Physics 122: Electricity & Magnetism – Lecture 5 Gauss’ Law

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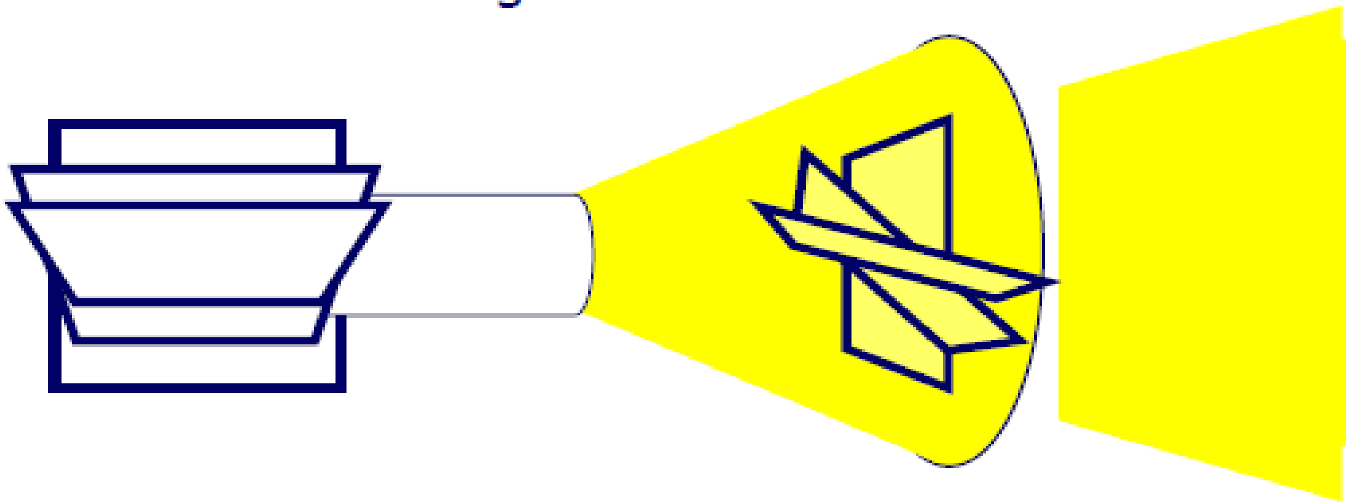
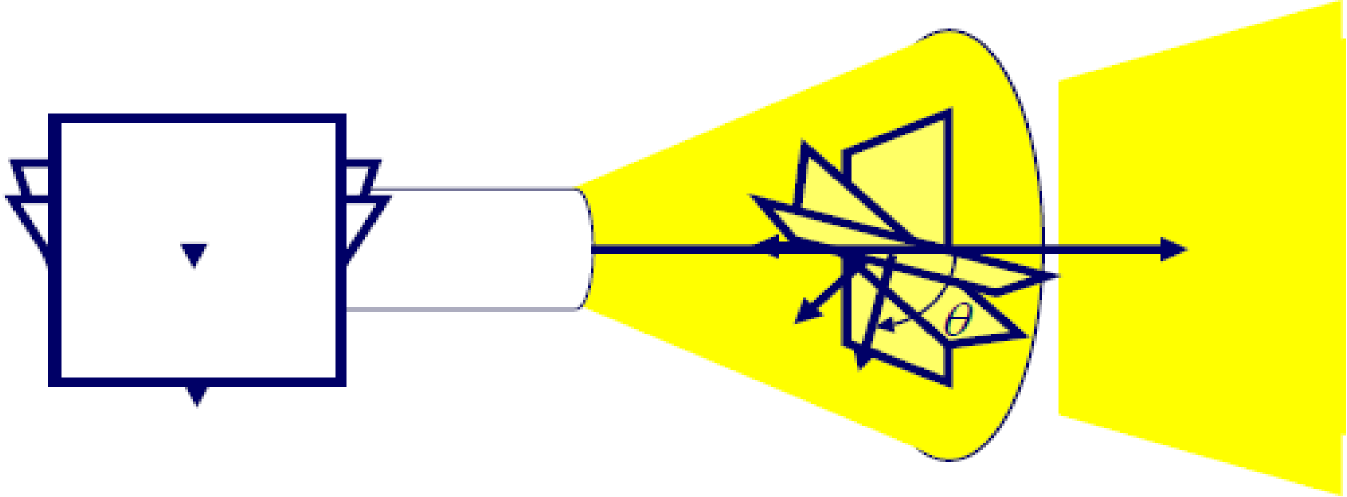
Flux

- Flux in Physics is used to two distinct ways.
- The first meaning is the rate of flow, such as the amount of water flowing in a river, i.e. volume per unit area per unit time. Or, for light, it is the amount of energy per unit area per unit time.
- Let's look at the case for light:



Area Vector

- Represent an area as a vector $\Delta\vec{A}$, of length equal to the area, and direction of the “outward normal” to the surface.
- The flux of light through a hole of area ΔA is proportional to the area, and the cosine of the angle between the light direction and this area vector.



- If we use a vector $\vec{L}$ to represent the light energy per unit time, then the light out of the hole is $L\Delta A\cos\theta = \vec{L}\cdot\Delta\vec{A}$. In this case it is *negative* $(\theta > 90°)$ which means the light flux is into the hole.

Flux of Electric Field

- Like the flow of water, or light energy, we can think of the electric field as flowing through a surface (although in this case nothing is actually moving).
- We represent the flux of electric field as Φ (greek letter phi), so the flux of the electric field through an element of area ΔA is

$$\Delta\Phi = \vec{E}\cdot\Delta\vec{A} = E\,\Delta A\cos\theta$$

- When $\theta < 90^\circ$, the flux is positive (out of the surface), and when $\theta > 90^\circ$, the flux is negative.
- When we have a complicated surface, we can divide it up into tiny elemental areas:

$$d\Phi = \vec{E}\cdot d\vec{A} = E\,dA\cos\theta$$

Gauss' Law

- We are going to be most interested in *closed* surfaces, in which case the outward direction becomes self-evident.
- We can ask, what is the electric flux out of such a closed surface? Just integrate over the closed surface: $\Phi = \oint d\Phi = \oint \vec{E} \cdot d\vec{A}$

  Flux positive => out
  Flux negative => in

- The $\oint$ symbol has a little circle to indicate that the integral is over a closed surface.
- The closed surface is called a gaussian surface, because such surfaces are used by Gauss' Law, which states that:

Gauss' Law

The flux of electric field through a closed surface is proportional to the charge enclosed.

Mathematical Statement of Gauss' Law

- The constant of proportionality in Gauss' Law is our old friend ε_0.

$$\varepsilon_0 \Phi = q_{enc}$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$

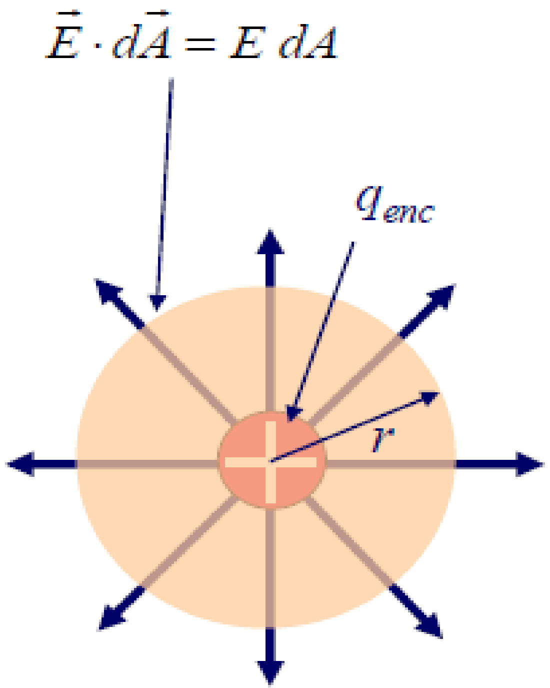
- Recall that I said that we would see later why Coulomb's constant is written $k_E = \dfrac{1}{4\pi\varepsilon_0}$?

- We can see it now by integrating the electric flux of a point charge over a spherical gaussian surface.

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 E \oint dA = \varepsilon_0 E 4\pi r^2 = q_{enc}$$

- Solving for E gives Coulomb's Law.

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q_{enc}}{r^2}$$



Field Inside a Conductor

- We can use Gauss' Law to show that the inside of a conductor must have no net charge.
- Take an arbitrarily shaped conductor, and draw a gaussian surface just inside.
- Physically, we expect that there is no electric field inside, since otherwise the charges would move to nullify it.
- Since E = 0 everywhere inside, E must be zero also on the gaussian surface, hence there can be no net charge inside.
- Hence, all of the charge must be on the surface (as discussed in the previous slide).
- If we make a hole in the conductor, and surround the hole with a gaussian surface, by the same argument there is no E field through this new surface, hence there is no net charge in the hole.

- We have the remarkable fact that if you try to deposit charge on the inside of the conductor…
- The charges all move to the outside and distribute themselves so that the electric field is everywhere normal to the surface.
- This is NOT obvious, but Gauss' Law allows us to show this!

There are two ideas here

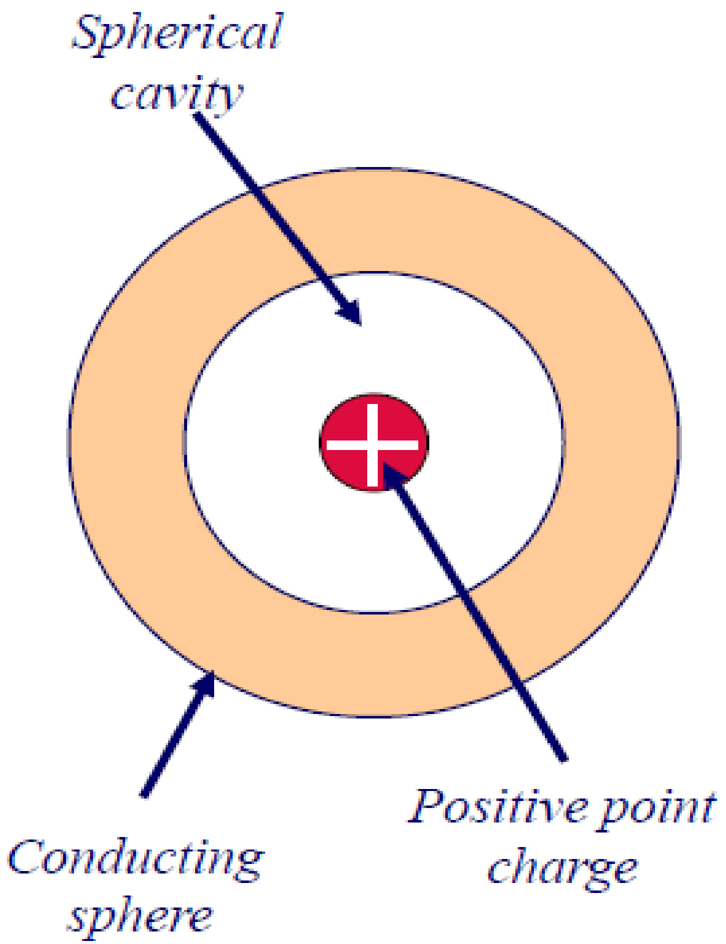
- Electric field is zero inside conductors
- Because that is true, from Gauss' Law, cavities in conductors have E = 0

Charge Distribution on Conductors

- For a conducting sphere, the charges spread themselves evenly around the surface.
- For other shapes, however, the charges tend to collect near sharp curvature.
- To see why, consider a line of charge.

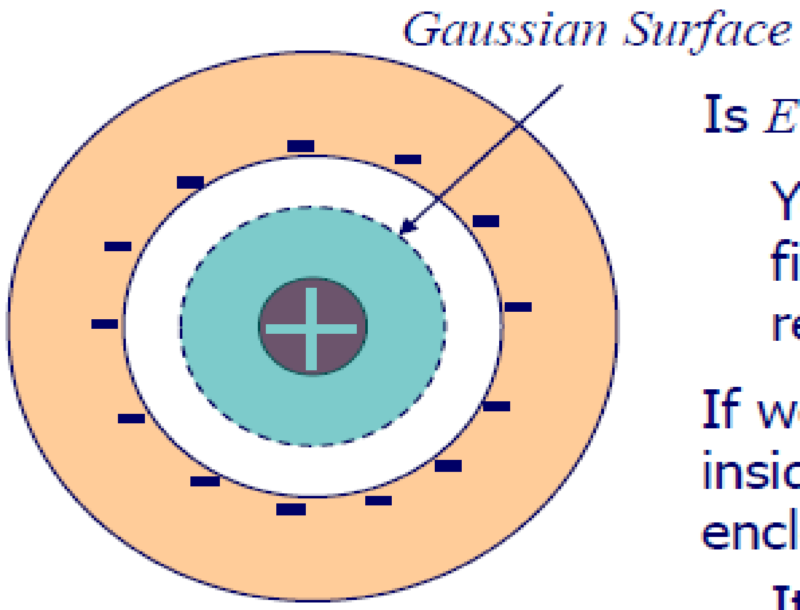
A Charge Inside a Conductor

What will happen when we add a charge inside a conductor?



Use Gauss' Law to Find Out

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$



Is $E = 0$ in the cavity?

No, because there is charge enclosed (Gauss' Law).

Is $E = 0$ in the conductor?

Yes, because as before, if there were an electric field in the conductor, the charges would move in response (NOT Gauss' Law).

If we enlarge the gaussian surface so that it is inside the conductor, is there any net charge enclosed?

It looks like there is, but there cannot be, because Gauss' Law says $E = 0$ implies $q_{enc} = 0$!

How do we explain this?

There must be an equal and opposite charge induced on the inner surface.