## ELE 321 Linear System Analysis

Ankara University

Faculty of Engineering

**Electrical and Electronics Engineering Department** 

# The Continuous-Time Fourier Transform

ELE321 Linear System Analysis

Lecture 11

#### Agenda

- Continuous-Time Fourier Transform for Aperiodic Signals
- Convergence of Fourier Transform
- Continuous-Time Fourier Transform for Periodic Signals

### CT Fourier Transform for Aperiodic Signals

- x(t) is an aperiodic signal
- $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$
- $X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

## Convergence of CT Fourier Transform

- Dirichlet conditions must be satisfied.
  - Aperiodic signal must be absolutely integrable.
  - Number of the maxima and minimas of the aperiodic signal must be finite.
  - Number of the discontinuities of the aperiodic signal must be finite.

#### CT Fourier Transform for Periodic Signals

- Fourier series coefficients,  $a_k$
- $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$  : periodic signal
- $X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega k\omega_0)$ 
  - Train of impulses

## Linearity

- $x(t) \leftrightarrow X(j\omega)$  and  $y(t) \leftrightarrow Y(j\omega)$
- $ax(t) + by(t) \iff aX(j\omega) + bY(j\omega)$

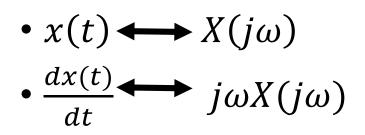
## Time Shifting

• 
$$x(t) \longleftrightarrow X(j\omega)$$
  
•  $x(t - t_0) \bigstar e^{-j\omega t_0} X(j\omega)$ 

## Conjugation and Conjugate Symmetry

- $x(t) \longleftrightarrow X(j\omega)$
- $x^*(t) \longleftrightarrow X^*(-j\omega)$

## Differentiation



#### Parseval's Relation

• 
$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$

### Convolution Property

• 
$$y(t) = h(t) * x(t) \leftrightarrow Y(j\omega) = H(j\omega)X(j\omega)$$

- Time domain: convolution
- Frequency domain: multiplication

### Multiplication (Modulation) Property

• 
$$r(t) = s(t)p(t) \longrightarrow R(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(j\theta) P(j(\omega - \theta)) d\theta$$

- Time domain: multiplication (amplitude modulation)
- Frequency domain: convolution

## Systems Characterized by Linear Constant Coefficient Differential Equations

• 
$$Y(j\omega) = H(j\omega)X(j\omega)$$
  
•  $H(j\omega) = \frac{\sum_{k=0}^{M} b_k(j\omega)^k}{\sum_{k=0}^{N} a_k(j\omega)^k}$ 



• Signals and Systems, 2nd Edition, Oppenheim, Willsky, Nawab