Physics 122: Electricity & Magnetism – Lecture 6 Gauss’ Law (continued)

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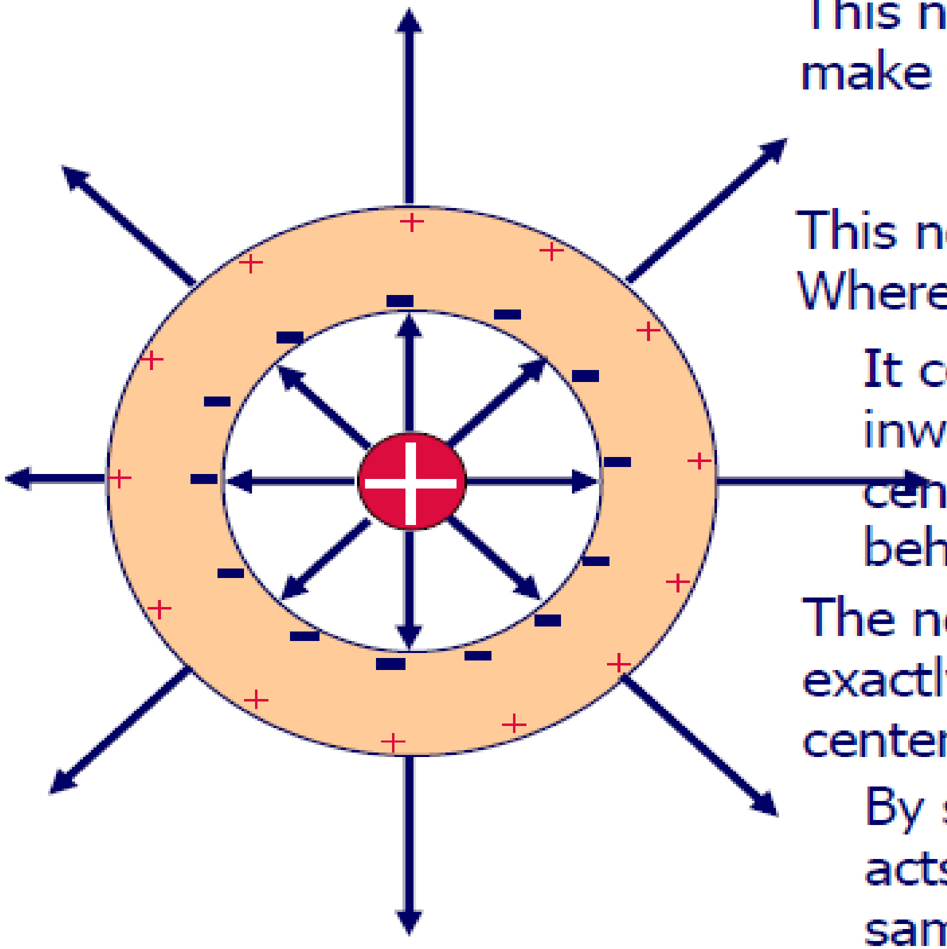
E Field of Charge In Conductor



This negative charge acts with the inner charge to make the field radial inside the cavity.

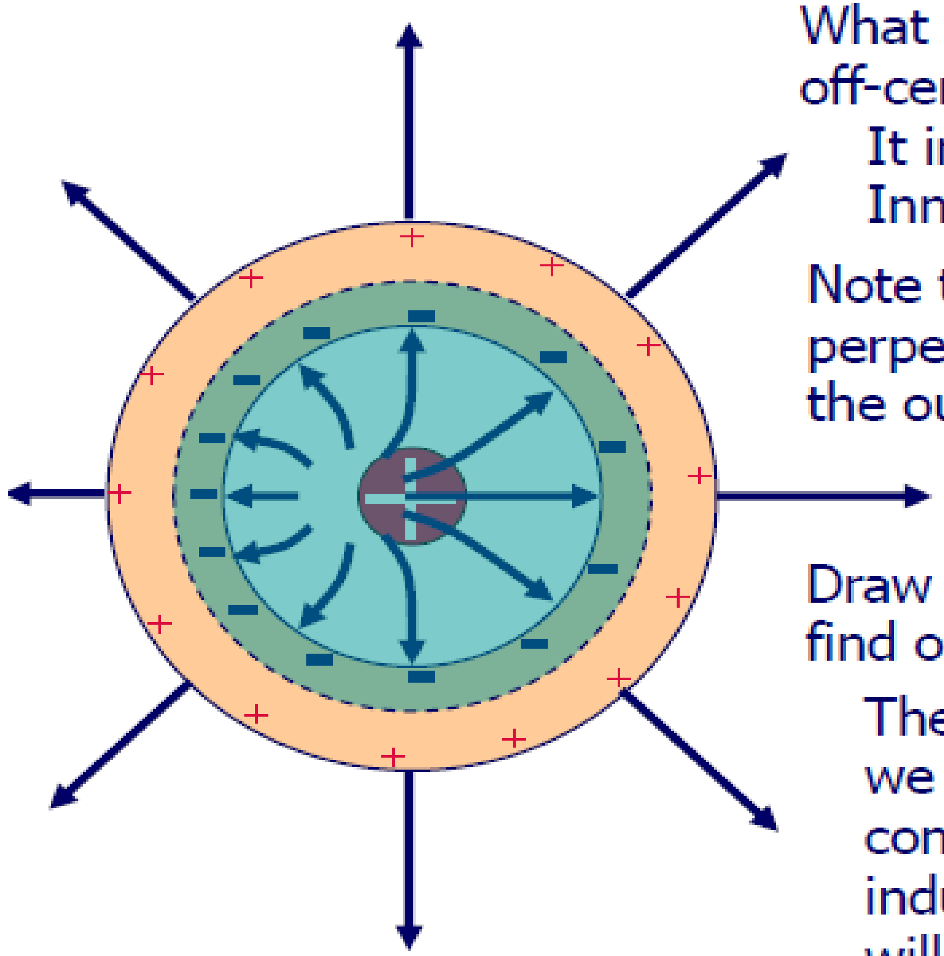
This negative charge cannot appear out of nowhere. Where does it come from?

It comes from the outer surface (electrons drawn inward, attracted to the positive charge in the center). Therefore, it leaves positive charge behind.

The net positive charge that appears conductor is exactly the same as the original charge in the center, so what do the field lines look like?

By spherical symmetry, the positive shell of charge acts like a point charge in the center, so field is the same as the field of the original point charge.

E Field of Charge In Conductor



What happens when we move the inner charge off-center?

It induces an off-center charge distribution on the Inner wall.

Note that the field lines distorted, so they remain perpendicular to the inner wall. What happens to the outer positive charge distribution?

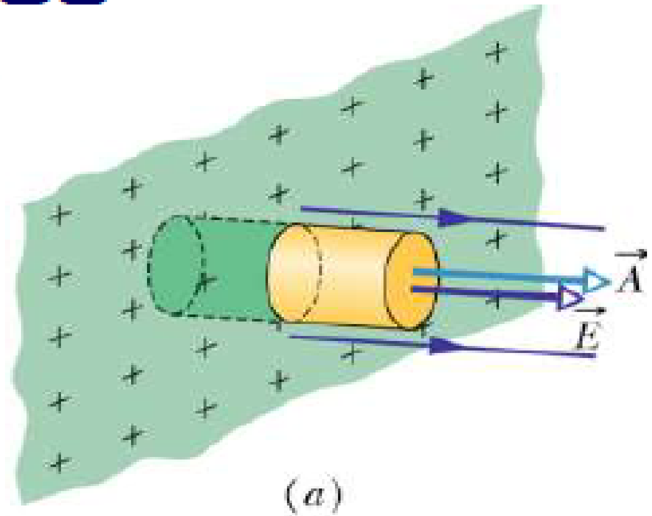
Draw a gaussian surface inside the conductor to find out.

The net charge enclosed is zero, so E = 0, which we already knew because it is inside the conductor. The inner charge is shielded by the induced charge distribution, so the outer charges will be evenly distributed.

Other Geometries

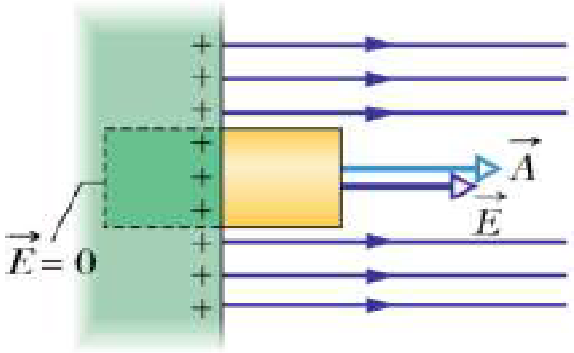
- Always use the symmetry of the problem to determine what shape to make your gaussian surface.
- Here is a plate (plane) geometry, where the charges are evenly distributed on a flat surface. If the total charge on the plate is Q, and the plate has a total area A_{tot}, then the *surface charge density* is

$$\sigma = Q / A_{tot} \quad \mathrm{C/m^2}$$



(a)

- The E field is everywhere perpendicular to the plate (again, if not, the charges will move until the part parallel to the surface is nullified). What is $\vec{E}$?
- Use a gaussian surface that is parallel to $\vec{E}$ on the sides (so no flux through side surfaces), and closes inside the conductor (no flux through that end).



- On the remaining side, the area vector $\vec{A}$ is parallel to the E field, so $\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 EA = \sigma A$ or $E = \dfrac{\sigma}{\varepsilon_0}$

Conducting Surface

Line of Charge

- In the previous chapter, we calculated the E field on the axis of a line of charge, but with Gauss' Law we can now handle finding E off the line axis.
- Here is a line geometry, where the charges are evenly distributed on a long line. If the total charge on the line is Q, and the line has a total length L_{tot}, then the *linear charge density* is

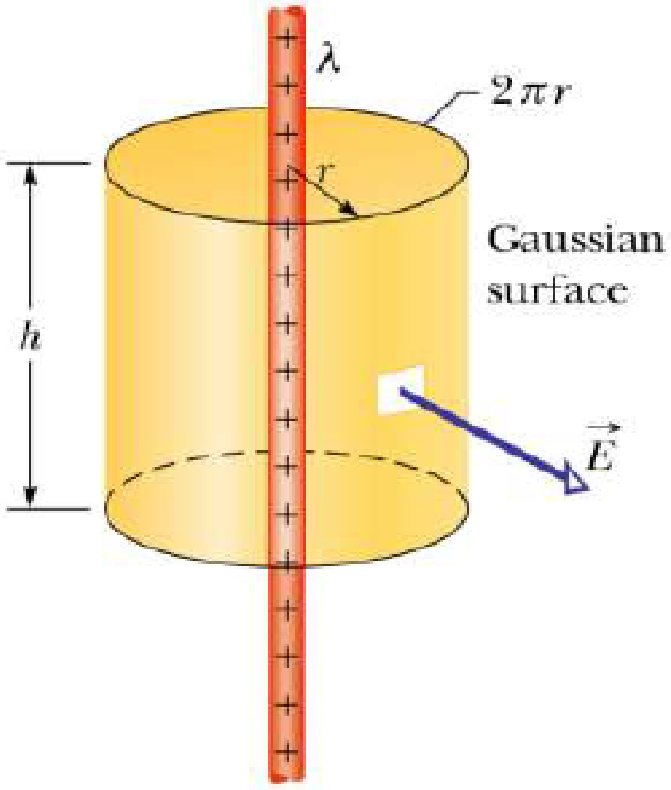
$$\lambda = Q / L_{tot} \quad \mathrm{C/m}$$

- The E field is everywhere perpendicular to the line (again, if not, the charges will move until the part parallel to the line is nullified).
- Use a cylindrical gaussian surface that is parallel to $\vec{E}$ on the top and bottom (so no flux through those surfaces), and is perpendicular to $\vec{E}$ elsewhere.
- The area vector $d\vec{A}$ is parallel to $\vec{E}$, and the total area is $2\pi rh$ so

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 E 2\pi r h = \lambda h$$

or

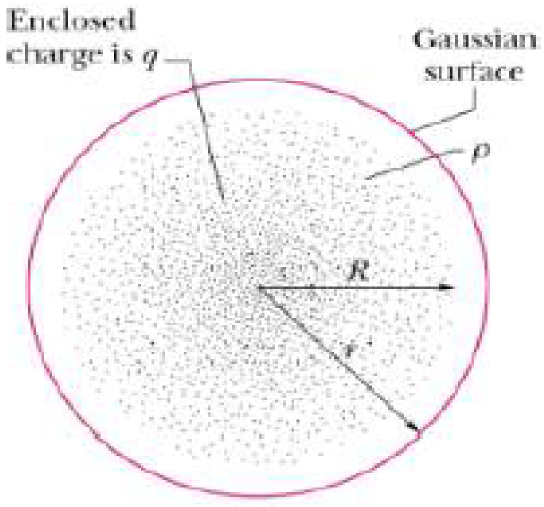
$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$



Line of Charge

Uniform Sphere of Charge

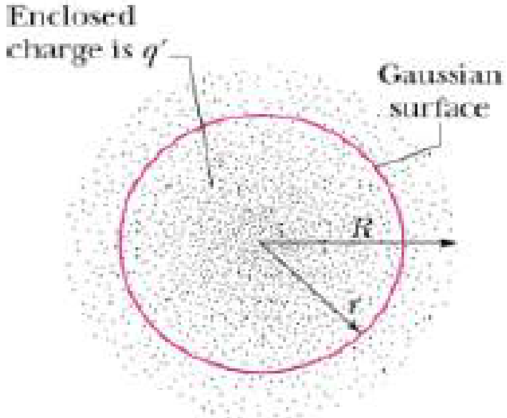
- Here is a spherical geometry, where the charges are evenly distributed throughout the volume. If the total charge in the sphere is Q, and the sphere has a radius R, then the *volume charge density* is $\rho = \dfrac{Q}{\frac{4}{3}\pi R^3}\ \ \mathrm{C/m^3}$



(a)

- By symmetry, the E field is everywhere radial from the center of the sphere.
- Use a spherical gaussian surface, which is perpendicular to $\vec{E}$ everywhere.
- The area vector $\vec{A}$ is parallel to $\vec{E}$, and the total area is $4\pi r^2$ so when the gaussian surface radius is $r < R$, then

$$\varepsilon_0 \oint \vec{E}\cdot d\vec{A} = \varepsilon_0 E 4\pi r^2 = \rho \frac{4}{3}\pi r^3 \quad \text{or} \quad E = \frac{\rho}{3\varepsilon_0} r$$



- When $r>R$, then the charge enclosed is just Q, so

$$\varepsilon_0 \oint \vec{E}\cdot d\vec{A} = \varepsilon_0 E 4\pi r^2 = Q \quad \text{or} \quad E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Coulomb's Law again

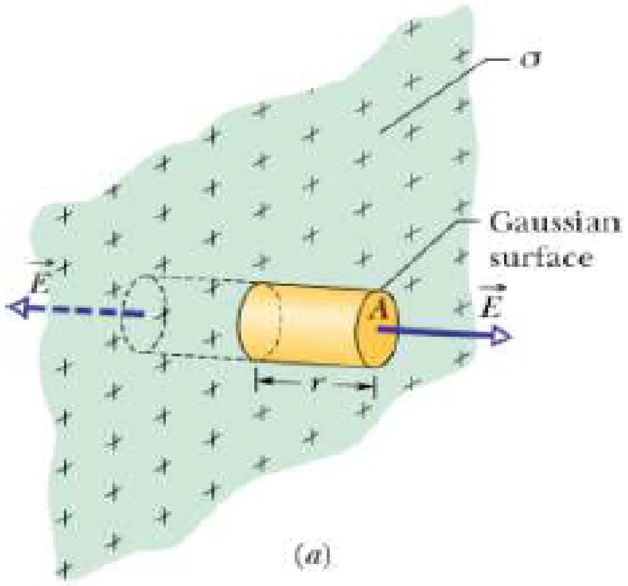
Nonconducting Sheet

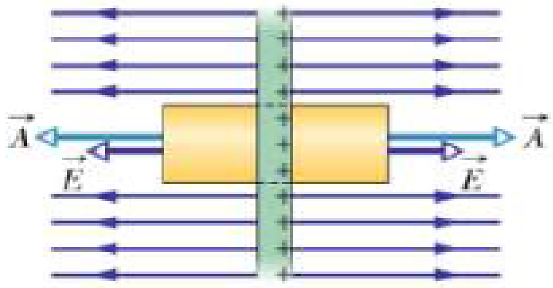
- A nonconducting sheet with a uniform surface charge density has the same geometry as for the conducting plate, so use the same gaussian surface.
- The only difference is that now one end cannot close in a conductor, so there is electric flux out both ends.
- As you may expect, the resulting electric field is half of what we got before.

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 2EA = \sigma A$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

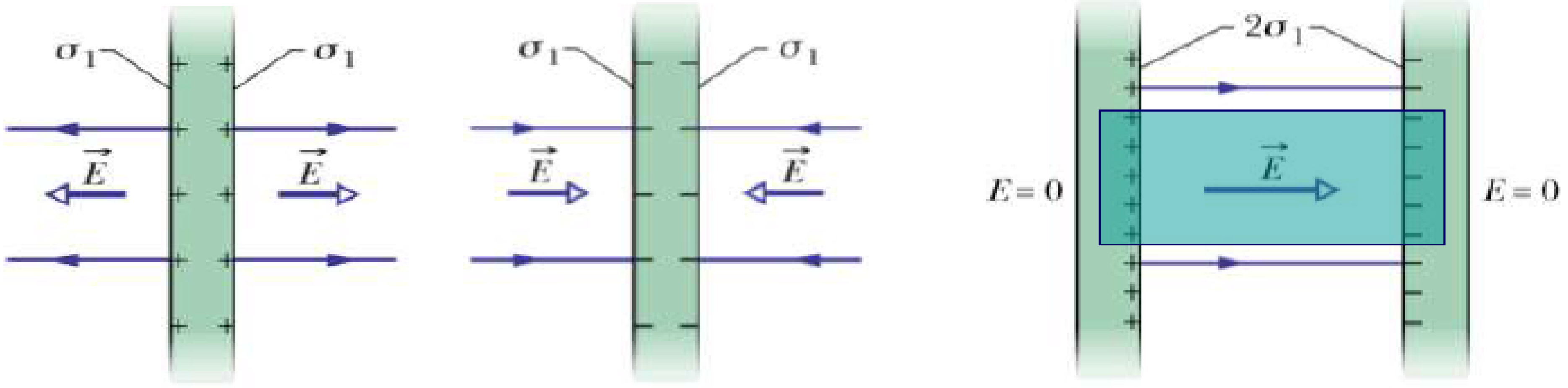
Sheet of Charge





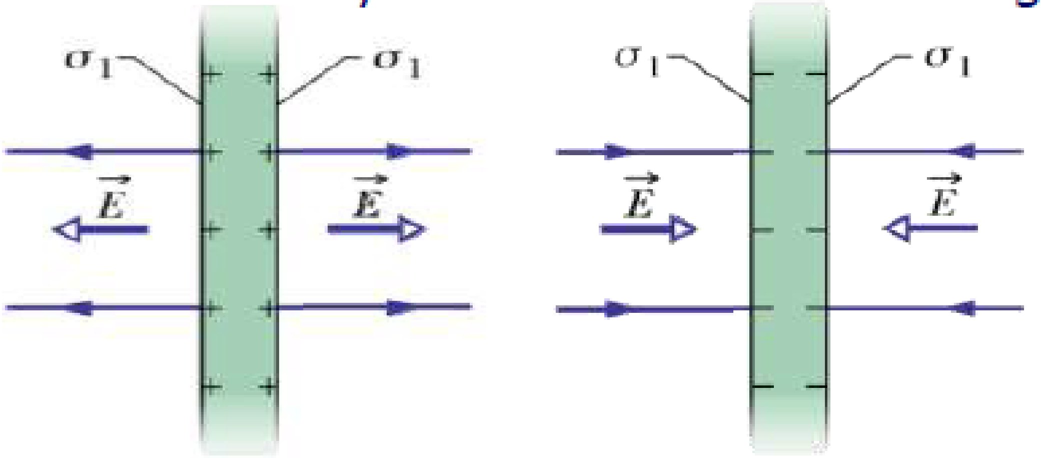
Two Parallel Conducting Plates

- When we have the situation shown in the left two panels (a positively charged plate and another negatively charged plate with the same magnitude of charge), both in isolation, they each have equal amounts of charge (surface charge density σ) on both faces.
- But when we bring them close together, the charges on the far sides move to the near sides, so on that inner surface the charge density is now 2σ.
- A gaussian surface shows that the net charge is zero (no flux through sides — dA perpendicular to E, or ends — $E = 0$). $E = 0$ outside, too, due to shielding, in just the same way we saw for the sphere.



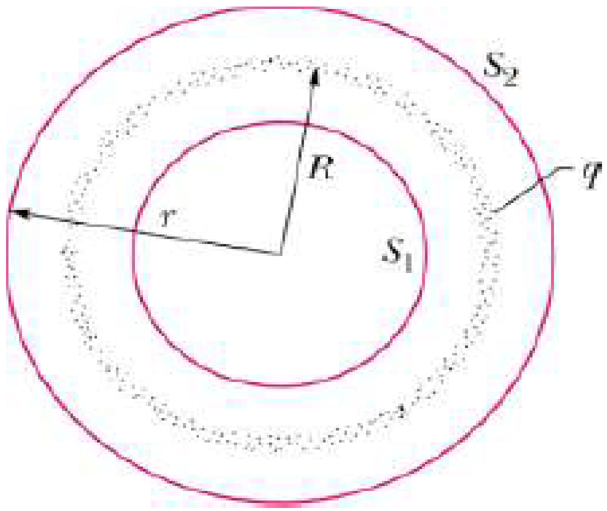
Two Parallel Nonconducting Sheets

- The situation is different if you bring two nonconducting sheets of charge close to each other.
- In this case, the charges cannot move, so there is no shielding, but now we can use the principle of superposition.
- In this case, the electric field on the left due to the positively charged sheet is canceled by the electric field on the left of the negatively charged sheet, so the field there is zero.
- Likewise, the electric field on the right due to the negatively charged sheet is canceled by the electric field on the right of the positively charged sheet.



- The result is much the same as before, with the electric field in between being twice what it was previously.

Spherical Symmetry



Spherical shell

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad (r \geq R)$$

$$E = 0 \quad (r < R)$$

We earlier said that a shell of uniform charge attracts or repels a charged particle that is outside the shell as if the shell's charge were concentrated at the center of the shell. We can now prove this using Gauss' Law.

We also said that a shell of uniform charge exerts no electrostatic force on a charged particle that is located inside the shell. Again, Gauss' Law can be used to prove this.

Summary

- Electric flux is the amount of electric field passing through a closed surface.
- Flux is positive when electric field is outward, and negative when electric field is inward through the closed surface.
- Gauss' Law states that the electric flux is proportional to the net charge enclosed by the surface, and the constant of proportionality is ε_0. In symbols, it is

$$\varepsilon_0 \Phi = q_{enc}$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$

- There are three geometries we typically deal with:

Geometry	Charge Density	Gaussian surface	Electric field
Linear	$\lambda = q/L$	Cylindrical, with axis along line of charge	$E = \frac{\lambda}{2\pi\varepsilon_0 r}$ Line of Charge
Sheet or Plane	$\sigma = q/A$	Cylindrical, with axis along E.	$E = \frac{\sigma}{\varepsilon_0}$ Conducting; $E = \frac{\sigma}{2\varepsilon_0}$ Nonconducting
Spherical	$\rho = q/V$	Spherical, with center on center of sphere	$E = \frac{q}{4\pi\varepsilon_0 r^2}$ $r \geq R$; $E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r$ $r < R$