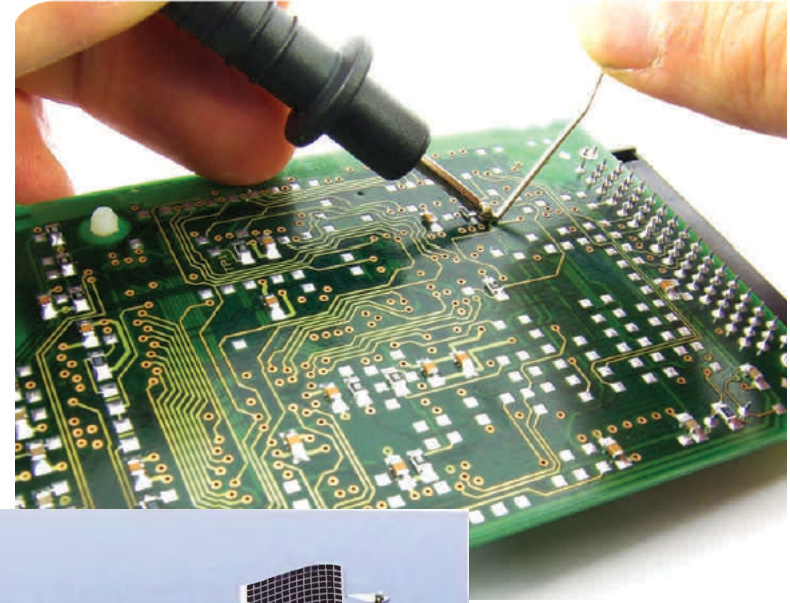


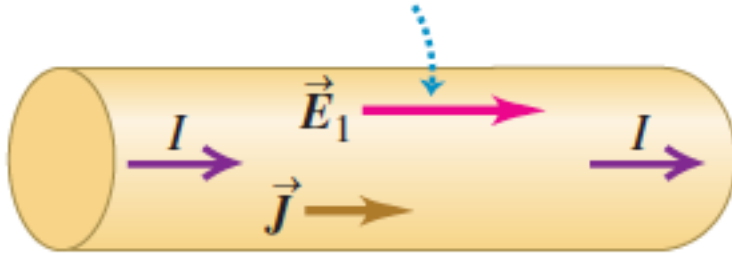
DC Current Circuits

- 7.1 Pumping Charges
- 7.2 Resistors in Series and Parallel
- 7.3 Kirchoff's Rules
- 7.4 RC Circuits



7.1 Pumping Charges

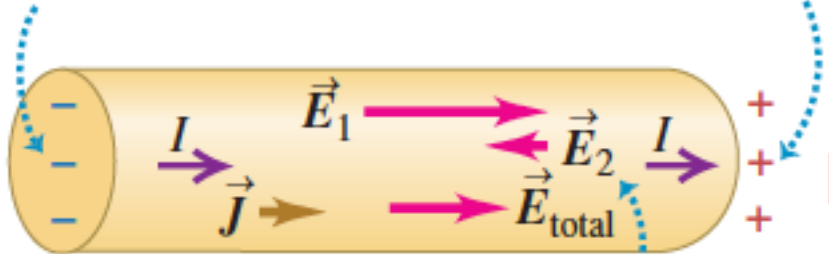
(a) An electric field \vec{E}_1 produced inside an isolated conductor causes a current.



How is it possible to maintain a steady current in a complete circuit?

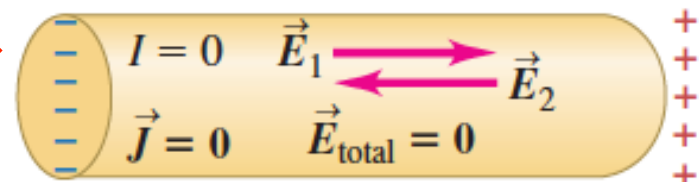


(b) The current causes charge to build up at the ends.



The charge buildup produces an opposing field \vec{E}_2 , thus reducing the current.

(c) After a very short time \vec{E}_2 has the same magnitude as \vec{E}_1 ; then the total field is $\vec{E}_{total} = 0$ and the current stops completely.



7.1 Pumping Charges

A steady current in a complete circuit :



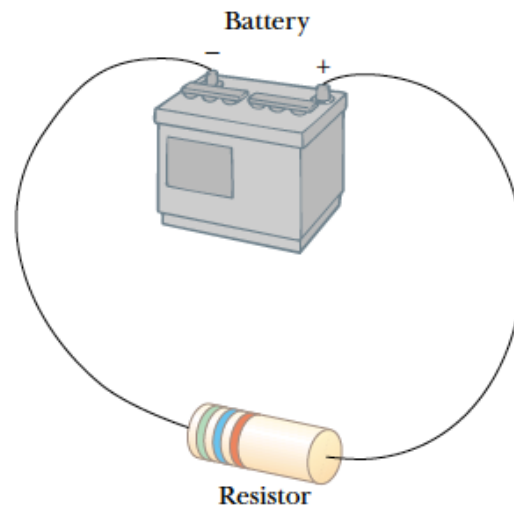
The water pours out of openings at the top, cascades down over the terraces and spouts (**moving in the direction of decreasing gravitational potential energy**), and collects in a basin in the bottom. A pump then lifts it back to the top (**increasing the potential energy**) for another trip

In an electric circuit there must be a device somewhere in the loop that acts like the water pump in a water fountain



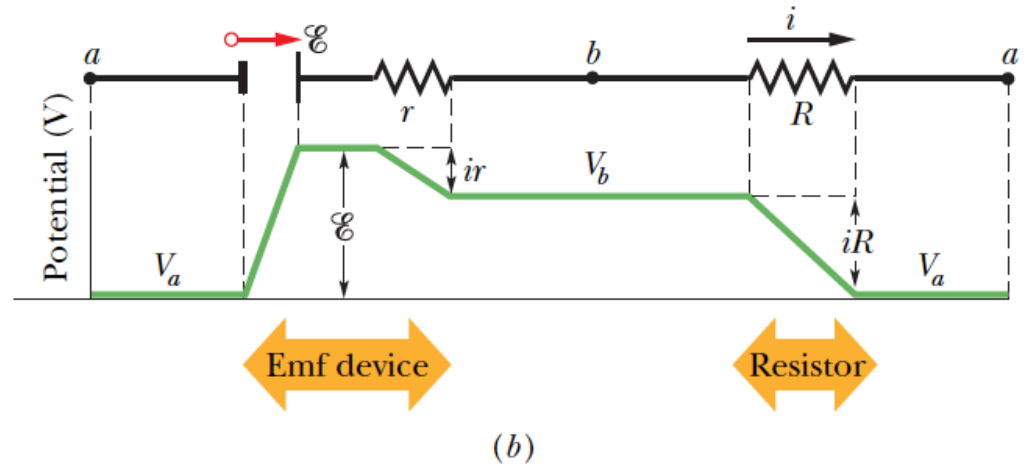
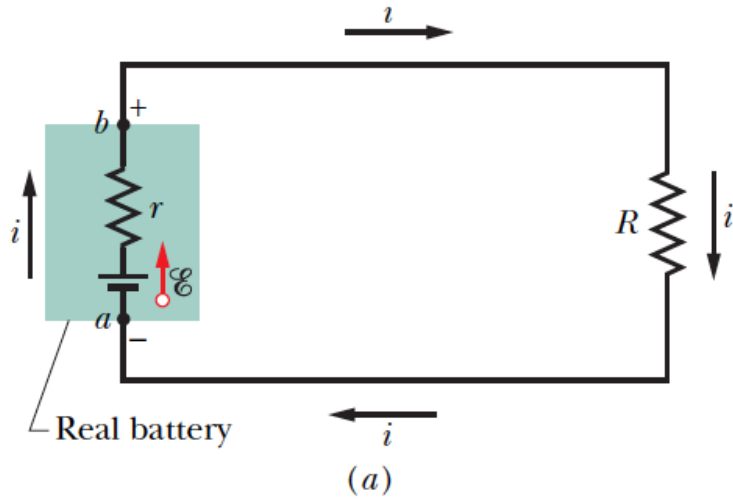
7.1 Pumping Charges

To produce a steady flow of charge, you need a “charge pump,” a device that—by doing work on the charge carriers—maintains a potential difference between a pair of terminals. We call such a device an emf device, and the device is said to provide an emf, which means that it does work on charge carriers.



The term **emf** comes from the outdated phrase **electromotive force**, which was adopted before scientists clearly understood the function of an emf device.

7.1 Pumping Charges

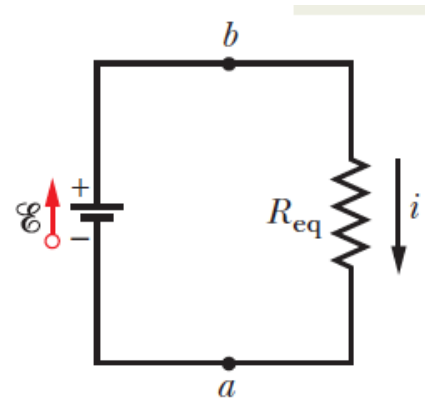
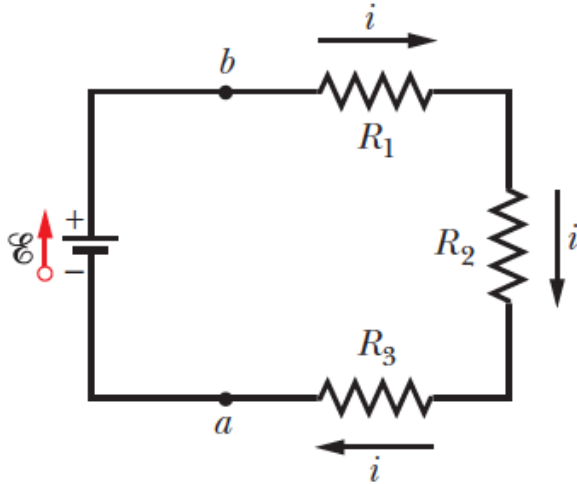


$$\mathcal{E} = ir + iR \Rightarrow i = \frac{\mathcal{E}}{R + r}$$



$$i\mathcal{E} = P = i^2 r + i^2 R$$

7.2 Resistors in Series and Parallel

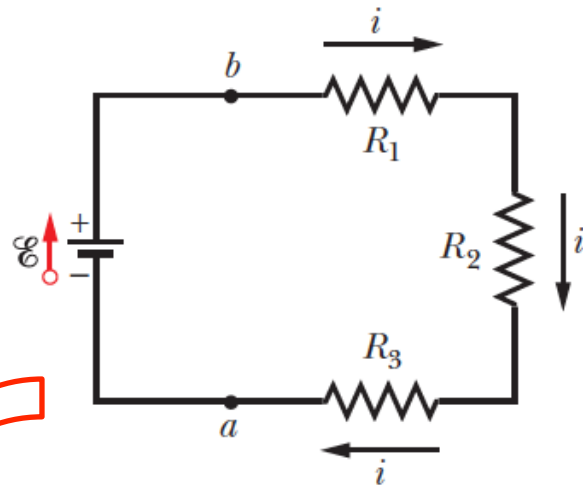


✓ When a potential difference V is applied across resistances connected in series, the resistances have identical currents i .

✓ The sum of the potential differences across the resistances is equal to the applied potential difference V .

Resistances connected in series can be replaced with an equivalent resistance R_{eq} that has the same current i and the same total potential difference V as the actual resistances.

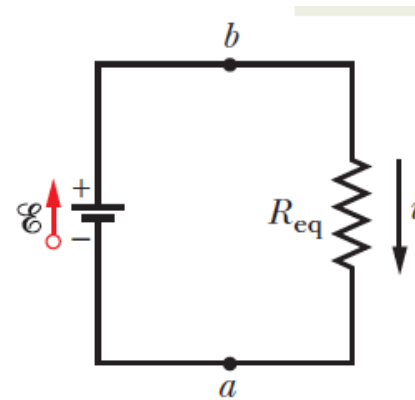
7.2 Resistors in Series and Parallel



$$\mathcal{E} = iR_1 + iR_2 + iR_3$$

$$i = \frac{\mathcal{E}}{R_1 + R_2 + R_3}$$

$$R_{eq} = R_1 + R_2 + R_3$$

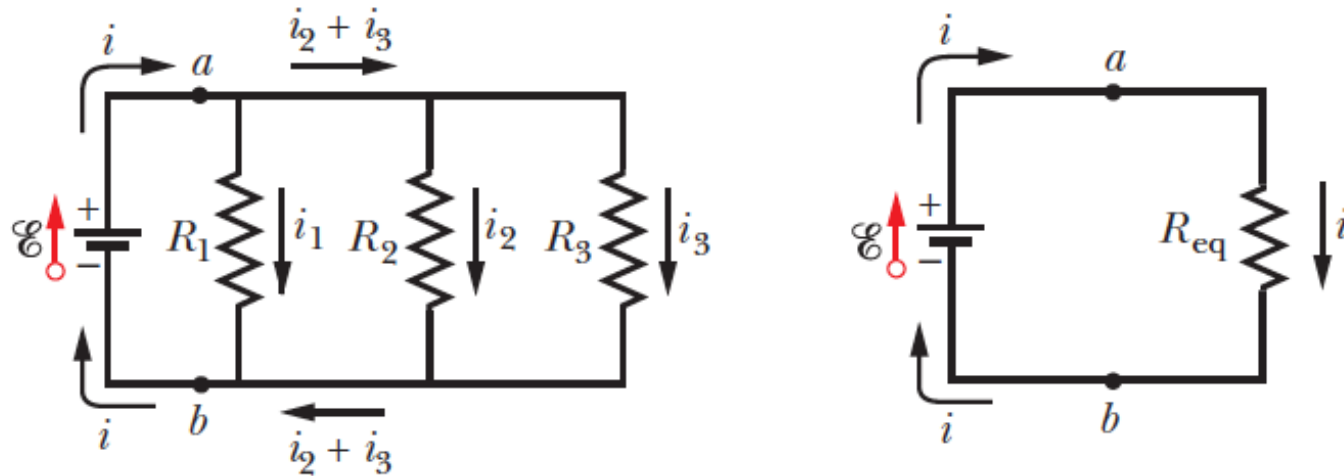


$$\mathcal{E} = iR_{eq}$$

$$i = \frac{\mathcal{E}}{R_{eq}}$$

$$R_{eq} = \sum_{j=1}^n R_j$$

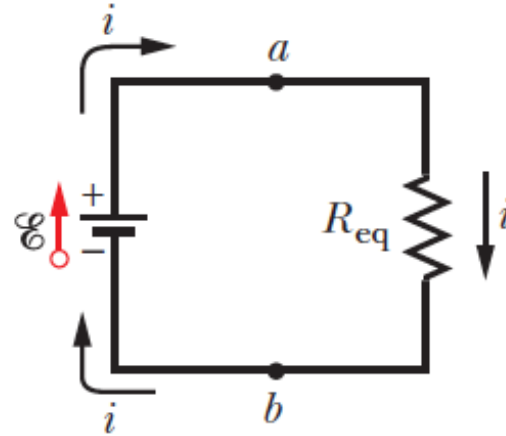
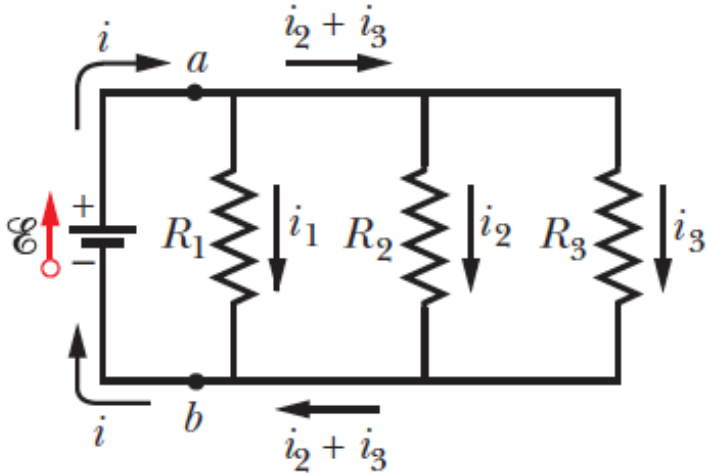
7.2 Resistors in Series and Parallel



✓ When a potential difference V is applied across resistances connected in parallel, the resistances all have that same potential difference V .

Resistances connected in parallel can be replaced with an equivalent resistance R_{eq} that has the same potential difference V and the same total current i as the actual resistances.

7.2 Resistors in Series and Parallel

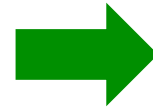


$$i = i_1 + i_2 + i_3$$

$$i = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$i = \frac{V}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$



$$\frac{1}{R_{eq}} = \sum_{j=1}^n \frac{1}{R_j}$$

7.2 Resistors in Series and Parallel

Table 27-1

Series and Parallel Resistors and Capacitors

Series	Parallel	Series	Parallel
<u>Resistors</u>		<u>Capacitors</u>	
$R_{\text{eq}} = \sum_{j=1}^n R_j$ Eq. 27-7 Same current through all resistors	$\frac{1}{R_{\text{eq}}} = \sum_{j=1}^n \frac{1}{R_j}$ Eq. 27-24 Same potential difference across all resistors	$\frac{1}{C_{\text{eq}}} = \sum_{j=1}^n \frac{1}{C_j}$ Eq. 25-20 Same charge on all capacitors	$C_{\text{eq}} = \sum_{j=1}^n C_j$ Eq. 25-19 Same potential difference across all capacitors

7.3 Kirchoff's Rules

Junction rule : The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction:

$$\sum_{\text{junction}} i = 0$$

Loop rule: The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum_{\text{closed loop}} \Delta V = 0$$

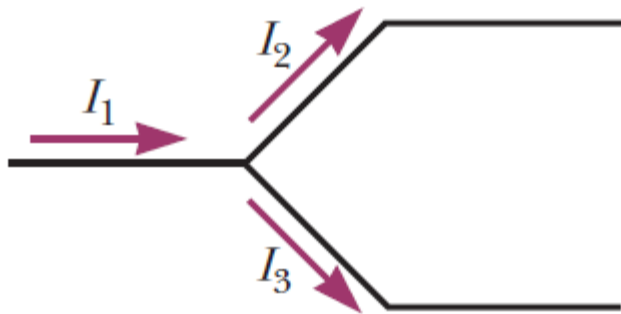
7.3 Kirchoff's Rules

Junction rule : The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction:

$$\sum_{\text{kavşak}} i = 0$$

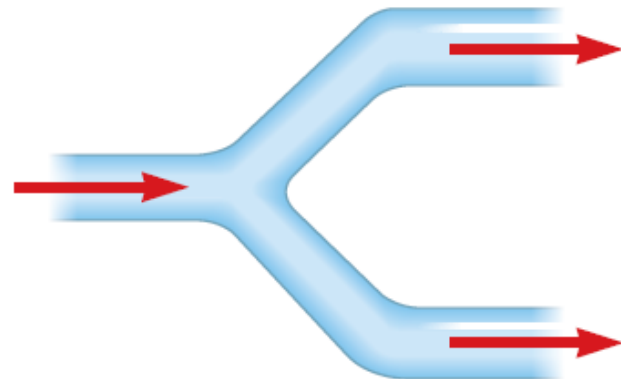
it is a statement of conservation of electric charge

The amount of charge flowing out of the branches on the right must equal the amount flowing into the single branch on the left.



a

The amount of water flowing out of the branches on the right must equal the amount flowing into the single branch on the left.



b

7.3 Kirchoff's Rules

Loop rule: The sum of the potential differences across all elements around any closed circuit loop must be zero:

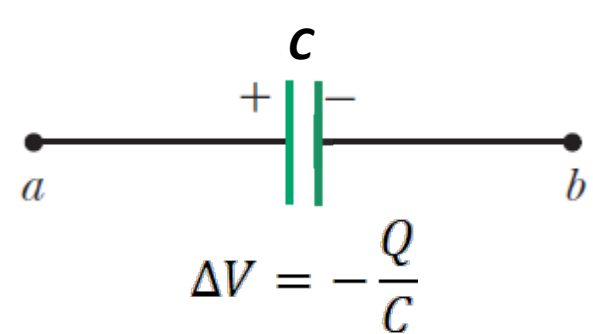
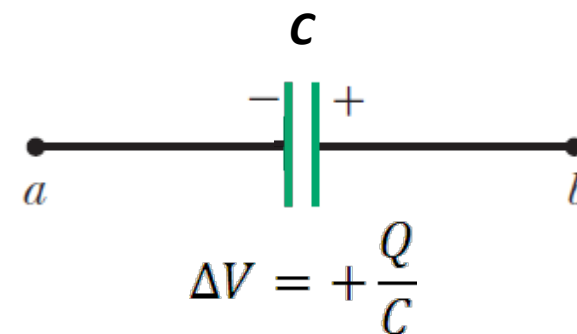
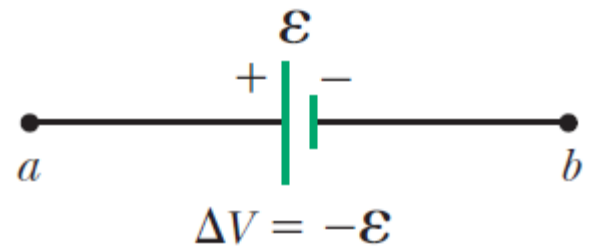
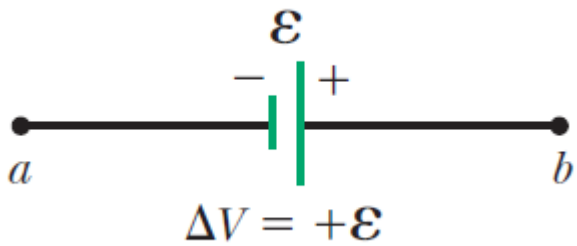
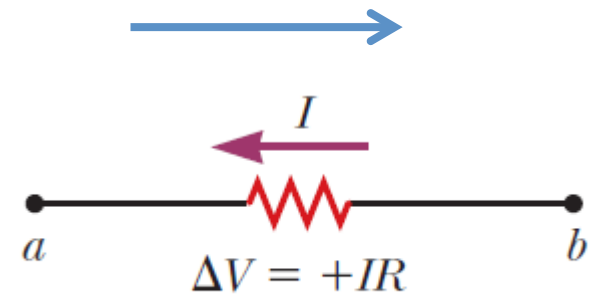
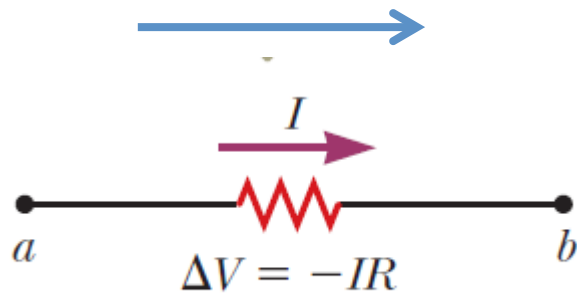
$$\sum_{\text{closed loop}} \Delta V = 0$$

It follows from the law of conservation of energy

When applying Kirchoff's second rule in practice, we imagine traveling around the loop and consider changes in electric potential, rather than the changes in potential energy described in the preceding paragraph.

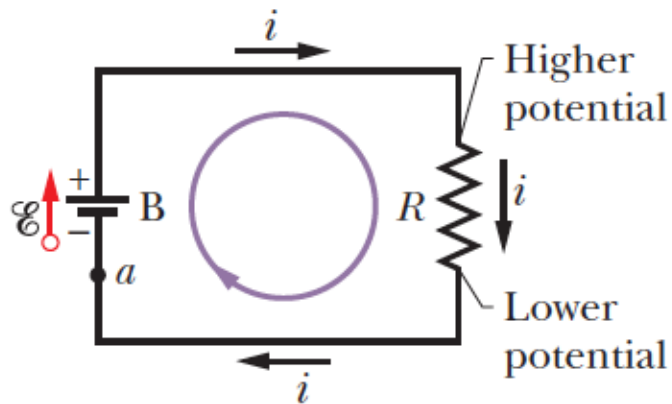
You should note the following sign conventions when using the second rule

7.3 Kirchoff's Rules



7.3 Kirchoff's Rules

The battery drives current through the resistor, from high potential to low potential.

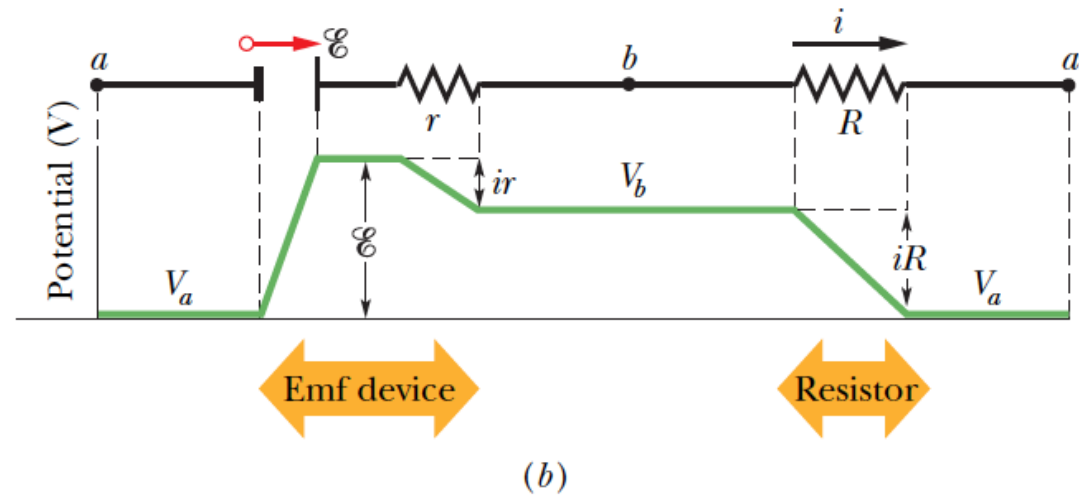
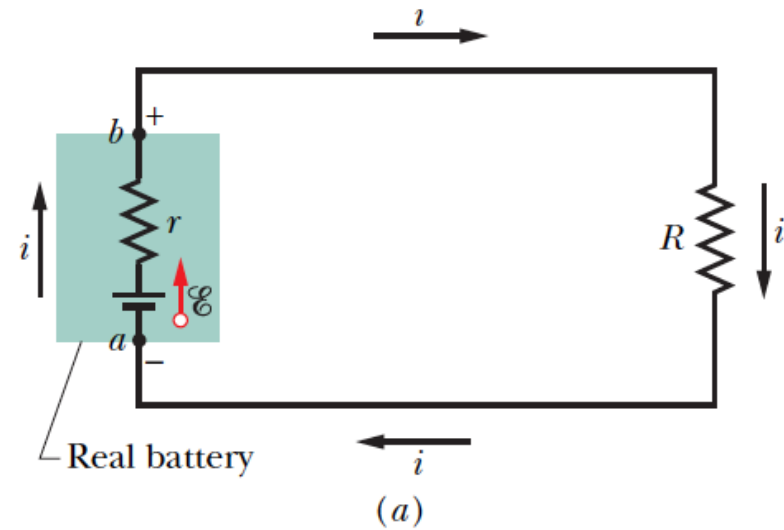


$$V_a + \varepsilon - iR = V_a$$

$$\varepsilon - iR = 0$$

$$\varepsilon = iR$$

7.3 Kirchoff's Rules



$$\mathcal{E} - ir - iR = 0 \Rightarrow i = \frac{\mathcal{E}}{r + R}$$

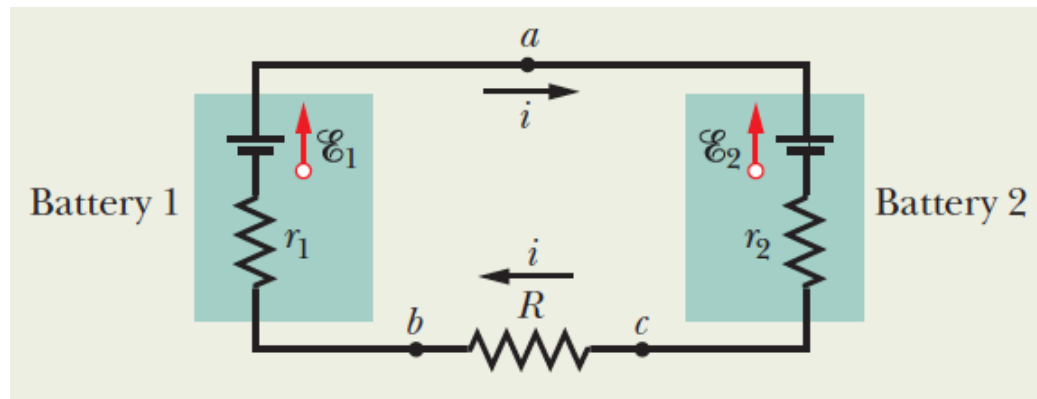
7.3 Kirchoff's Rules

Example: The emfs and resistances in the circuit of Fig. below have the following values:

$$\varepsilon_1 = 4.4V, \quad \varepsilon_2 = 2.1V,$$

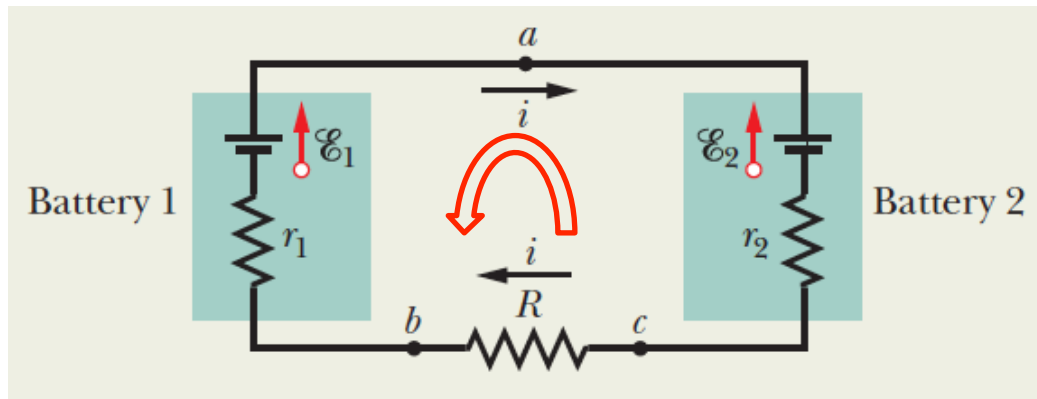
$$r_1 = 2.3\Omega, \quad r_2 = 1.8\Omega, \quad R = 5.5\Omega$$

- What is the current i in the circuit?
- What is the potential difference between the terminals of battery 1 in Fig. below ?



7.3 Kirchoff's Rules

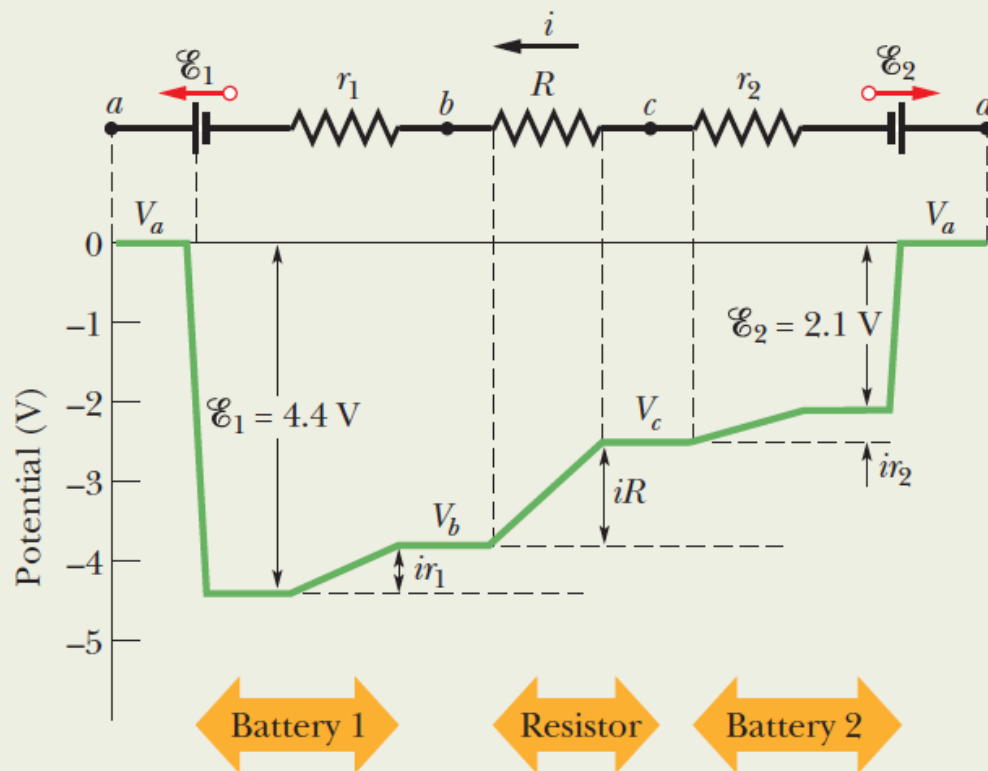
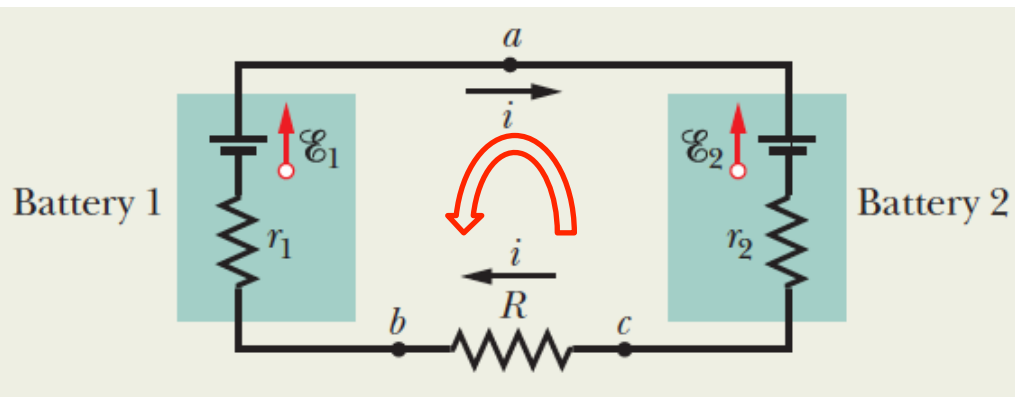
Solution a)



$$-\varepsilon_1 + ir_1 + iR + ir_2 + \varepsilon_2 = 0$$

$$i = \frac{\varepsilon_1 - \varepsilon_2}{R + r_1 + r_2} \approx 240 \text{ mA}$$

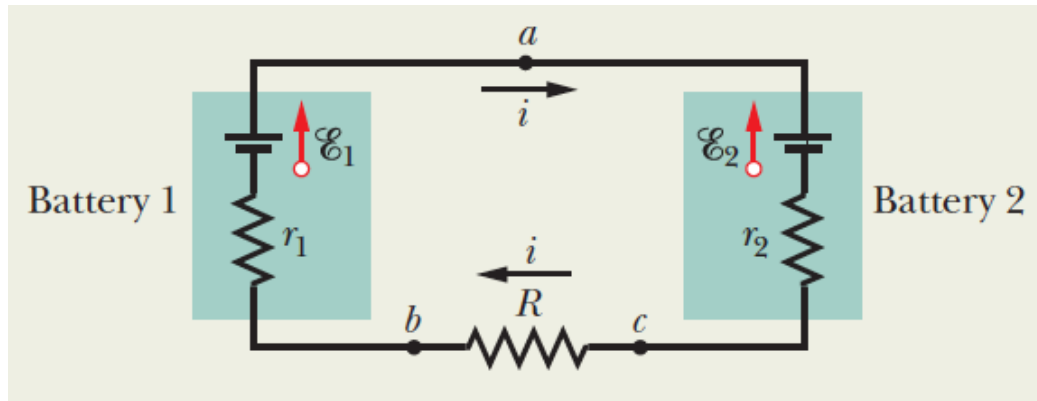
7.3 Kirchoff's Rules



$$-\mathcal{E}_1 + ir_1 + iR + ir_2 + \mathcal{E}_2 = 0$$

7.3 Kirchoff's Rules

Solution b)



$$V_b - ir_1 + \mathcal{E}_1 = V_a$$

$$V_a - V_b = -ir_1 + \mathcal{E}_1 \approx 3.8V$$

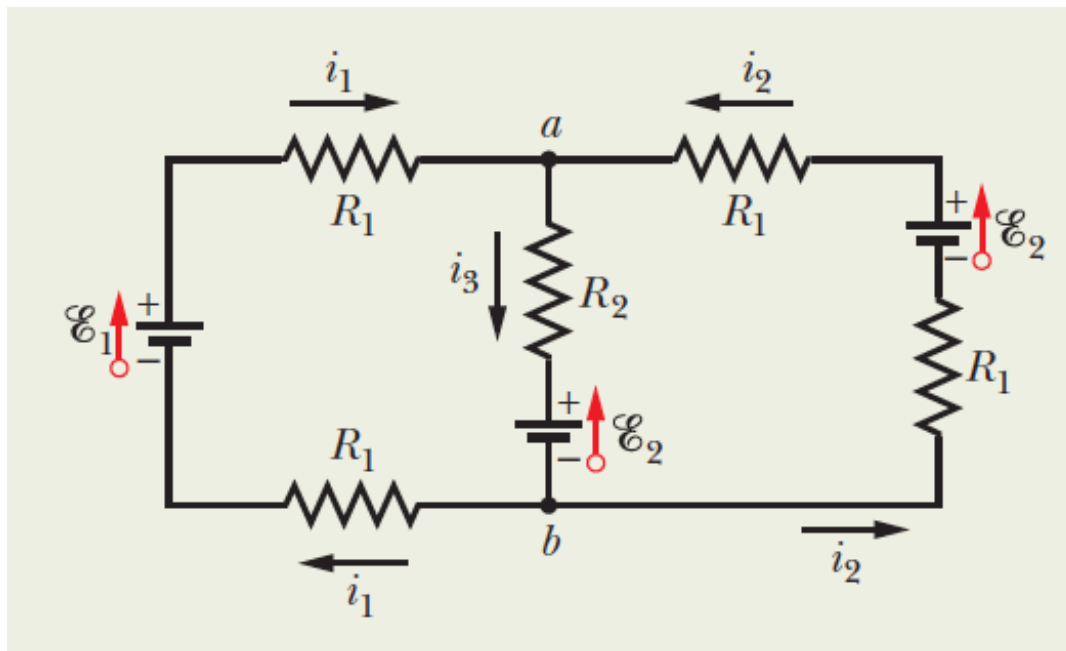
7.3 Kirchoff's Rules

Example: The Fig. below shows a circuit whose elements have the following values:

$$\varepsilon_1 = 3.0V, \quad \varepsilon_2 = 6.0V,$$

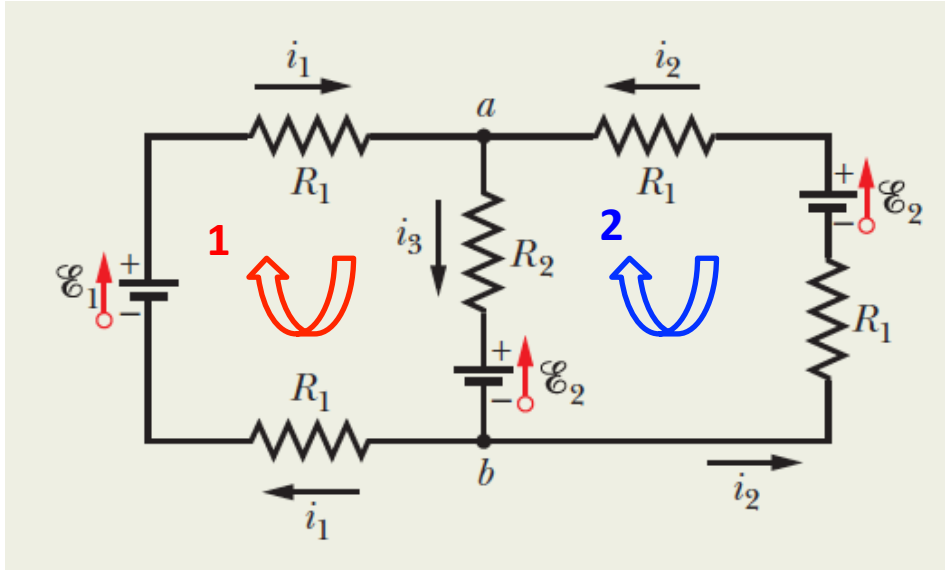
$$R_1 = 2.0\Omega, \quad R_2 = 4.0\Omega$$

The three batteries are ideal batteries. Find the magnitude and direction of the current in each of the three branches.



7.3 Kirchoff's Rules

Solution:



Junction rule for point a

$$i_1 + i_2 = i_3$$

For the loop on the left side:

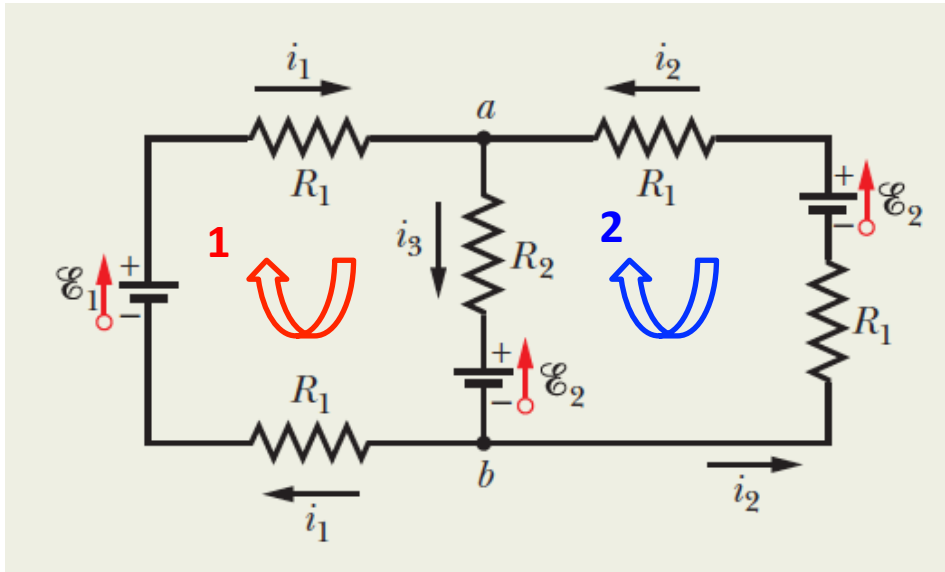
$$-i_1 R_1 + \varepsilon_1 - i_1 R_1 - i_3 R_2 - \varepsilon_2 = 0$$

For the loop on the right side:

$$i_2 R_1 - \varepsilon_2 + i_2 R_1 + \varepsilon_2 + i_3 R_2 = 0$$

7.3 Kirchoff's Rules

Solution:



$$8i_1 + 4i_2 = -3$$

$$4i_1 + 8i_2 = 0$$

$$i_1 = -0.5A$$

$$i_2 = 0.25A$$

$$i_3 = -0.25A$$

The minus sign signals that our arbitrary choice of direction for that current was wrong. Therefore the correct answers are:

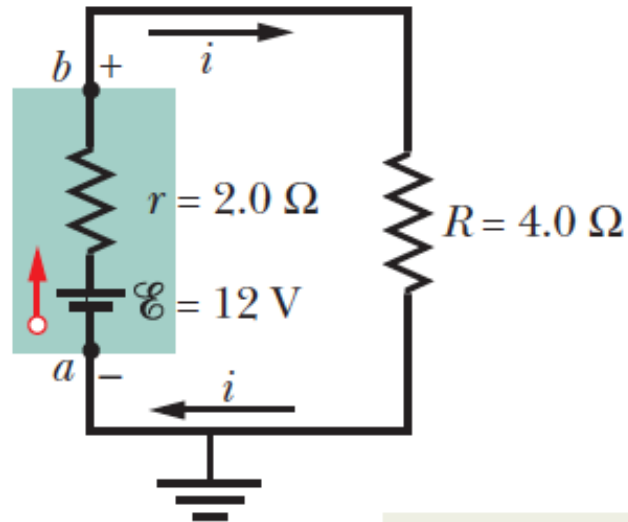
$$i_1 = 0.5A$$

$$i_2 = 0.25A$$

$$i_3 = 0.25A$$

7.3 Kirchoff's Rules

Grounding a circuit



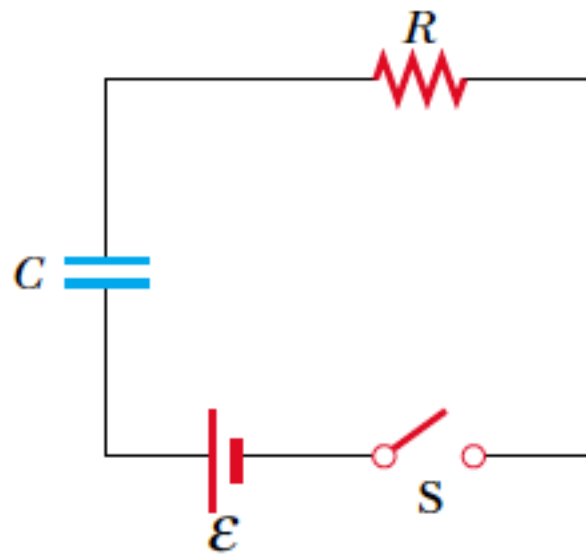
(a)

Ground is taken to be zero potential.

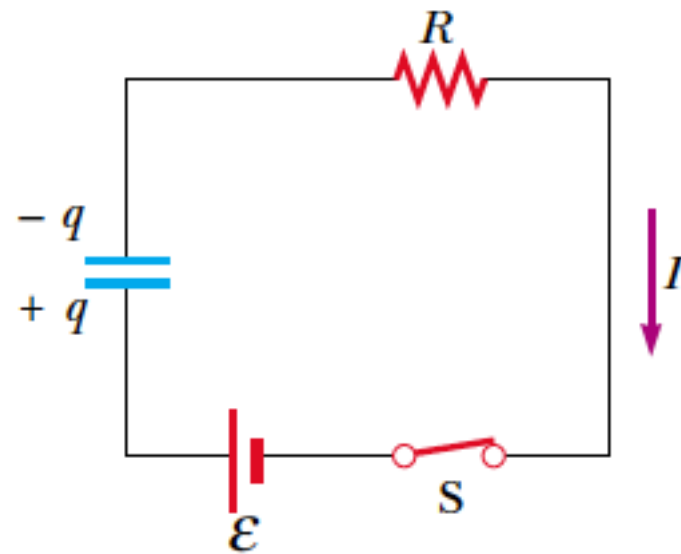
Grounding a circuit usually means connecting the circuit to a conducting path to Earth's surface. Here, such a connection means only that the potential is defined to be **zero** at the grounding point in the circuit.

7.4 RC Circuits

Charging a Capacitor



(b) $t < 0$

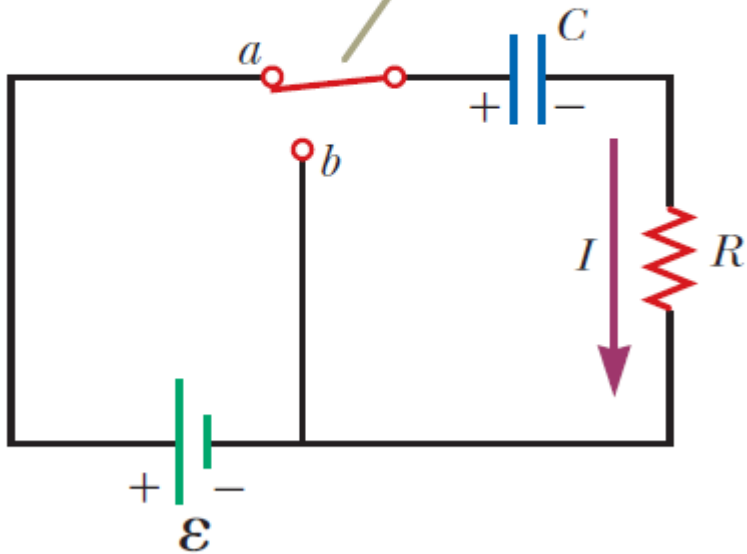


(c) $t > 0$

7.4 RC Circuits

Charging a Capacitor

When the switch is thrown to position *a*, the capacitor begins to charge up.



By applying Kirchhoff's loop rule to the circuit , we get

$$\varepsilon - \frac{q}{C} - iR = 0$$

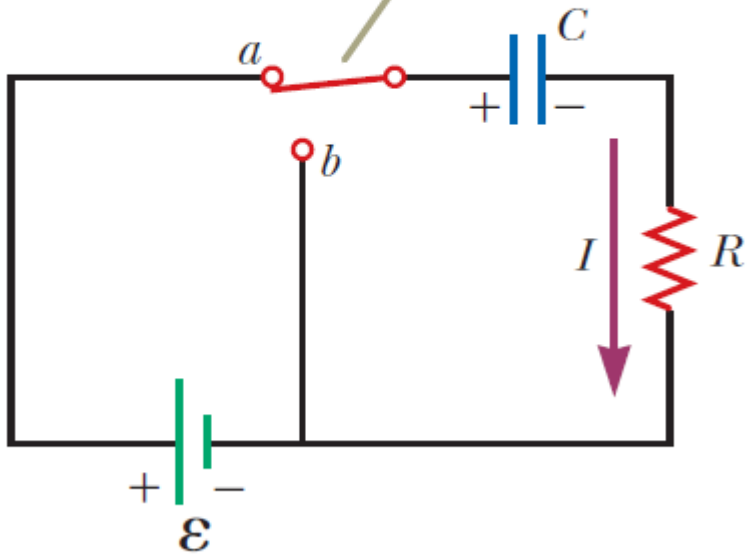
By using definition of current as $i = dq/dt$

$$\varepsilon - \frac{q}{C} - \frac{dq}{dt} R = 0$$
$$R \frac{dq}{dt} + \frac{q}{C} = \varepsilon$$

7.4 RC Circuits

Charging a Capacitor

When the switch is thrown to position *a*, the capacitor begins to charge up.



The solution to this equation is:

$$q(t) = C\varepsilon(1 - e^{-t/RC})$$

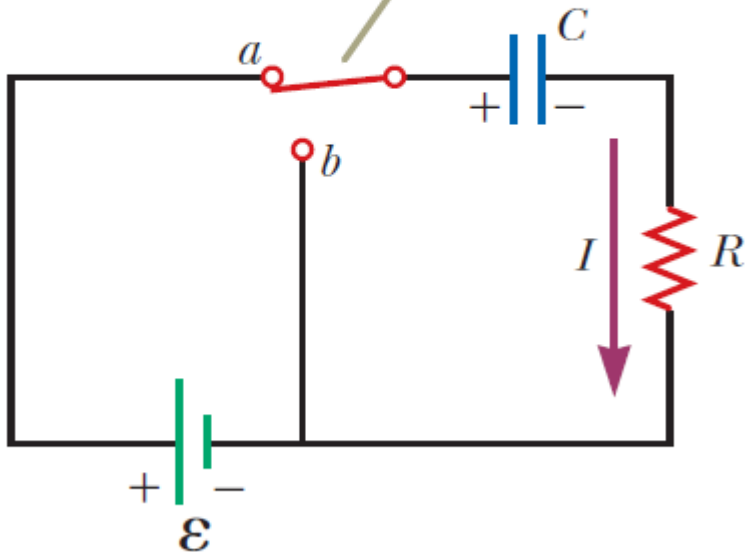
The derivative of $q(t)$ is the current $i(t)$ charging the capacitor:

$$i = \frac{dq}{dt} = \frac{\varepsilon}{R} e^{-t/RC}$$

7.4 RC Circuits

Charging a Capacitor

When the switch is thrown to position *a*, the capacitor begins to charge up.



The potential difference $V_C(t)$ across the capacitor during the charging process is

$$V_C = \frac{q}{C} = \epsilon(1 - e^{-t/RC})$$

The product RC is called the capacitive time constant of the circuit and is represented with the symbol τ

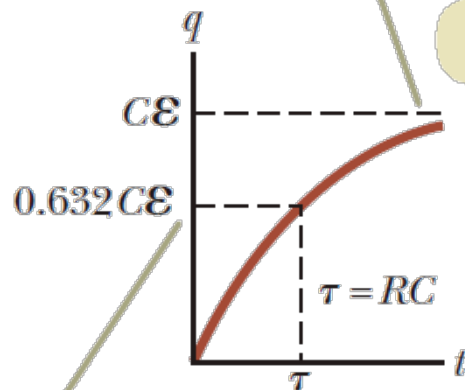
$$\tau = RC$$

At $t = \tau$ the charge has reached to 63% of its final value $C\epsilon$

7.4 RC Circuits

Charging a Capacitor

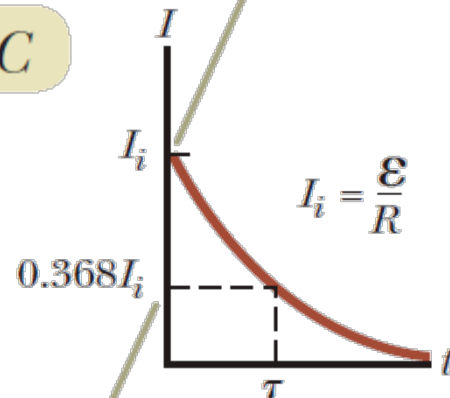
The charge approaches its maximum value $C\mathcal{E}$ as t approaches infinity.



After a time interval equal to one time constant τ has passed, the charge is 63.2% of the maximum value $C\mathcal{E}$.

a

The current has its maximum value $I_i = \mathcal{E}/R$ at $t = 0$ and decays to zero exponentially as t approaches infinity.

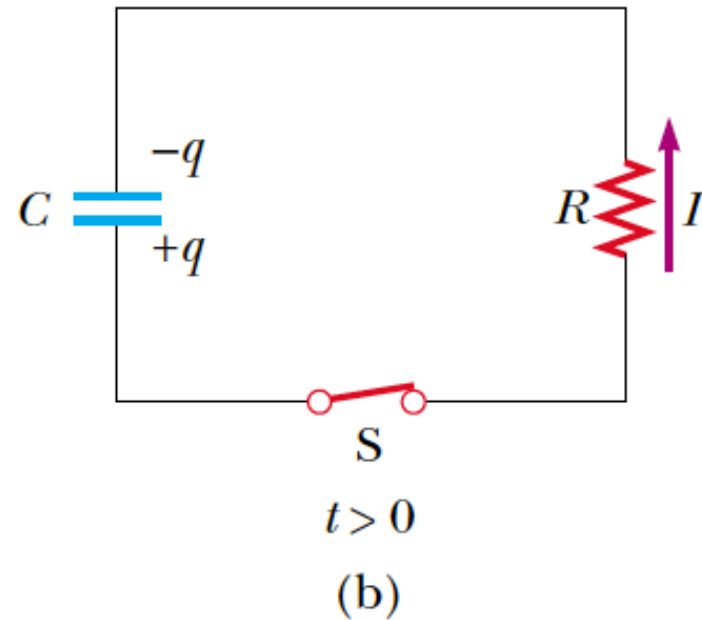
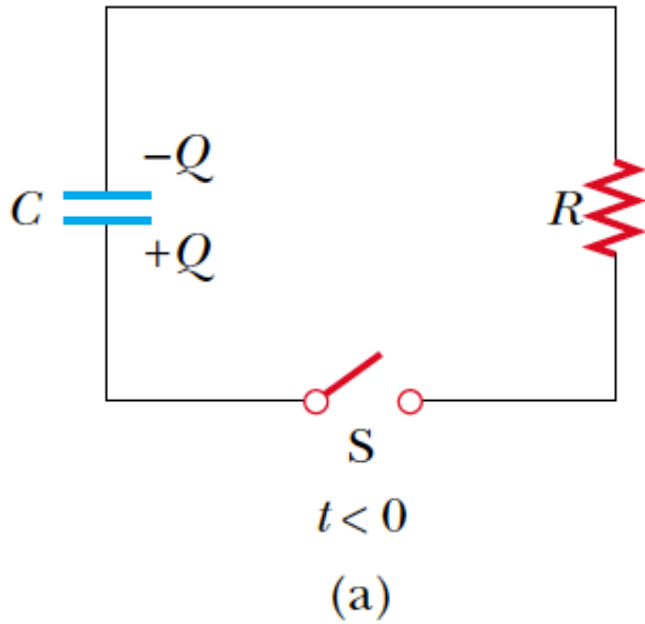


After a time interval equal to one time constant τ has passed, the current is 36.8% of its initial value.

b

7.4 RC Circuits

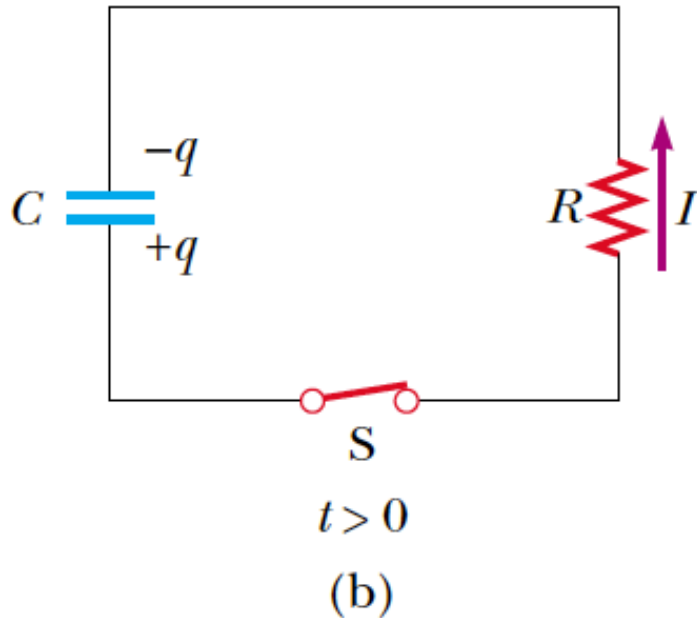
Discharging a Capacitor



7.4 RC Circuits

Discharging a Capacitor

By applying Kirchhoff's loop rule to the circuit, we get



$$-\frac{q}{C} + iR = 0$$

$$-\frac{q}{C} + \frac{dq}{dt} R = 0$$

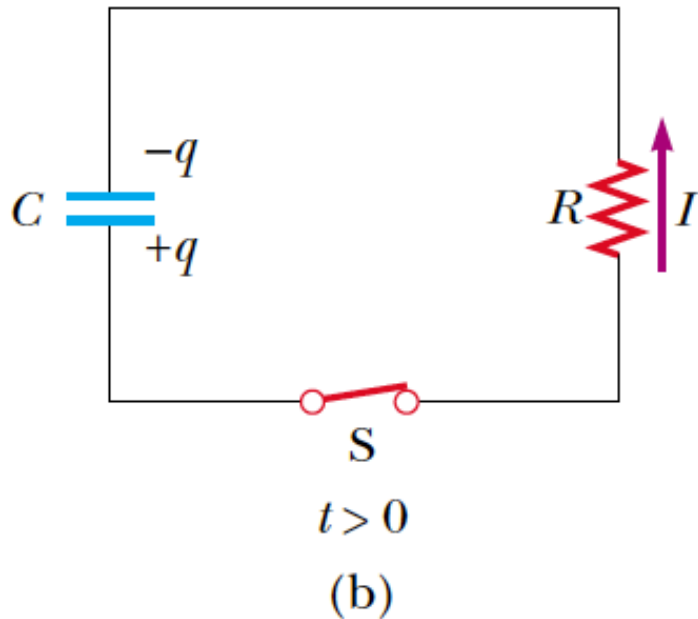
The solution to this equation is:

$$q(t) = C\epsilon e^{-t/RC} = q_0 e^{-t/RC}$$

q decreases exponentially with time, at a rate that is set by the capacitive time constant RC

7.4 RC Circuits

Discharging a Capacitor



The current $i(t)$:

$$i(t) = -\frac{q_0}{RC} e^{-t/RC}$$

The current also decreases exponentially with time, at a rate set by RC

The negative sign indicates that as the capacitor discharges, the current direction is opposite its direction when the capacitor was being charged.