Physics 122: Electricity & Magnetism – Lecture 8 Capacitance

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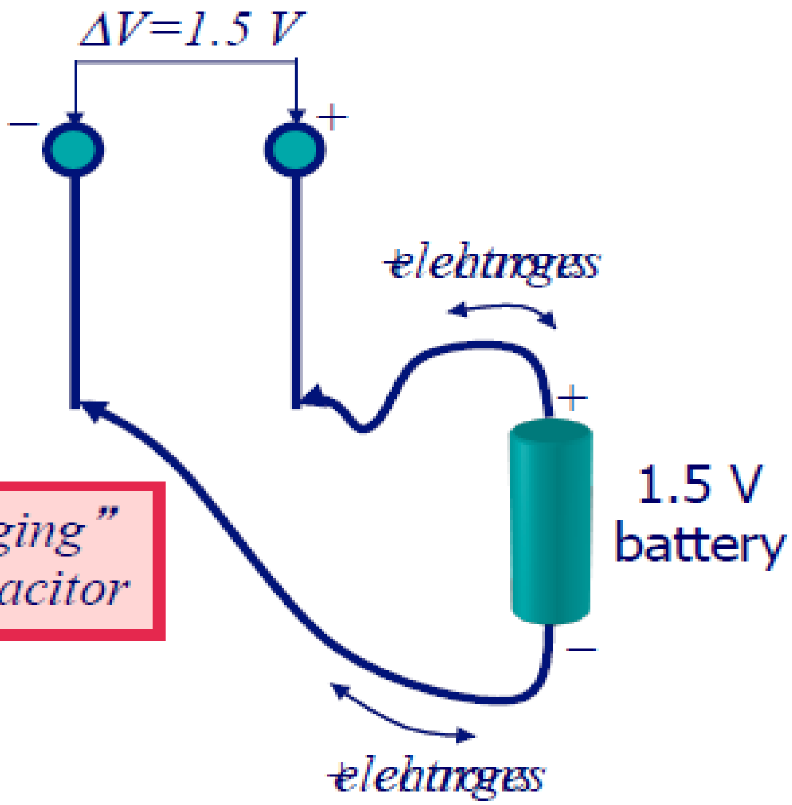
What is Capacitance?

- From the word "capacity," it describes how much charge an arrangement of conductors can hold for a given voltage applied.
- Charges will flow until the right conductor's potential is the same as the + side of the battery, and the left conductor's potential is the same as the – side of the battery.

"Charging" the capacitor

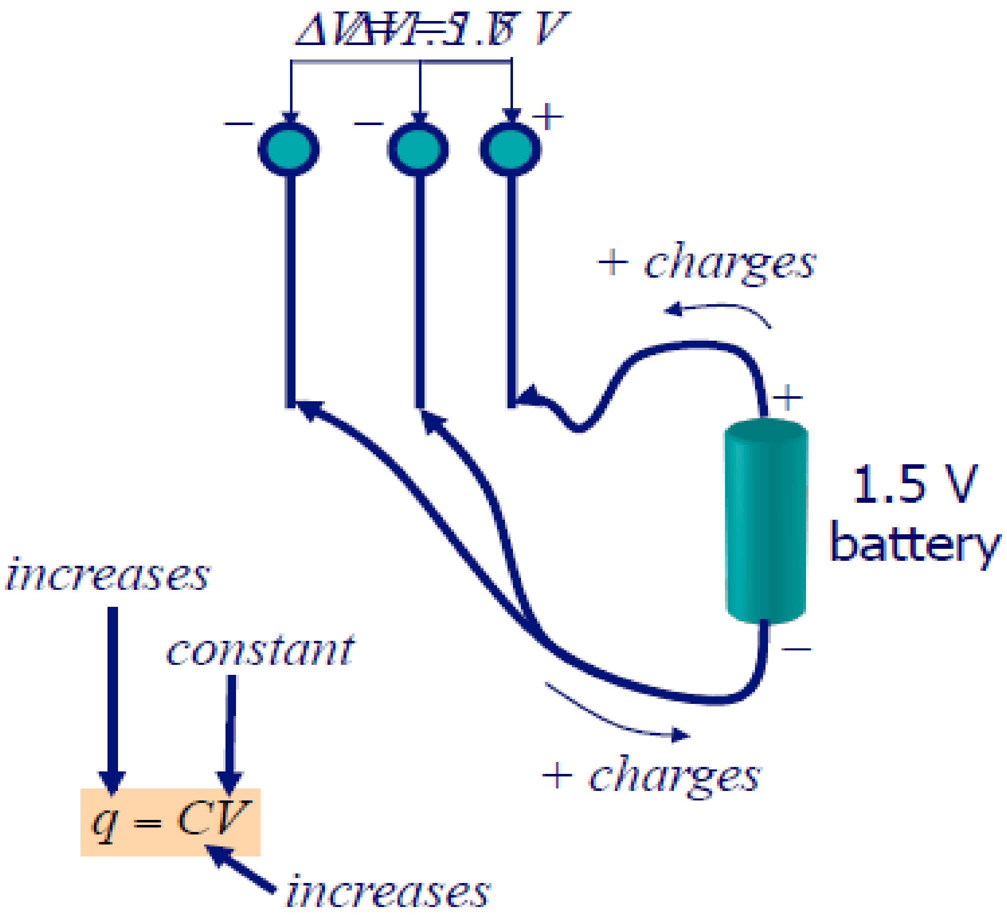


- How much charge is needed to produce an electric field whose potential difference is 1.5 V?
- Depends on capacitance: $q = CV$

definition of capacitance

Capacitance Depends on Geometry

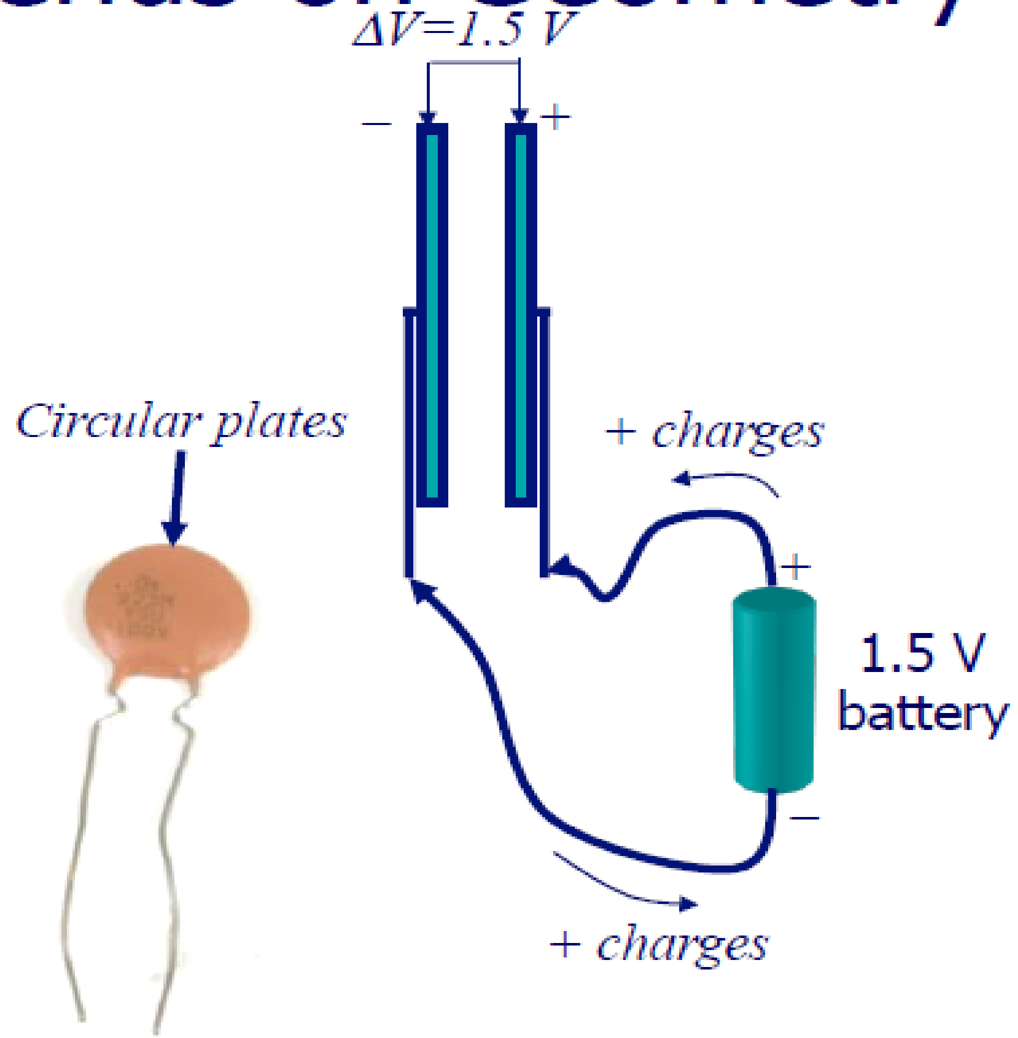
- What happens when the two conductors are moved closer together?
- They are still connected to the battery, so the potential difference cannot change.
- But recall that $V = -\int \vec{E} \cdot d\vec{s}$.
- Since the distance between them decreases, the E field has to increase.
- Charges have to flow to make that happen, so now these two conductors can hold more charge. I.e. the capacitance increases.



Capacitance Depends on Geometry

- What happens if we replace the small conducting spheres with large conducting plates?
- The plates can hold a lot more charge, so the capacitance goes way up.

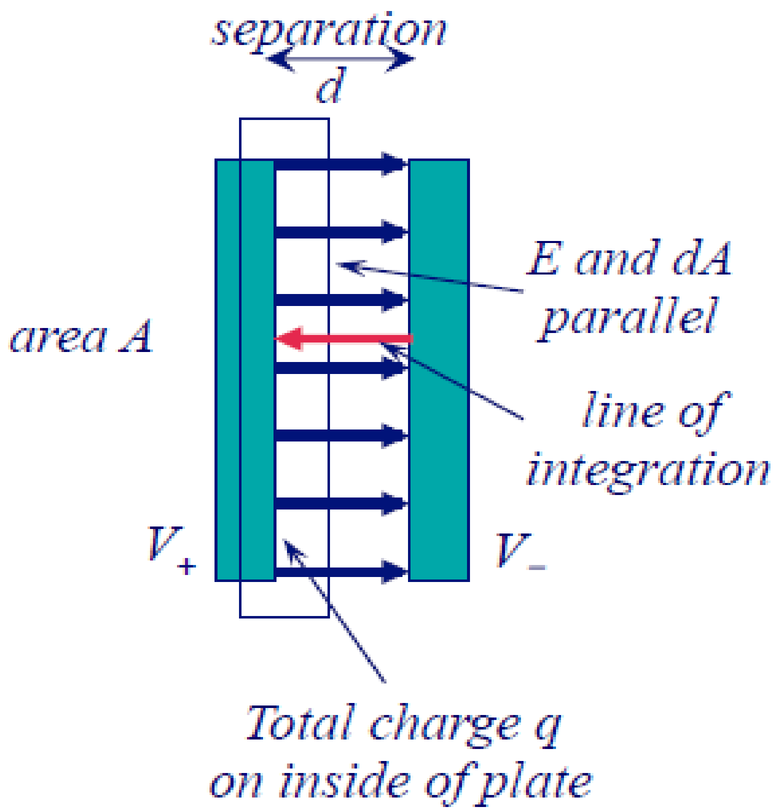
- Here is a capacitor that you can use in an electronic circuit.
- We will discuss several ways in which capacitors are useful.
- But first, let's look in more detail at what capacitance is.



Capacitance for Parallel Plates

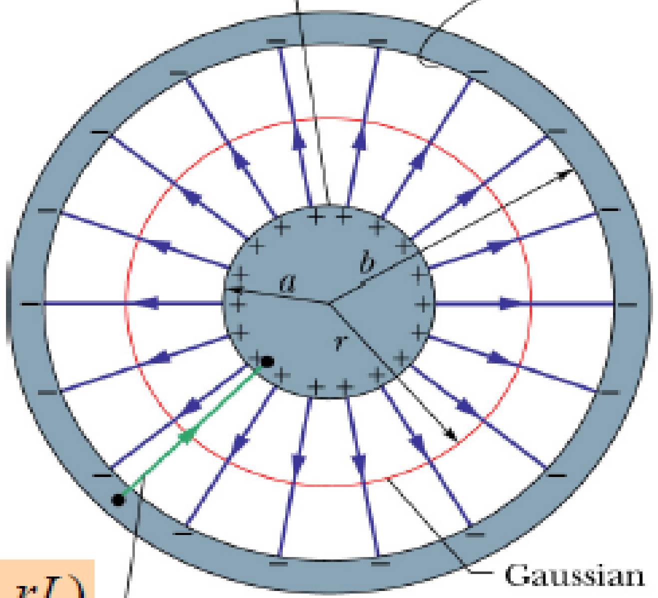
- Parallel plates make a great example for calculating capacitance, because
  - The E field is constant, so easy to calculate.
  - The geometry is simple, only the area and plate separation are important.
- To calculate capacitance, we first need to determine the E field between the plates. We use Gauss' Law, with one end of our gaussian surface closed inside one plate, and the other closed in the region between the plates (neglect fringing at ends):

  $\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q$ so $q = \varepsilon_0 EA$



- Need to find potential difference $V = V_+ - V_- = -\int \vec{E} \cdot d\vec{s}$
- Since $E = constant$, we have $V = Ed$, so the capacitance is $C = q/V = \frac{\varepsilon_0 EA}{Ed} = \frac{\varepsilon_0 A}{d}$

Capacitance for Other Configurations (Cylindrical)



- Cylindrical capacitor
  - The E field falls off as $1/r$.
  - The geometry is fairly simple, but the V integration is slightly more difficult.
- To calculate capacitance, we first need to determine the E field between the plates. We use Gauss' Law, with a cylindrical gaussian surface closed in the region between the plates (neglect fringing at ends):

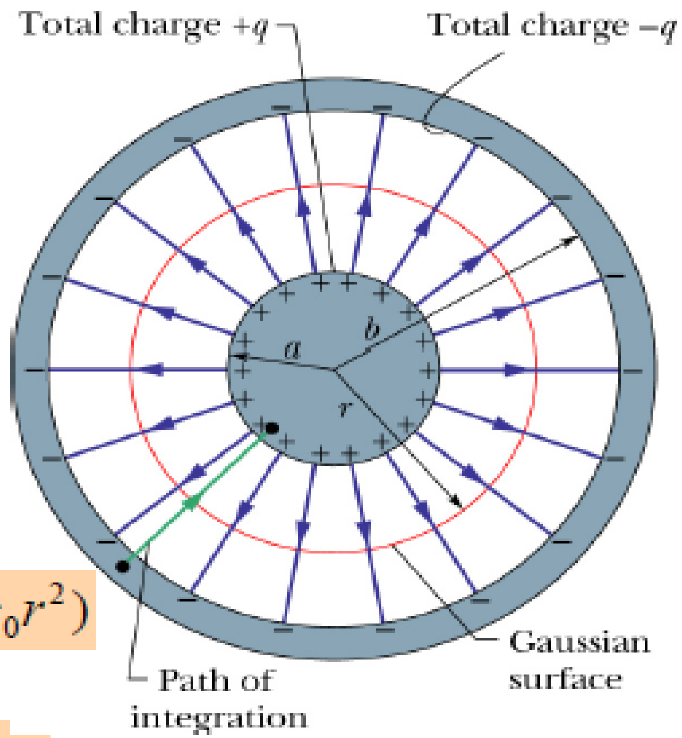
$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q$ So $q = \varepsilon_0 EA = \varepsilon_0 E(2\pi r L)$ or $E = q/(2\pi\varepsilon_0 r L)$

- Need to find potential difference $V = V_+ - V_- = -\int \vec{E} \cdot d\vec{s}$
- Since $E \sim 1/r$, we have $V = \dfrac{q}{2\pi\varepsilon_0 L}\int_b^a \dfrac{dr}{r} = \dfrac{q}{2\pi\varepsilon_0 L}\ln\left(\dfrac{b}{a}\right)$, so the capacitance is

$$C = q/V = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$$

Capacitance for Other Configurations (Spherical)

- Spherical capacitor
  - The E field falls off as $1/r^2$.
  - The geometry is fairly simple, and the V integration is similar to the cylindrical case.
- To calculate capacitance, we first need to determine the E field between the spheres. We use Gauss' Law, with a spherical gaussian surface closed in the region between the spheres:



$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q$ So $q = \varepsilon_0 EA = \varepsilon_0 E(4\pi r^2)$ or $E = q/(4\pi\varepsilon_0 r^2)$

- Need to find potential difference $V = V_+ - V_- = -\int \vec{E} \cdot d\vec{s}$
- Since $E \sim 1/r^2$, we have $V = \frac{q}{4\pi\varepsilon_0} \int_b^a \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$, so the capacitance is

$$C = q/V = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

Capacitance Summary

- Parallel Plate Capacitor $$C = \frac{\varepsilon_0 A}{d}$$
- Cylindrical (nested cylinder) Capacitor $$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$$
- Spherical (nested sphere) Capacitor $$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$
- Capacitance for isolated Sphere $$C = 4\pi\varepsilon_0 R$$
- Units: $\varepsilon_0 \times$length = C^2/Nm = F (farad), named after Michael Faraday. [note: ε_0 = 8.85 pF/m]

Units of Capacitance

2. Given these expressions, and $\varepsilon_0 = 8.85 \times 10^{-12}$ C²/N·m², what are the units of capacitance?

$$C = \frac{\varepsilon_0 A}{d} \qquad C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)} \qquad C = 4\pi\varepsilon_0 \frac{ab}{b-a} \qquad C = 4\pi\varepsilon_0 R$$

A. The units are different in the different expressions.

B. The units are $C^2/N\cdot m^2$.

C. The units are $C^2/N\cdot m$.

D. The units are C^2/N.

E. The units are C/V.

- Units: $\varepsilon_0 \times \text{length} = C^2/N\cdot m = F$ (farad), named after Michael Faraday. [note: $\varepsilon_0 = 8.85$ pF/m]