Physics 122: Electricity & Magnetism – Lecture 9 Capacitance (continued)

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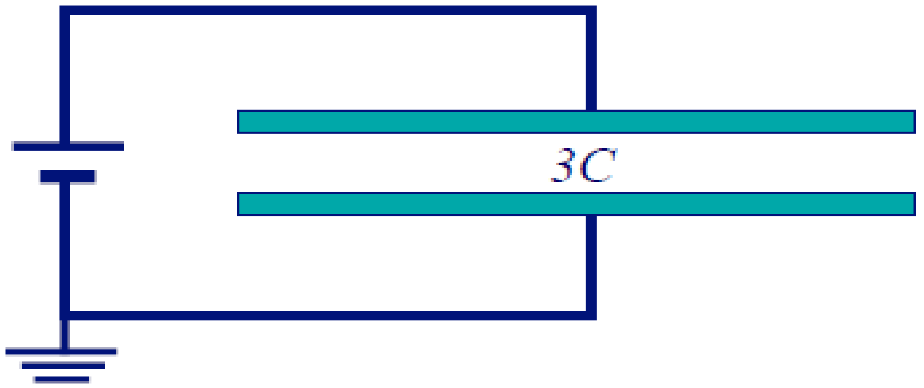
Capacitors in Parallel

- No difference between



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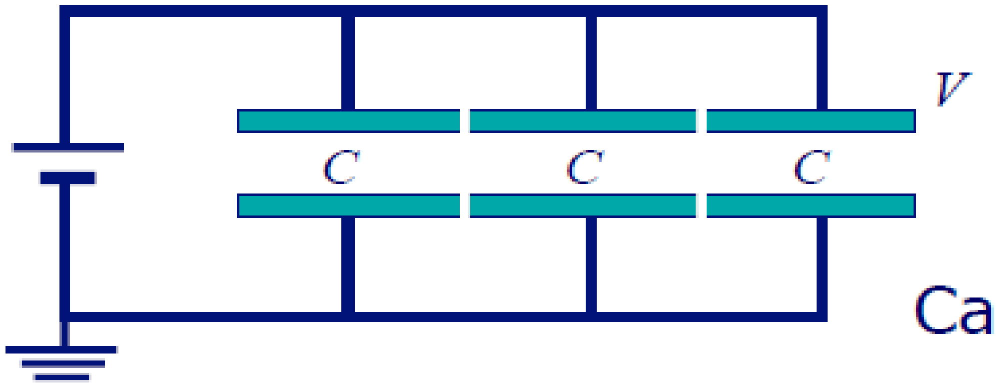
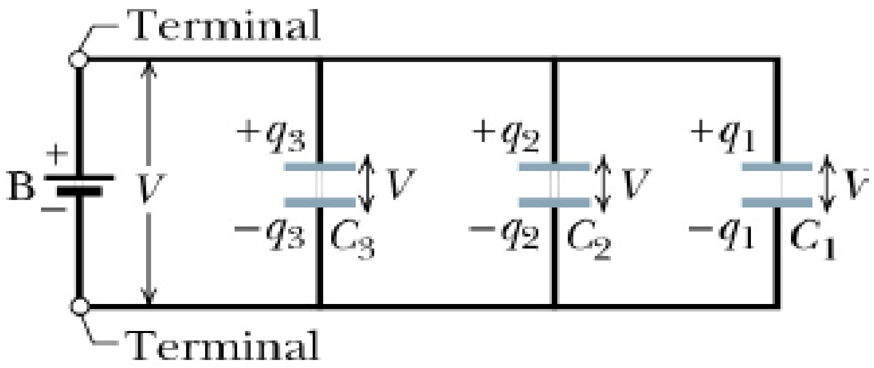
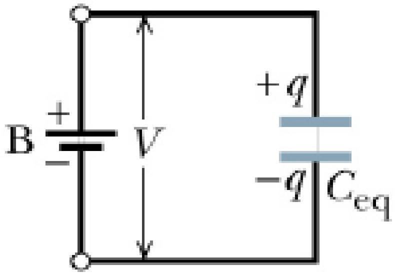




(a)

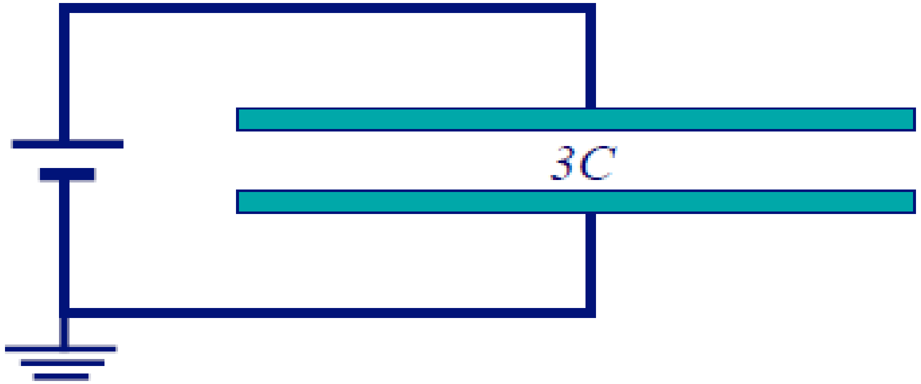


(b)

Capacitors in parallel: $C_{eq} = \sum_{j=1}^{n} C_j$

Capacitors in Series

- There *is* a difference between



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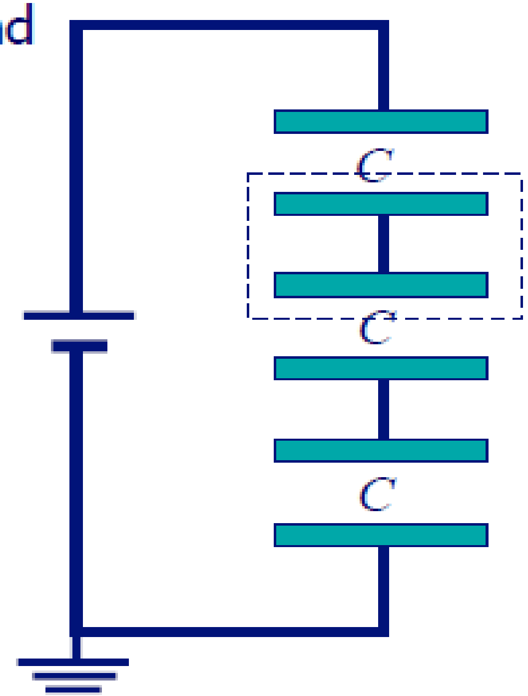


- Charge on lower plate of one and upper plate of next are equal and opposite. (show by gaussian surface around the two plates).
- Total charge is q, but voltage on each is only $V/3$.

Capacitors in series: $$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$$

Capacitors in Series

- To see the series formula, consider the individual voltages across each capacitor

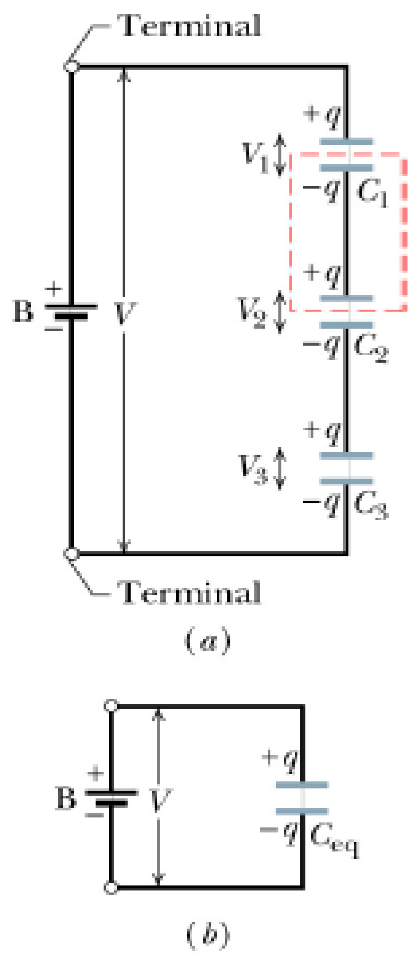
$$V_1 = \frac{q}{C_1},\ V_2 = \frac{q}{C_2},\ V_3 = \frac{q}{C_3}$$

- The sum of these voltages is the total voltage of the battery, V

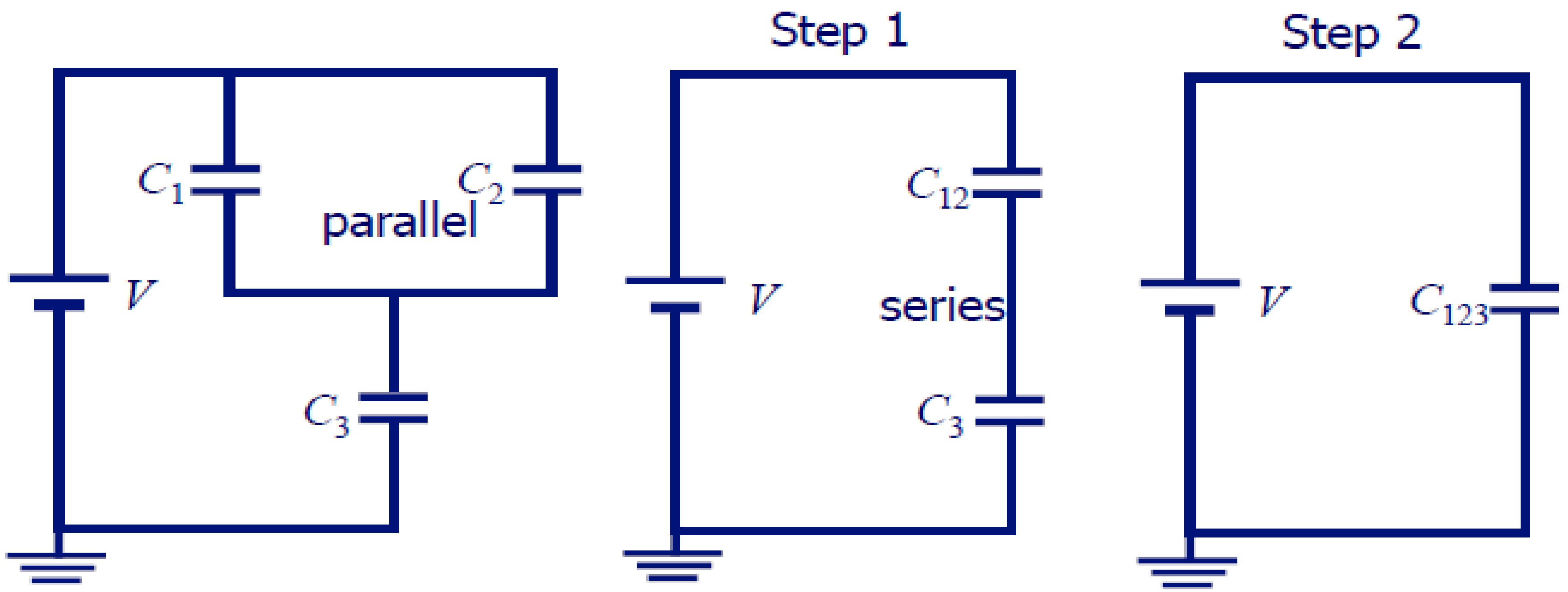
$$V = V_1 + V_2 + V_3 = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

- Since $V/q = 1/C_{eq}$, we have

$$\frac{V}{q} = \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



Example Capacitor Circuit



$$C_{12} = C_1 + C_2$$

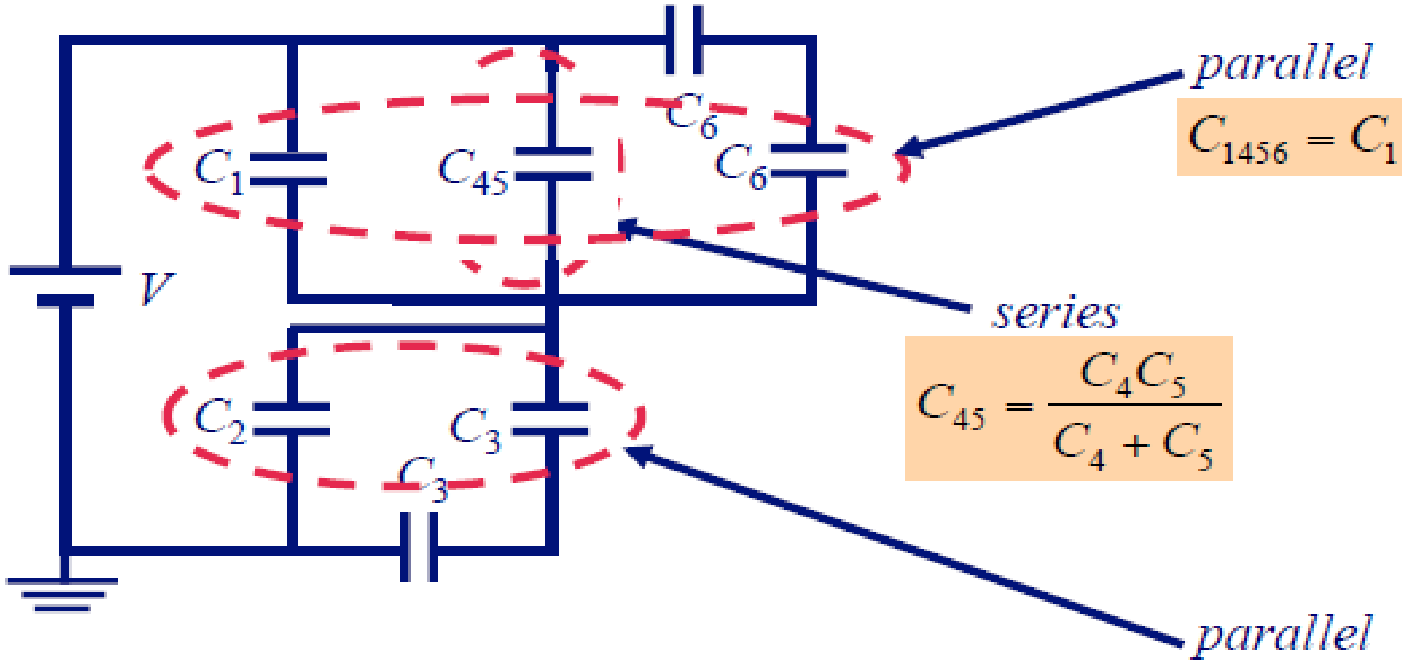
$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3}$$

$$C_{123} = \frac{C_{12}C_3}{C_{12} + C_3}$$

$C_1 = 12.0\ \mu\text{F}$, $C_2 = 5.3\ \mu\text{F}$, $C_3 = 4.5\ \mu\text{F}$

$C_{123} = (12 + 5.3)4.5/(12+5.3+4.5)\ \mu\text{F} = 3.57\ \mu\text{F}$

Another Example

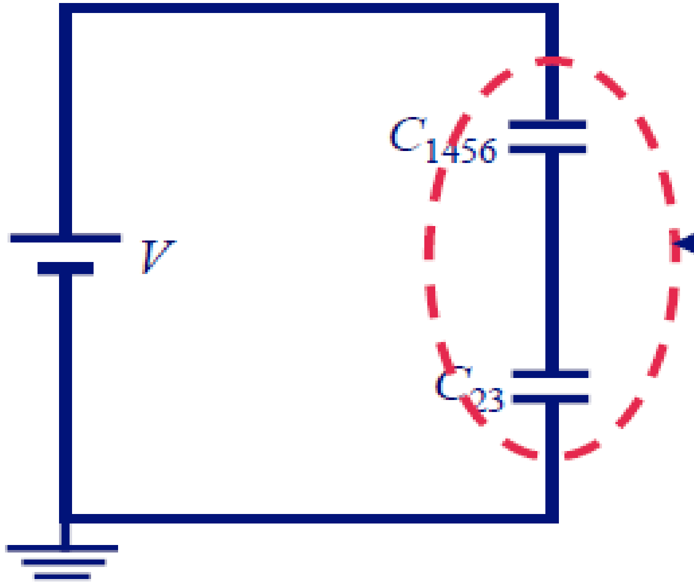


$$C_{1456} = C_1 + C_{45} + C_6$$

$$C_{45} = \frac{C_4 C_5}{C_4 + C_5}$$

$$C_{23} = C_2 + C_3$$

Another Example



$$C_{45} = \frac{C_4 C_5}{C_4 + C_5}$$

$$C_{1456} = C_1 + C_{45} + C_6$$

$$C_{23} = C_2 + C_3$$

$$C_{123456} = \frac{C_{1456} C_{23}}{C_{1456} + C_{23}}$$

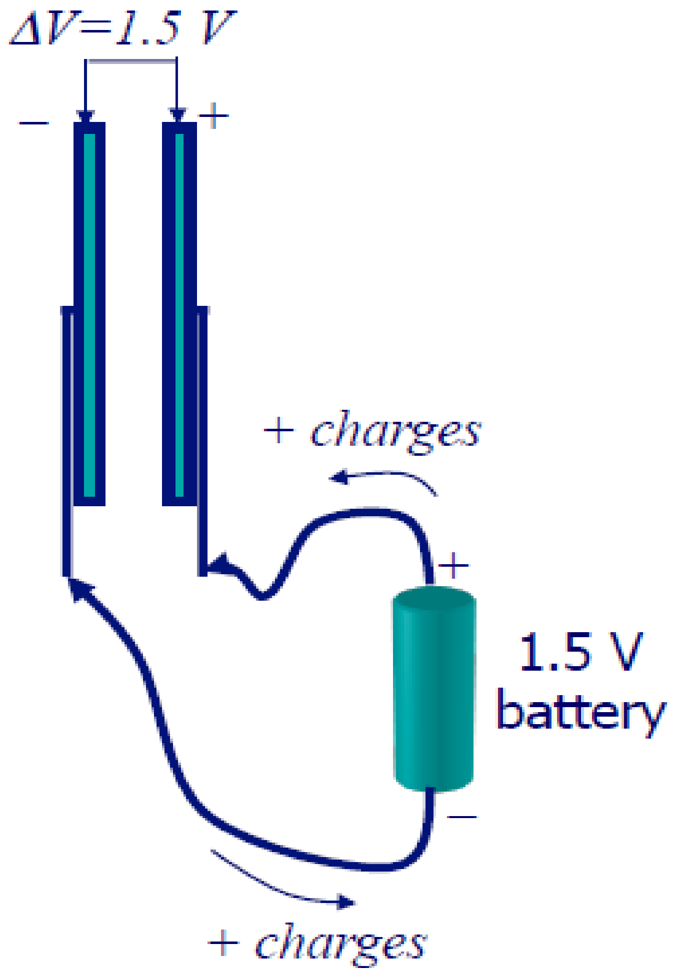
Complete solution

$$C_{123456} = \frac{\left(C_1 + \frac{C_4 C_5}{C_4 + C_5} + C_6\right)(C_2 + C_3)}{C_1 + \frac{C_4 C_5}{C_4 + C_5} + C_6 + C_2 + C_3}$$

Capacitors Store Energy

- When charges flow from the battery, energy stored in the battery is lost. Where does it go?
- We learned last time that an arrangement of charge is associated with potential energy. One way to look at it is that the charge arrangement stores the energy.
- Recall the definition of electric potential $V = U/q$
- For a distribution of charge on a capacitor, a small element dq will store potential energy $dU = V\,dq$
- Thus, the energy stored by charging a capacitor from charge 0 to q is

$$U = \frac{1}{C}\int_0^q q'\,dq' = \frac{q^2}{2C} = \tfrac{1}{2}CV^2$$



Movie 1

Movie 2

Capacitors Store Energy

- Another way to think about the stored energy is to consider it to be stored in the electric field itself.
- The total energy in a parallel plate capacitor is

$$U = \tfrac{1}{2} C V^2 = \frac{\varepsilon_0 A}{2d} V^2$$

- The volume of space filled by the electric field in the capacitor is $vol = Ad$, so the *energy density* is

$$u = \frac{U}{vol} = \frac{\varepsilon_0 A}{2dAd} V^2 = \tfrac{1}{2} \varepsilon_0 \left(\frac{V}{d} \right)^2$$

- But $V = -\int \vec{E} \cdot d\vec{s} = Ed$ for a parallel plate capacitor, so

$$u = \tfrac{1}{2} \varepsilon_0 E^2$$

Energy stored in electric field

Dielectrics

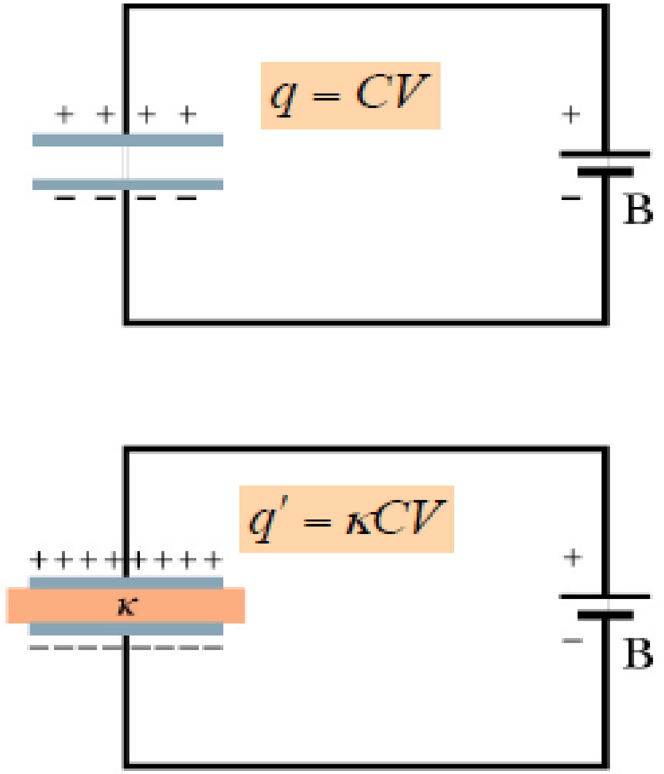
- You may have wondered why we write ε_0 (permittivity of free space), with a little zero subscript. It turns out that other materials (water, paper, plastic, even air) have different permittivities $\varepsilon = \kappa\varepsilon_0$. The κ is called the *dielectric constant*, and is a unitless number. For air, κ = 1.00054 (so ε for air is for our purposes the same as for "free space.")
- In all of our equations where you see ε_0, you can substitute $\kappa\varepsilon_0$ when considering some other materials (called dielectrics).
- The nice thing about this is that we can increase the capacitance of a parallel plate capacitor by filling the space with a dielectric:

$$C' = \frac{\kappa \varepsilon_0 A}{d} = \kappa C$$

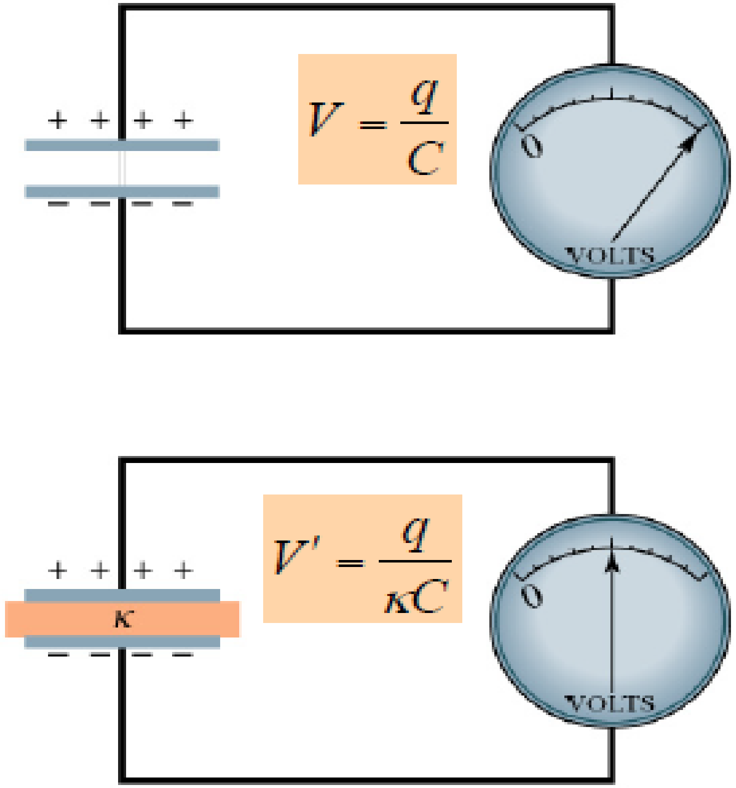
Material	**Dielectric Constant κ**	**Dielectric Strength (kV/mm)**
Air	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer Oil	4.5	
Pyrex	4.7	14
Ruby Mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20º C)	80.4	
Water (50º C)	78.5	
Titania Ceramic	130	
Strontium Titanate	310	8

What Happens When You Insert a Dielectric?

- With battery attached, V=const, so more charge flows to the capacitor
- With battery disconnected, q=const, so voltage (for given q) drops.



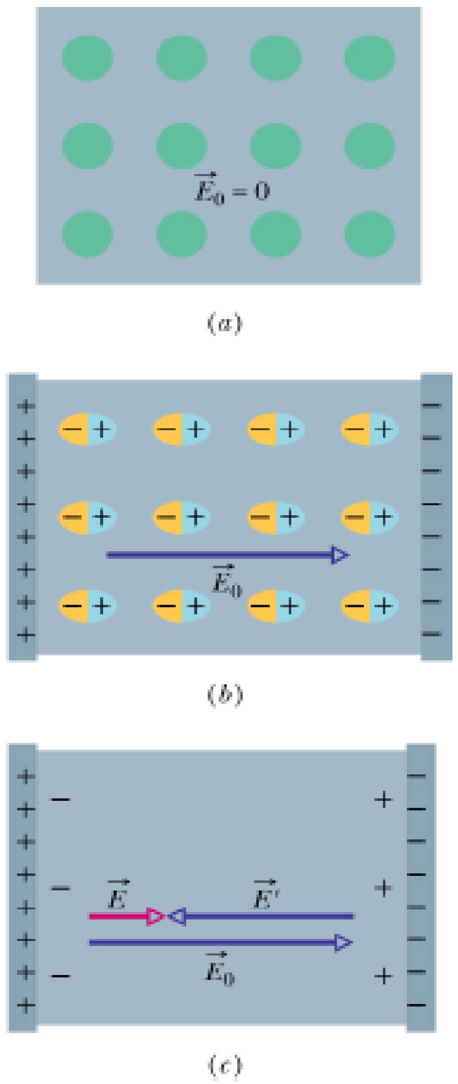
V = a constant



q = a constant

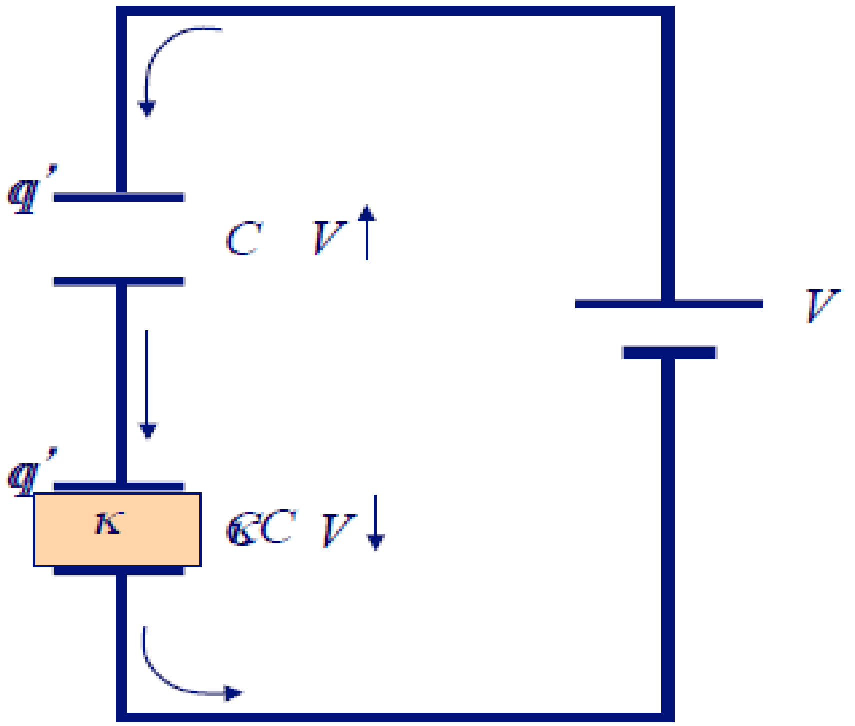
What Does the Dielectric Do?

- A dielectric material is made of molecules.
- Polar dielectrics already have a dipole moment (like the water molecule).
- Non-polar dielectrics are not naturally polar, but actually stretch in an electric field, to become polar.
- The molecules of the dielectric align with the applied electric field in a manner to oppose the electric field.
- This reduces the electric field, so that the net electric field is less than it was for a given charge on the plates.
- This lowers the potential (case b of the previous slide).
- If the plates are attached to a battery (case a of the previous slide), more charge has to flow onto the plates.



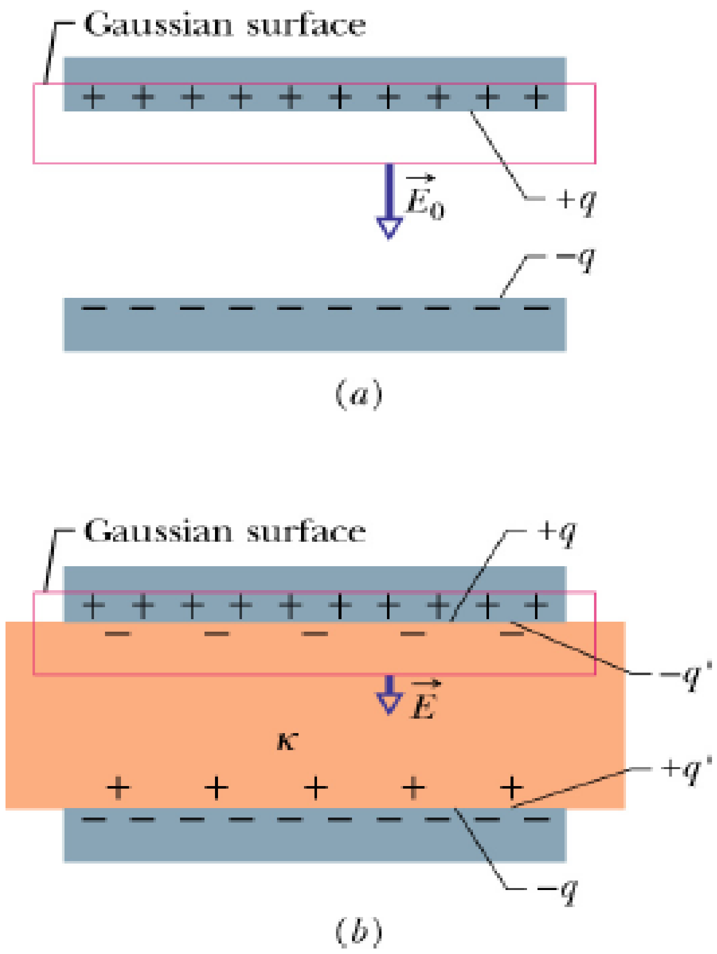
A Closer Look

- Insert dielectric
- Capacitance goes up by κ
- Charge increases
- Charge on upper plate comes from upper capacitor, so its charge also increases.
- Since $q' = CV_1$ increases on upper capacitor, V_1 must increase on upper capacitor.
- Since total $V = V_1 + V_2$ = constant, V_2 must decrease.



Dielectrics and Gauss' Law

- Gauss' Law holds without modification, but notice that the charge enclosed by our gaussian surface is less, because it includes the induced charge q' on the dielectric.
- For a given charge q on the plate, the charge enclosed is $q - q'$, which means that the electric field must be smaller. The effect is to weaken the field.
- When attached to a battery, of course, more charge will flow onto the plates until the electric field is again E_0.



Summary

- Capacitance says how much charge is on an arrangement of conductors for a given potential. $q = CV$
- Capacitance depends only on geometry
  - Parallel Plate Capacitor $C = \dfrac{\varepsilon_0 A}{d}$
  - Cylindrical Capacitor $C = 2\pi\varepsilon_0 \dfrac{L}{\ln(b/a)}$
  - Spherical Capacitor $C = 4\pi\varepsilon_0 \dfrac{ab}{b-a}$
  - Isolated Sphere $C = 4\pi\varepsilon_0 R$
- Units, F (farad) = C^2/Nm or C/V (note ε_0 = 8.85 pF/m)
- Capacitors in parallel $C_{eq} = \sum_{j=1}^{n} C_j$ in series $\dfrac{1}{C_{eq}} = \sum_{j=1}^{n} \dfrac{1}{C_j}$
- Energy and energy density stored by capacitor

  $U = \frac{1}{2} CV^2$ $u = \frac{1}{2} \varepsilon_0 E^2$
- Dielectric constant increases capacitance due to induced, opposing field. $C' = \kappa C$ κ is a unitless number.