EEE 321
Signals and Systems

Ankara University

Faculty of Engineering

Electrical and Electronics Engineering Department

Unit Impulse and Unit Step Functions

EEE321 Signals and Systems

Lecture 5

Agenda

- General Discrete-Time Complex Exponential Signals
- Unit impulse function
- Unit step function
- Sampling property of unit impulse function

General Discrete-Time Complex Exponential Signals

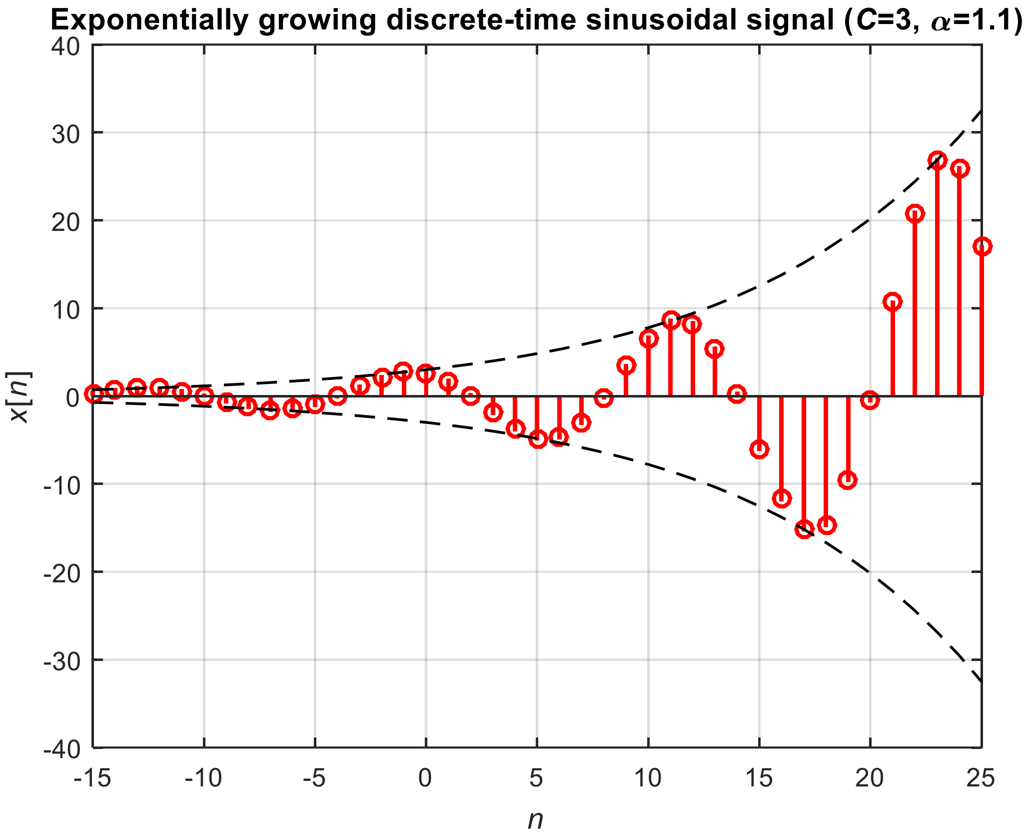
Let $C = |C|e^{j\theta}$ and α $= |\alpha|e^{j\omega_o}$ then,

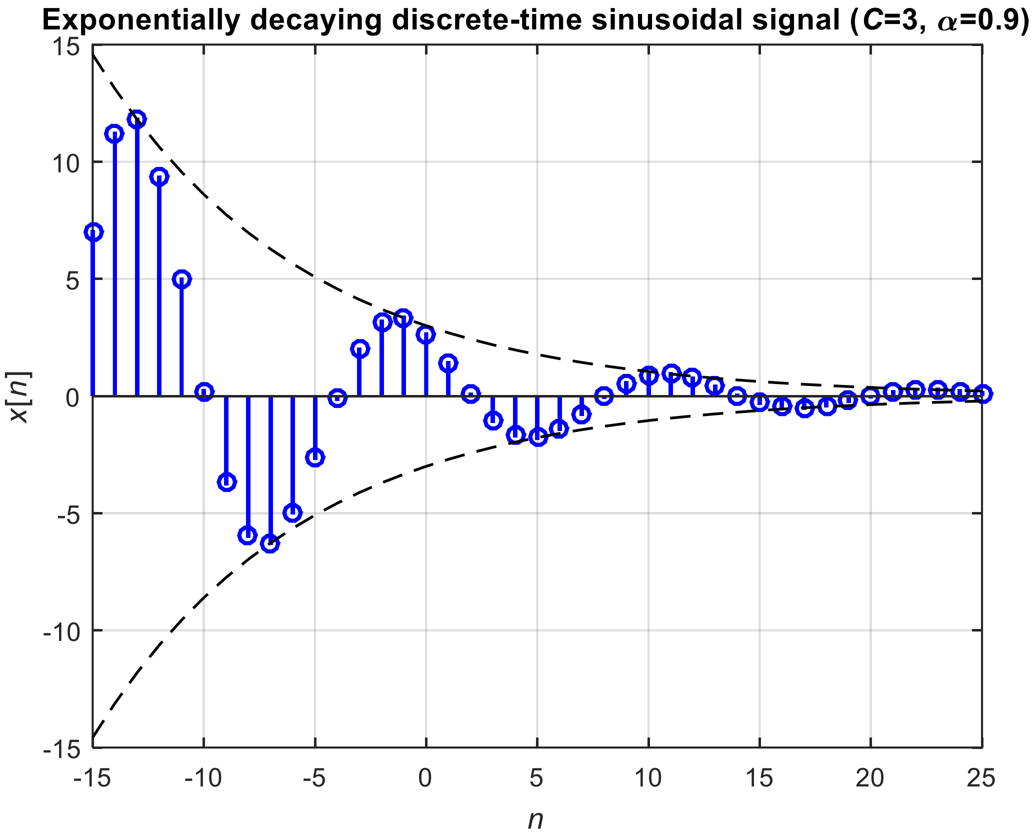
$$C\alpha^n = |C||\alpha|^n \cos(\omega_o n + \theta) + j|C||\alpha|^n \sin(\omega_o n + \theta)$$

For $|\alpha| = 1$, the real and imaginary parts of a complex sequence are sinusoidal.

For $|\alpha| < 1$, they correspond to sinusoidal sequences multiplied by a decaying exponential.

For $|\alpha| > 1$, they correspond to sinusoidal sequences multiplied by a growing exponential.


Exponentially growing discrete-time sinusoidal signal (C=3, α=1.1)
x[n]
n
40
30
20
10
0
-10
-20
-30
-40
-15
-10
-5
0
5
10
15
20
25


Exponentially decaying discrete-time sinusoidal signal (C=3, α=0.9)
x[n]
n
15
10
5
0
-5
-10
-15
-15
-10
-5
0
5
10
15
20
25

Periodicity Properties of Dicrete-Time Complex Exponentials

Consider the discrete-time complex exponential with frequency $\omega_o + 2\pi$:

$$e^{j(\omega_o+2\pi)n} = e^{j2\pi n}e^{j\omega_o n} = e^{j\omega_o n}$$

So the exponential at frequency $\omega_o + 2\pi$ is the same as that at frequency ω_o. This is very different situation from the continuous-time case, in which signals $e^{j\omega_o t}$ are all disctinct for distinct values of ω_o. However, in discrete-time case, these signals are not disctint. Signal with frequency ω_o is idectical to the signals with frequencies $\omega_o \pm 2\pi$, $\omega_o \pm 4\pi$, $\omega_o \pm 6\pi$, …

Fundamental frequency of the periodic signal $e^{j\omega_o n}$ is

$$\frac{2\pi}{N} = \frac{\omega_o}{m}$$

Fundamental frequency is

$$N = m\frac{2\pi}{\omega_o}$$

Discrete-Time Unit Impulse (Unit Sample)

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

Discrete-Time Unit Step

$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$

Discrete-Time Unit Step – Unit Impulse Relationship

$$\delta[n] = u[n] - u[n-1]$$

$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$

Also

$$u[n] = \sum_{k=\infty}^{0} \delta[n-k]$$

or

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

Sampling Property of Discrete-Time Unit Impulse

$$u[n]\delta[n] = x[0]\delta[n]$$

Generally, at $n = n_o$

$$x[n]\delta[n - n_o] = x[n_o]\delta[n - n_o]$$

Continuous-Time Unit Impulse

$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ 1, & t = 0 \end{cases}$$

Continuous-Time Unit Step

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

Continuous-Time Unit Step – Unit Impulse Relationship

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$$

and

$$\delta(t) = \frac{du(t)}{dt}$$

Sampling Property of Continuous-Time Unit Impulse

$$x(t)\delta(t) = x(0)\delta(t)$$

In general, at $t = t_o$

$$x(t)\delta(t - t_o) = x(t_o)\delta(t - t_o)$$

References

- Signals and Systems, 2nd Edition, Oppenheim, Willsky, Nawab