ELE 321
Linear System Analysis

Ankara University

Faculty of Engineering

Electrical and Electronics Engineering Department

Sampling

ELE321 Linear System Analysis

Lecture 14

Agenda

- Sampling Theorem
- Impulse Train Sampling
- Reconstruction of a Sampled Signal
- Aliasing

Sampling Theorem

- Band-limited signal, $x(t) \longleftrightarrow X(j\omega)$
- $X(j\omega) = 0$ for $|\omega| > \omega_M$
- Sampled signal, $x(nT), n = 0, \pm 1, \pm 2, \ldots$
  - Condition: $\omega_s > 2\omega_M$
- Sampling frequency: $\omega_s = \frac{2\pi}{T}$

Impulse Train Sampling

- $x(t)$: continuous time signal
- $p(t)$: periodic impulse train
- $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$
- To sample the continuous time signal
- $x_p(t) = x(t)p(t)$: multiplication in the time domain
- $X_p(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$: shifting in the frequency domain

Reconstruction of a Sampled Signal

- Interpolation
- $x_r(t)$: reconstructed signal
- $h(t)$: ideal low pass filter
- $h(t) = \frac{\omega_c T \sin(\omega_c t)}{\pi \omega_c t}$
- $x_r(t) = x_p(t) * h(t)$
- $x_r(t) = \sum_{n=-\infty}^{\infty} x(nT) \frac{\omega_c T}{\pi} \frac{\sin(\omega_c (t-nT))}{\omega_c (t-nT)}$

Aliasing

- Overlapping
- $\omega_s < 2\omega_M$ ---> aliasing occurs
- $\omega_s > 2\omega_M$ ---> perfect reconstruction, no aliasing

References

- Signals and Systems, 2nd Edition, Oppenheim, Willsky, Nawab