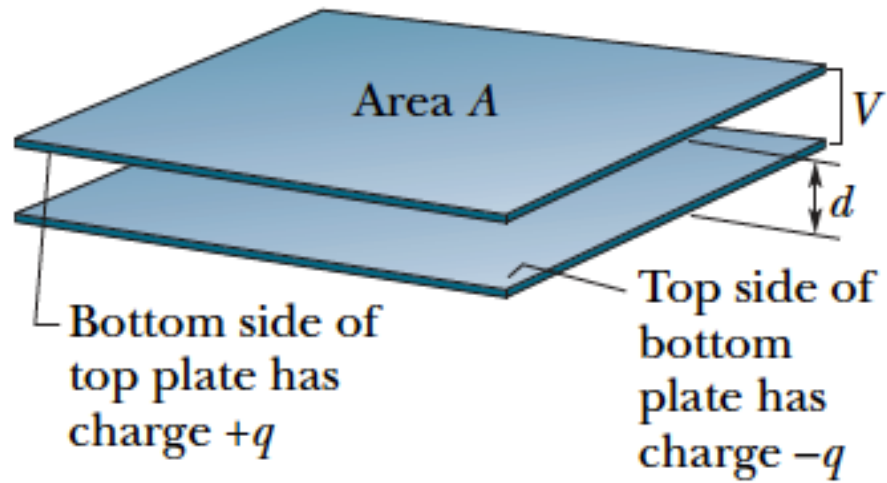
$$C = \frac{q}{\Delta V}$$

The capacitance has SI units of coulombs per volt. The SI unit of capacitance is the **farad (F)**, which was named in honor of Michael Faraday

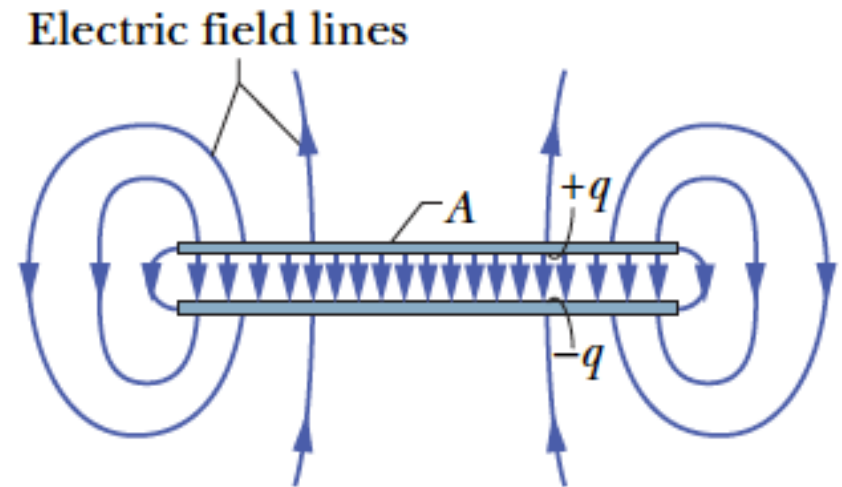
$$1 \text{ F} = 1 \text{ C} / 1 \text{ V}$$

Although the total charge on the capacitor is zero (because there is as much excess positive charge on one conductor as there is excess negative charge on the other), it is common practice to refer to the magnitude of the charge on either conductor as “the charge on the capacitor.”

25.1 Capacitance



(a)



(b)

25.2 Calculating Capacitance

To calculate the capacitance of an capacitor, our plan is as follows:

- (1) Assume a charge q on the plates;
- (2) Calculate the electric field between the plates in terms of this charge, using Gauss' law
- (3) Knowing electric field, calculate the potential difference V between the plates from Eq.

$$V_B - V_A = - \int_a^b \vec{E} \cdot d\vec{s} \Rightarrow V = \int_{-}^{+} E \cdot ds$$

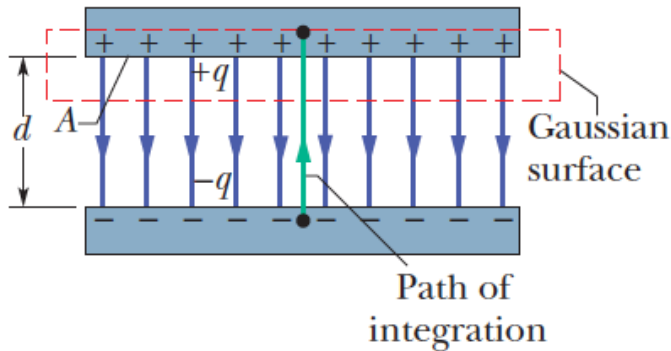
- (4) Calculate C from Eq.

$$C = \frac{q}{\Delta V}$$

25.2 Calculating Capacitance

A Parallel-Plate Capacitor

We use Gauss' law to relate q and E . Then we integrate the E to get the potential difference.



$$q = \epsilon_0 EA$$

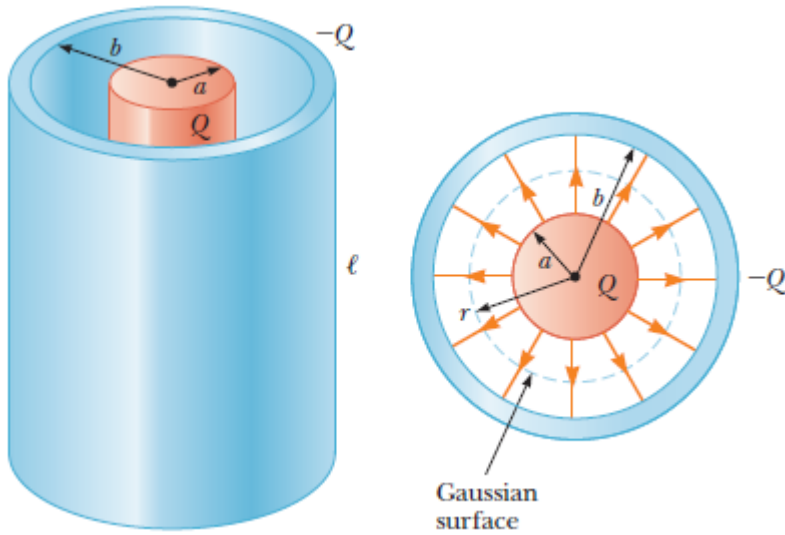
$$V = \int_{-}^{+} E ds = Ed$$

$$C = \frac{q}{V} = \frac{\epsilon_0 EA}{Ed}$$

$$C = \frac{\epsilon_0 A}{d}$$

25.2 Calculating Capacitance

A Cylindrical Capacitor



$$Q = \epsilon_0 EA = \epsilon_0 E(2\pi rl)$$

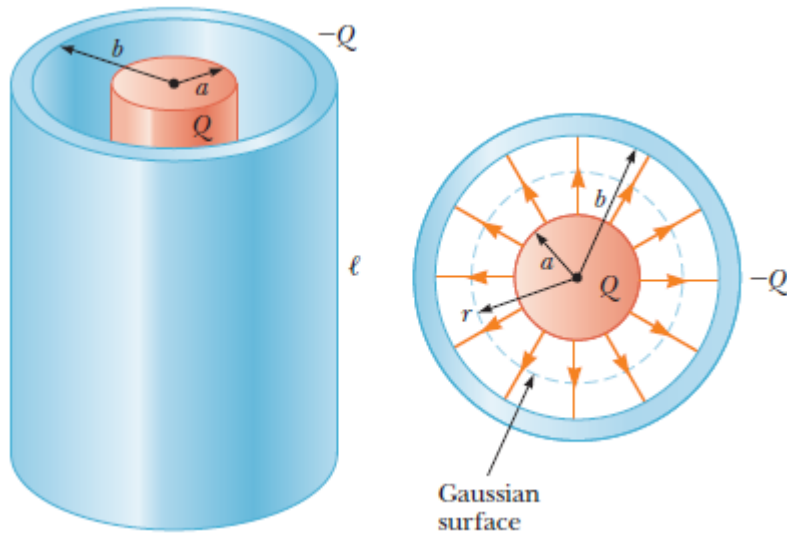
$$E = \frac{Q}{2\epsilon_0 \pi rl}$$

$$V = \int_{-}^{+} E ds = \int_a^b \frac{Q}{2\epsilon_0 \pi rl} dr$$

$$V = \frac{Q}{2\epsilon_0 \pi l} \int_a^b \frac{dr}{r} = \frac{Q}{2\epsilon_0 \pi l} \ln\left(\frac{b}{a}\right)$$

25.2 Calculating Capacitance

A Cylindrical Capacitor

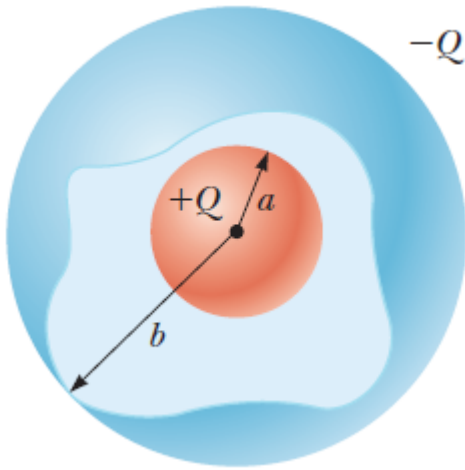


$$C = \frac{Q}{V}$$

$$C = 2\epsilon_0\pi \frac{l}{\ln\left(\frac{b}{a}\right)}$$

25.2 Calculating Capacitance

A Spherical Capacitor



$$V = \int_{-}^{+} E ds = \int_a^b \frac{Q}{4\pi\epsilon_0 r^2} dr$$

$$V = \frac{Q}{4\pi\epsilon_0} \int_a^b \frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0} \left(-\frac{1}{r}\right) \Big|_a^b = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{b} - \frac{1}{a}\right)$$

$$C = \frac{Q}{V} = 4\pi\epsilon_0 \frac{ab}{b-a}$$

25.2 Calculating Capacitance

An Isolated Sphere

While the most common situation is that of two conductors, a single conductor also has a capacitance. For example, imagine a spherical charged conductor. The electric field lines around this conductor are exactly the same as if there were a conducting shell of infinite radius, concentric with the sphere and carrying a charge of the same magnitude but opposite sign.

$$C = \frac{Q}{\Delta V} = \frac{Q}{kQ/R}$$

$$C = 4\pi\epsilon_0 R$$

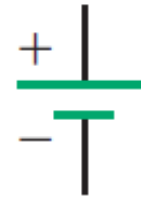
25.3 Capacitors in Parallel and Series

In studying electric circuits, we use a simplified pictorial representation called a circuit diagram. Such a diagram uses circuit symbols to represent various circuit elements.

Capacitor
symbol



Battery
symbol



Switch
symbol



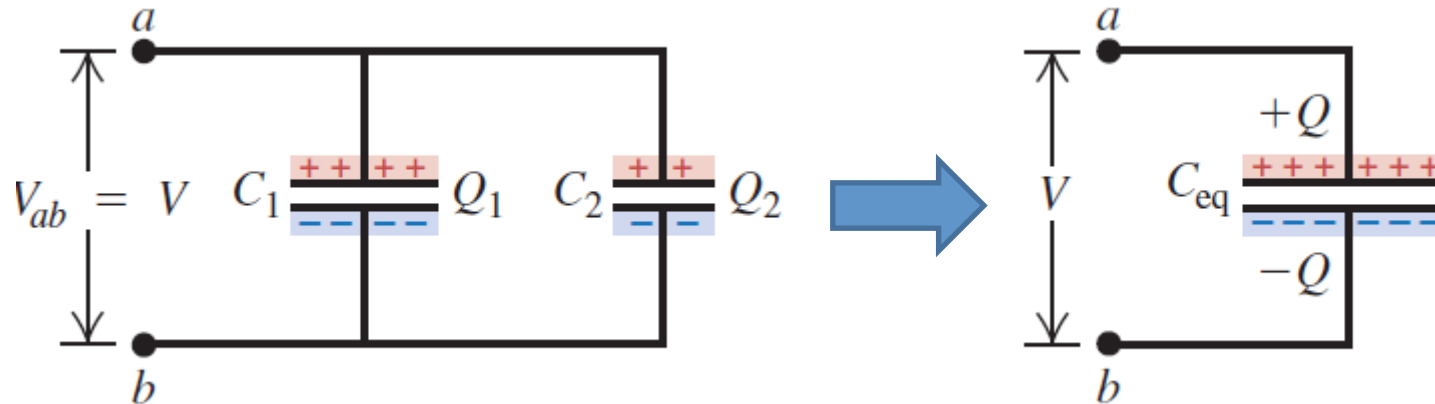
Open



Closed

25.3 Capacitors in Parallel and Series

Parallel Combination



In this case the upper plates of the two capacitors are connected by conducting wires to form an equipotential surface, and the lower plates form another. Hence in a parallel connection the potential difference for all individual capacitors is the same

$$V_1 = V_2 = V$$

Because the voltages across the capacitors are the same, the charges that they carry are:

$$Q_2 = C_2 V \quad Q_1 = C_1 V$$

we wish to replace these two capacitors by one equivalent capacitor.

25.3 Capacitors in Parallel and Series

Parallel Combination

The effect this equivalent capacitor has on the circuit must be exactly the same as the effect of the combination of the two individual capacitors. That is, the equivalent capacitor must store Q units of charge when connected to the battery.

$$Q_{top} = Q_1 + Q_2$$

Thus, for the equivalent capacitor:

$$Q_{top} = C_{eq} V$$

$$C_{eq} V = Q_1 + Q_2 = C_1 V + C_2 V$$

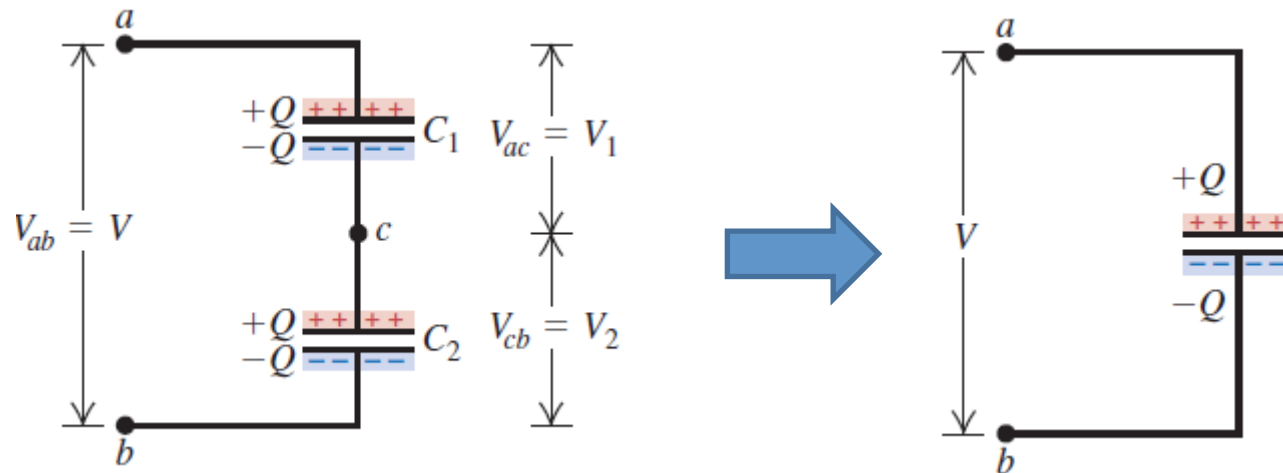
$$C_{eq} = C_1 + C_2$$

The properties of capacitor in parallel combination:

- **Capacitors have same potential.**
- **The charge on capacitors depends on the capacitance of each capacitor.**
- **Equivalent capacitor : $C_1 + C_2 + C_3 + \dots$**

25.3 Capacitors in Parallel and Series

Series Combination



When a potential difference V is applied across several capacitors connected in series, the capacitors have identical charge Q :

$$Q_1 = Q_2 = Q$$

The total potential difference across any number of capacitors connected in series is the sum of the potential differences across the individual capacitors.

$$V_{top} = V_1 + V_2$$

25.3 Capacitors in Parallel and Series

Series Combination

The equivalent single capacitor has the same effect on the circuit as the series combination when it is connected to the battery.

$$V_{top} = \frac{Q}{C_{eq}} \quad \frac{Q}{C_{eq}} = V_1 + V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

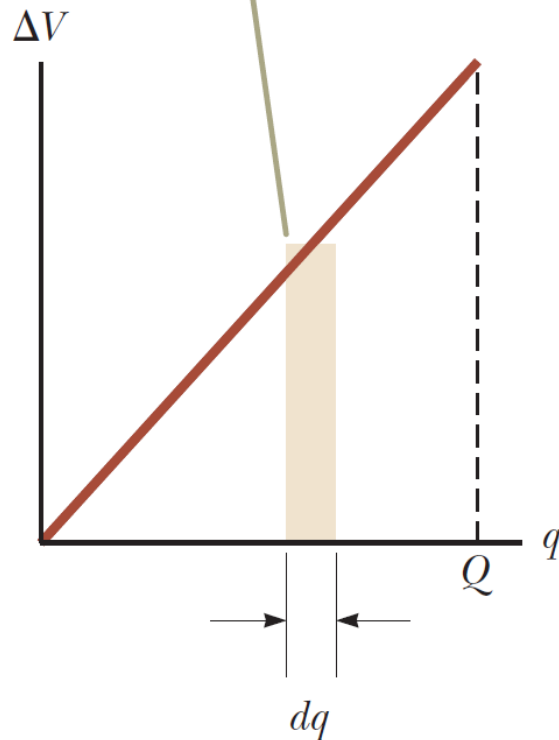
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

The properties of capacitors in series combination

- **Capacitors have same charge**
- The total potential difference across any number of capacitors connected in series is the sum of the potential differences across the individual capacitors.
- **Equivalent capacitance : $1/C_1 + 1/C_2 + 1/C_3 + \dots$**

25.4 Energy Stored in Capacitor

The work required to move charge dq through the potential difference ΔV across the capacitor plates is given approximately by the area of the shaded rectangle.



$$dW = \Delta V dq = \frac{q}{C} dq$$

The work required to bring the total capacitor charge up to a final value to Q is

$$W = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$

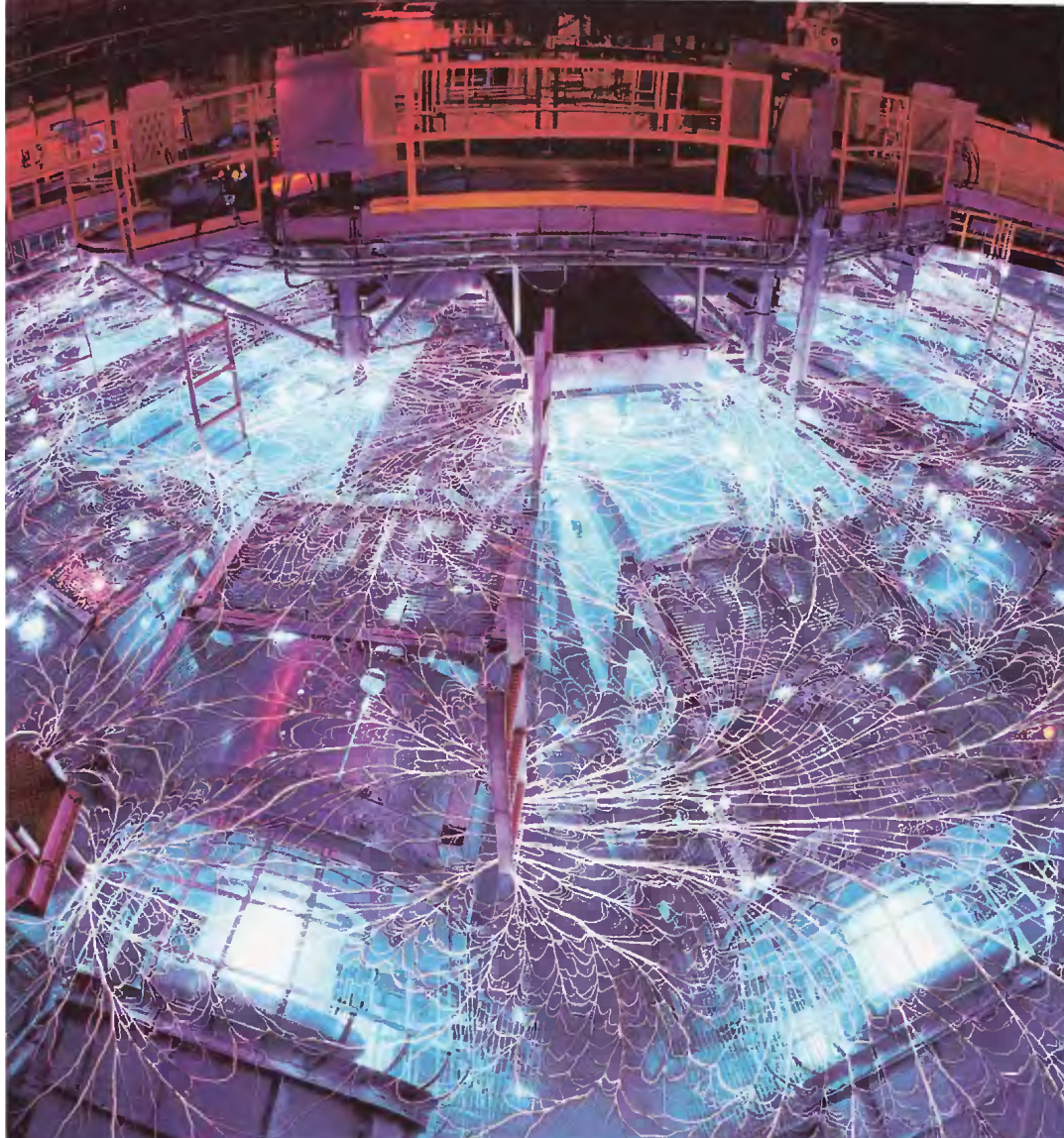
This work is stored as potential energy U in the capacitor, so that

$$U = \frac{Q^2}{2C} = \frac{1}{2} QV = \frac{1}{2} CV^2$$

25.4 Energy Stored in Capacitor



25.4 Energy Stored in Capacitor



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Lawrence Livermore National Lab.

25.4 Energy Stored in Capacitor

The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates. For parallel plates:

$$\Delta V = Ed \qquad C = \frac{\epsilon_0 A}{d}$$

By using this:

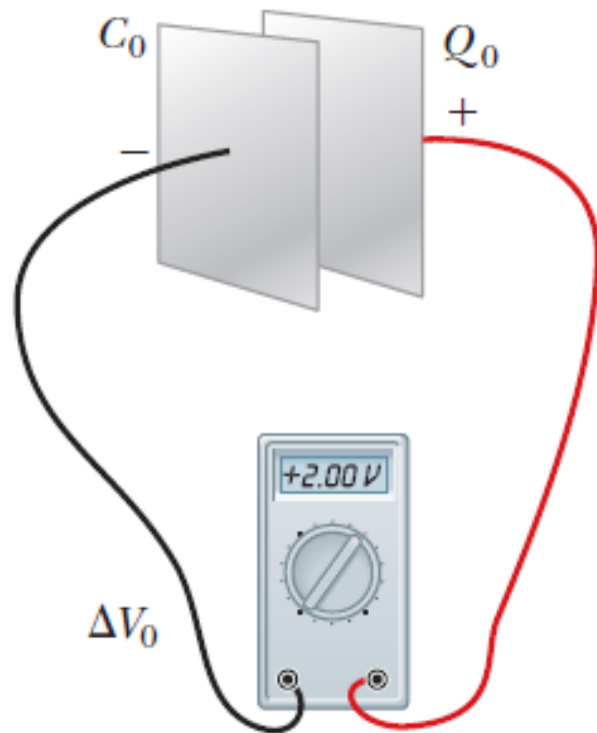
$$U = \frac{1}{2} \frac{\epsilon_0 A}{d} (Ed)^2 = \frac{1}{2} (\epsilon_0 Ad) E^2$$

In a parallel-plate capacitor, neglecting fringing, the electric field has the same value at all points between the plates. Because the volume occupied by the electric field is Ad , the energy per unit volume $u_E = U / (Ad)$ known as the energy density, is

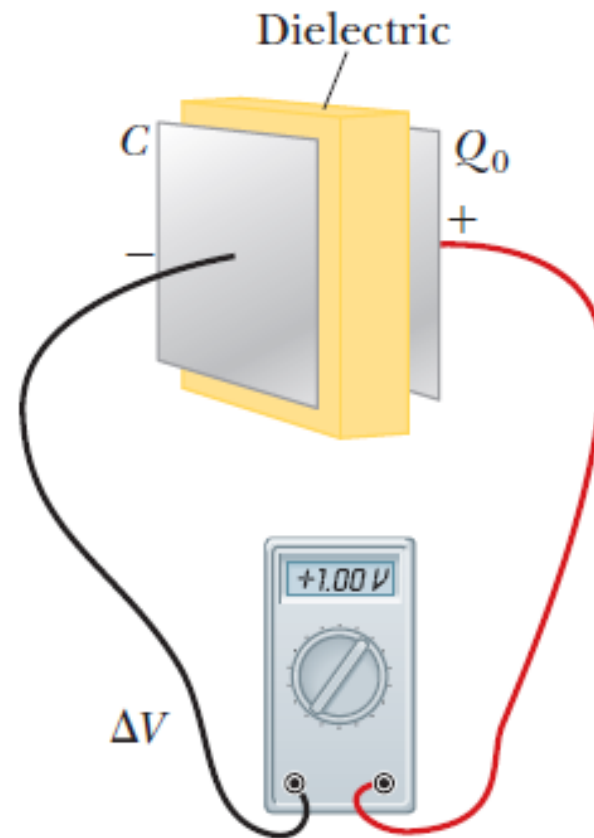
$$u_E = \frac{1}{2} \epsilon_0 E^2$$

25.5 Capacitors with Dielectrics

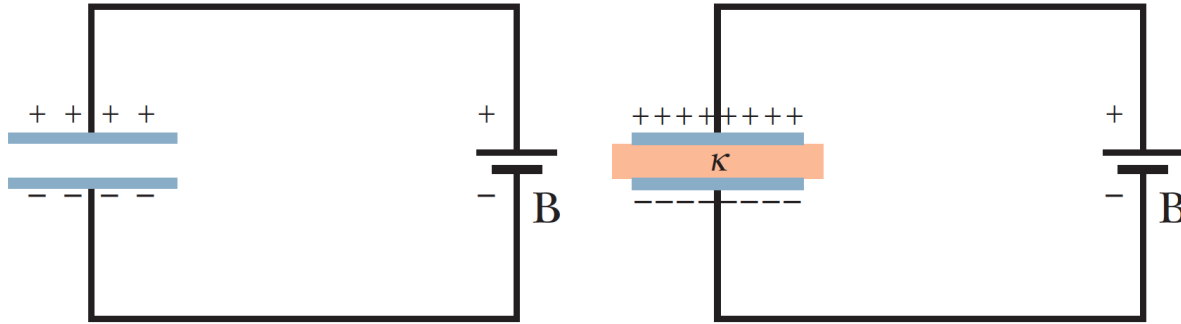
The potential difference across the charged capacitor is initially ΔV_0 .



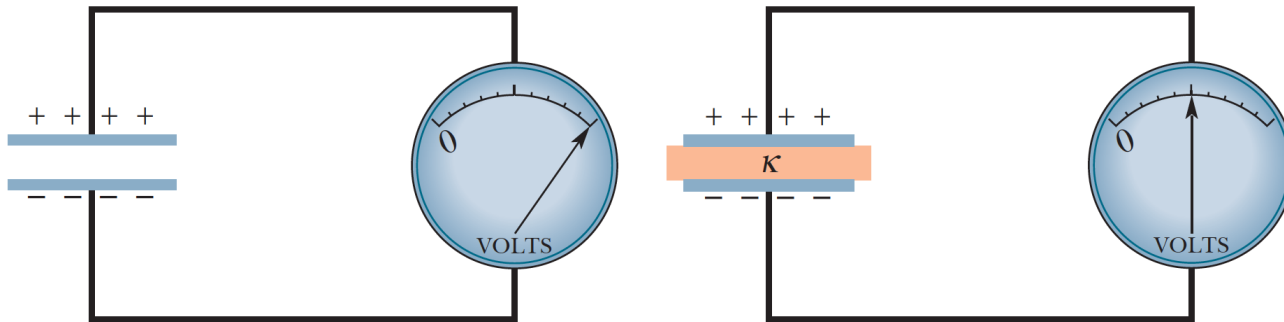
After the dielectric is inserted between the plates, the charge remains the same, but the potential difference decreases and the capacitance increases.



25.5 Capacitors with Dielectrics



$V = \text{a constant}$



$q = \text{a constant}$

25.5 Capacitors with Dielectrics

The potential difference across the capacitor without dielectric is:

$$\Delta V_0 = Q_0 / C_0$$

If a dielectric is inserted between the plates, then the potential difference decreases to a value ΔV . The voltages with and without the dielectric are related by the factor κ as follows :

$$\Delta V = \frac{\Delta V_0}{\kappa}$$

κ : dielectric constant

Since $\Delta V < \Delta V_0$, then $\kappa > 1$

25.5 Capacitors with Dielectrics

Because the charge Q_0 on the capacitor does not change, we conclude that the capacitance must change to the value

$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0/\kappa} = \kappa \frac{Q_0}{\Delta V_0}$$

$$C = \kappa C_0$$

The capacitance increases by the factor κ when the dielectric completely fills the region between the plates

25.5 Capacitors with Dielectrics

For capacitors with parallel plates, if there is a dielectric between the plates:

$$C = \kappa \frac{\epsilon_0 A}{d}$$

For any given separation d , the maximum voltage that can be applied to a capacitor without causing a discharge depends on the **dielectric strength** (maximum electric field) of the dielectric.

25.5 Capacitors with Dielectrics

A dielectric provides the following advantages:

- Increase in capacitance
- Increase in maximum operating voltage
- Possible mechanical support between the plates, which allows the plates to be close together without touching, thereby decreasing d and increasing C .

25.5 Capacitors with Dielectrics

Some Properties of Dielectrics^a

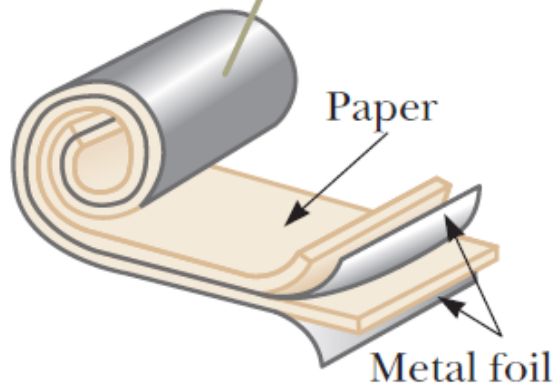
Material	Dielectric Constant κ	Dielectric Strength (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8

For a vacuum, $\kappa = \text{unity}$.

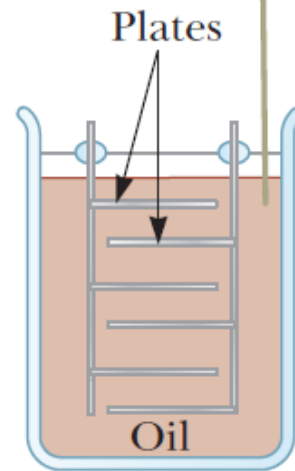
^aMeasured at room temperature, except for the water.

25.5 Capacitors with Dielectrics

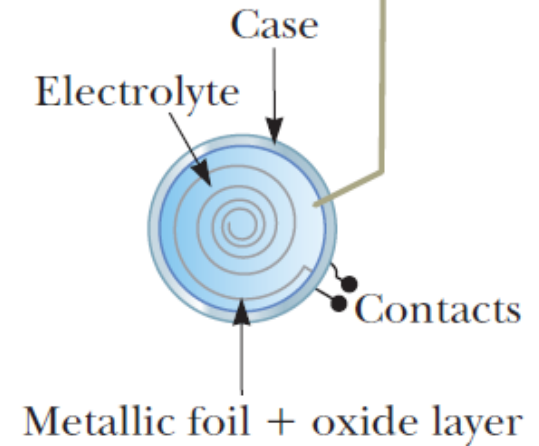
A tubular capacitor whose plates are separated by paper and then rolled into a cylinder



A high-voltage capacitor consisting of many parallel plates separated by insulating oil



An electrolytic capacitor



25.5 Capacitors with Dielectrics



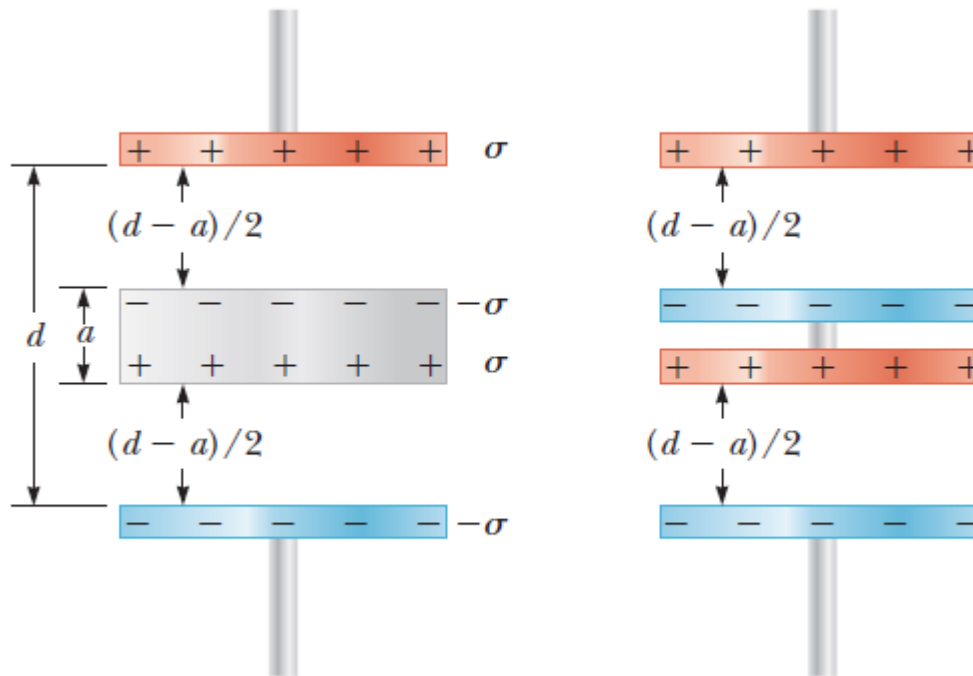
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When one set of metal plates is rotated so as to lie between a fixed set of plates, the capacitance of the device changes.

25.5 Capacitors with Dielectrics

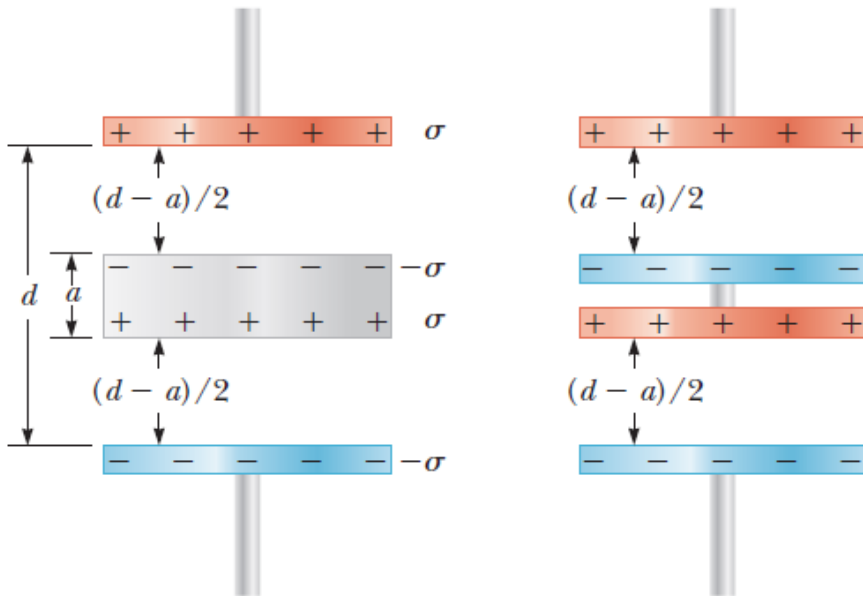
Example:

A parallel-plate capacitor has a plate separation d and plate area A . An uncharged metallic slab of thickness a is inserted midway between the plates.



25.5 Capacitors with Dielectrics

Solution:



$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

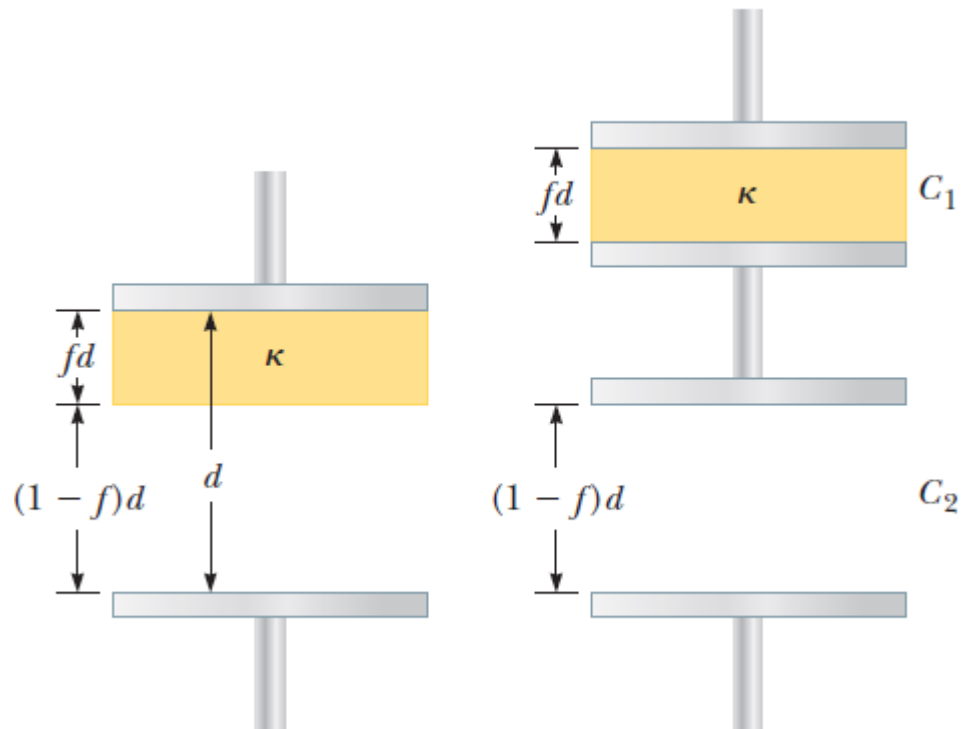
$$C = \frac{1}{\frac{\epsilon_0 A}{(d - a)/2}} + \frac{1}{\frac{\epsilon_0 A}{(d - a)/2}}$$

$$C = \frac{\epsilon_0 A}{d - a}$$

25.5 Capacitors with Dielectrics

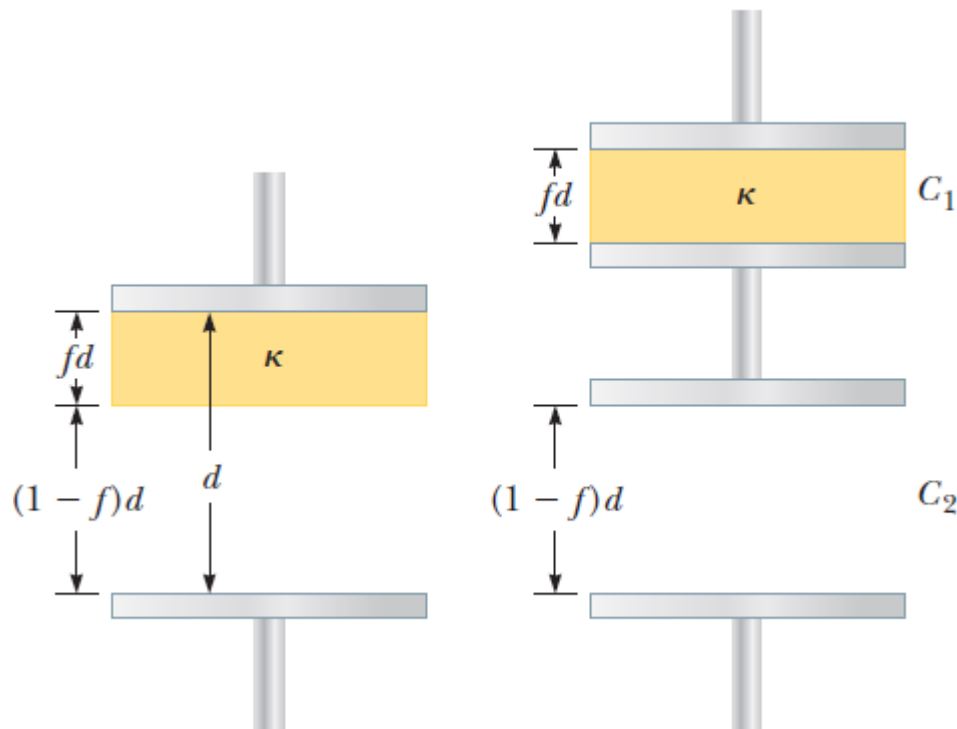
Example:

A parallel-plate capacitor with a plate separation d has a capacitance C_0 in the absence of a dielectric. What is the capacitance when a slab of dielectric material of dielectric constant κ and thickness fd ($0 < f < 1$) is inserted between the plates



25.5 Capacitors with Dielectrics

Solution:



$$C_1 = \frac{\kappa \epsilon_0 A}{fd} \quad C_2 = \frac{\epsilon_0 A}{(1-f)d}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C = \frac{\kappa}{f + \kappa(1-f)} C_0$$