ELE 321
Linear System Analysis

Ankara University

Faculty of Engineering

Electrical and Electronics Engineering Department

Properties of Discrete-Time Fourier Transform

ELE321 Linear System Analysis

Lecture 13

Agenda

- Properties of Discrete Time Fourier Transform
- Systems Characterized By Linear Constant-Coefficient Difference Equations

Periodicity

- $X\left(e^{j\omega}\right) = X\left(e^{j(\omega+2\pi)}\right)$
- Periodic in ω with period 2π

Linearity

- $x[n] \longleftrightarrow X(e^{j\omega})$ and $y[n] \longleftrightarrow Y(e^{j\omega})$
- $ax[n] + by[n] \longleftrightarrow aX(e^{j\omega}) + bY(e^{j\omega})$

Time Shifting and Frequency Shifting

- $x[n] \longleftrightarrow X\left(e^{j\omega}\right)$
- $x[n-n_0] \longleftrightarrow e^{-j\omega n_0} X(e^{j\omega})$

- $e^{j\omega_0 n} x[n] \longleftrightarrow X(e^{j(\omega-\omega_0)})$

Conjugation and Conjugate Symmetry

- $x[n] \longleftrightarrow X(e^{j\omega})$
- $x^*[n] \longleftrightarrow X^*(e^{-j\omega})$

Differencing

- $x[n] \longleftrightarrow X(e^{j\omega})$
- $x[n] - x[n-1] \longleftrightarrow (1 - e^{-j\omega})X(e^{j\omega})$

Parseval's Relation

- $\sum_{n=-\infty}^{\infty}|x[n]|^2 = \frac{1}{2\pi}\int_0^{2\pi}\left|X(e^{j\omega})\right|^2$

Convolution Property

- $y[n] = h[n] * x[n] \longleftrightarrow Y\left(e^{j\omega}\right) = H(e^{j\omega})X(e^{j\omega})$
- Time domain: convolution
- Frequency domain: multiplication

Multiplication Property

- $r[n] = s[n]p[n] \longleftrightarrow R(e^{j\omega}) = \frac{1}{2\pi}\int_0^{2\pi} S(e^{j\theta})P(e^{j(\omega-\theta)})d\theta$
- Time domain: multiplication
- Frequency domain: periodic convolution

Systems Characterized by Linear Constant Coefficient Difference Equations

- $Y\left(e^{j\omega}\right) = H\left(e^{j\omega}\right)X(e^{j\omega})$
- $H\left(e^{j\omega}\right) = \frac{\sum_{k=0}^{M} b_k e^{-jk\omega}}{\sum_{k=0}^{N} a_k e^{-jk\omega}}$

References

- Signals and Systems, 2nd Edition, Oppenheim, Willsky, Nawab