ELE 321
Linear System Analysis

Ankara University

Faculty of Engineering

Electrical and Electronics Engineering Department

The Continuous-Time Fourier Transform

ELE321 Linear System Analysis

Lecture 11

Agenda

- Continuous-Time Fourier Transform for Aperiodic Signals
- Convergence of Fourier Transform
- Continuous-Time Fourier Transform for Periodic Signals

CT Fourier Transform for Aperiodic Signals

- $x(t)$ is an aperiodic signal
- $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$
- $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$

Convergence of CT Fourier Transform

- Dirichlet conditions must be satisfied.
  - Aperiodic signal must be absolutely integrable.
  - Number of the maxima and minimas of the aperiodic signal must be finite.
  - Number of the discontinuities of the aperiodic signal must be finite.

CT Fourier Transform for Periodic Signals

- Fourier series coefficients, a_k
- $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$: periodic signal
- $X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$
  - Train of impulses

Linearity

- $x(t) \longleftrightarrow X(j\omega)$ and $y(t) \longleftrightarrow Y(j\omega)$
- $ax(t) + by(t) \longleftrightarrow aX(j\omega) + bY(j\omega)$

Time Shifting

- $x(t) \longleftrightarrow X(j\omega)$
- $x(t - t_0) \longleftrightarrow e^{-j\omega t_0} X(j\omega)$

Conjugation and Conjugate Symmetry

- $x(t) \longleftrightarrow X(j\omega)$
- $x^*(t) \longleftrightarrow X^*(-j\omega)$

Differentiation

- $x(t) \longleftrightarrow X(j\omega)$
- $\frac{dx(t)}{dt} \longleftrightarrow j\omega X(j\omega)$

Parseval's Relation

- $\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$

Convolution Property

- $y(t) = h(t) * x(t) \longleftrightarrow Y(j\omega) = H(j\omega)X(j\omega)$
- Time domain: convolution
- Frequency domain: multiplication

Multiplication (Modulation) Property

- $r(t) = s(t)p(t) \longleftrightarrow R(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(j\theta)P\big(j(\omega-\theta)\big)d\theta$
- Time domain: multiplication (amplitude modulation)
- Frequency domain: convolution

Systems Characterized by Linear Constant Coefficient Differential Equations

- $Y(j\omega) = H(j\omega)X(j\omega)$
- $H(j\omega) = \frac{\sum_{k=0}^{M} b_k (j\omega)^k}{\sum_{k=0}^{N} a_k (j\omega)^k}$

References

- Signals and Systems, 2nd Edition, Oppenheim, Willsky, Nawab