Physics 122: Electricity & Magnetism – Lecture 12
DC Circuits

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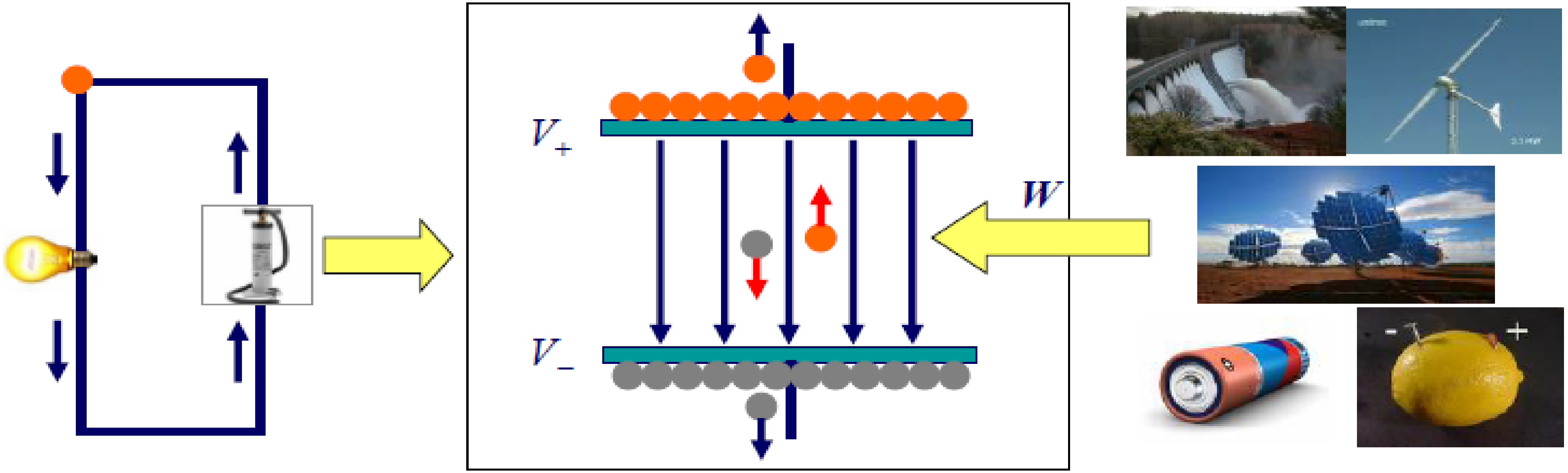
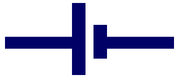
emf and emf devices

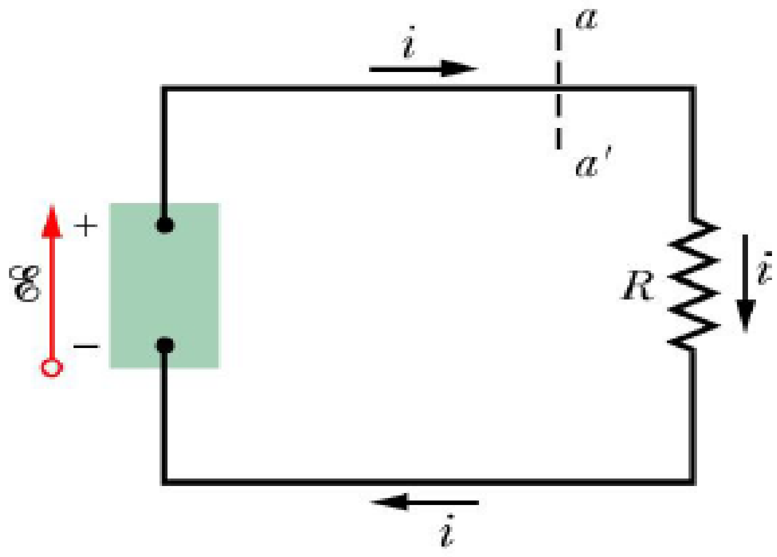


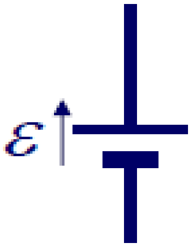
- The term emf comes from the outdated phrase electromovitive force.
- emf devices include battery, electric generator, solar cell, fuel cell,……
- emf devices are sources of charge, but also sources of voltage (potential difference).
- emf devices must do work to pump charges from lower to higher terminals.
- Source of emf devices: chemical, solar, mechanical, thermal-electric energy.

Emf $\mathcal{E}$

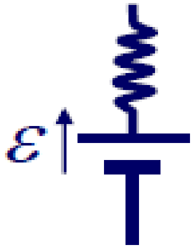
- We need a symbol  for emf, and we will use a script $\mathcal{E}$ to represent emf. ε is the potential difference between terminals of an emf device.
- The SI unit for emf is Volt (V).
- We earlier saw that there is a relationship between energy, charge, and voltage $dqV = dW$
- So, $$\varepsilon = \frac{dW}{dq}$$
- Power $dW = \varepsilon dq = \varepsilon i dt = P dt$

$$P = \varepsilon i$$



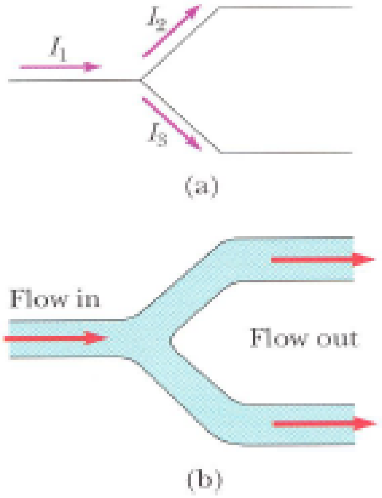


Ideal emf device:
$V = \mathcal{E}$
(open or close loop)



Real emf device:
$V = \mathcal{E}$ *(open loop)*
$V < \mathcal{E}$ *(close loop)*

Kirchhoff's Rules

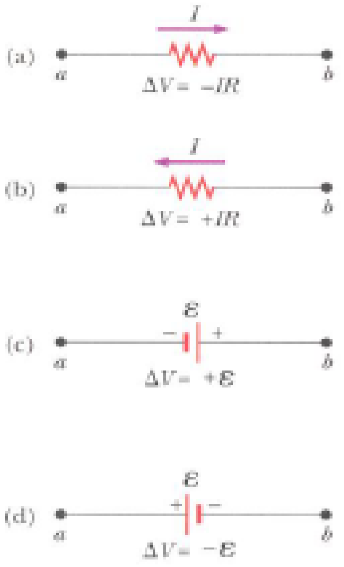


- Junction Rule: At any junction, the sum of the currents must equal zero:

$$\sum_{junction} i = 0$$

- Loop Rule: The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum_{closed\,loop} \Delta V = 0$$



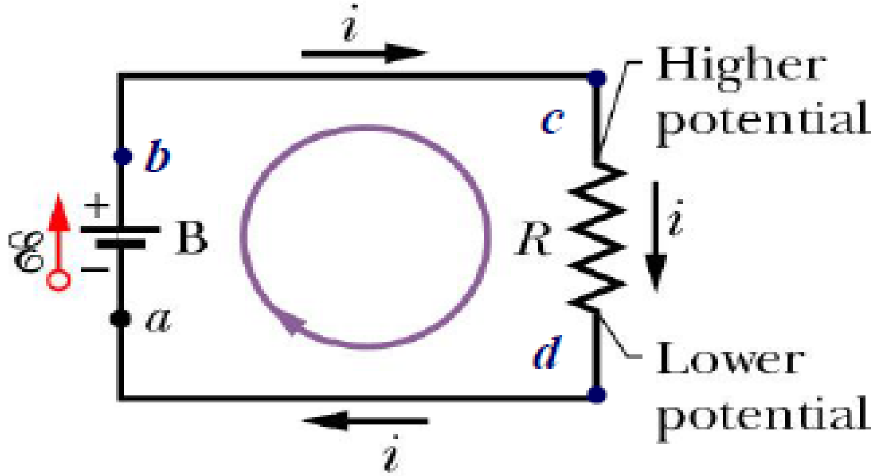
- For a move through a resistance in the direction of current, the change in potential is –iR; in the opposite direction it is +iR.
- For a move through an ideal emf device in the direction of the emf arrow, the change in potential is +ε; in the opposite direction it is - ε.

A Single-Loop Circuit

- Travel clockwise from a:

$$\Delta V_{ba} = V_b - V_a = \varepsilon$$
$$\Delta V_{cb} = V_c - V_b = 0$$
$$\Delta V_{dc} = V_d - V_c = -iR$$
$$\Delta V_{ad} = V_a - V_d = 0$$
$$\sum_{closedloop} \Delta V = \varepsilon + 0 - iR + 0 = 0$$

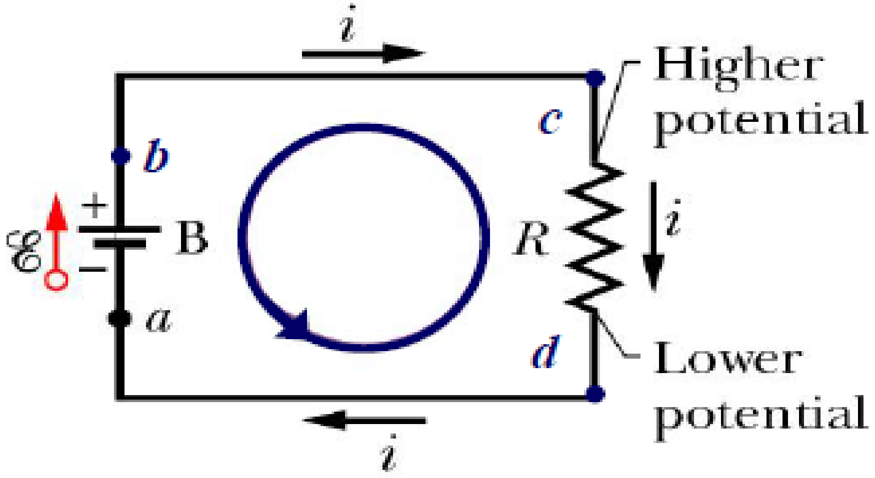
$$\varepsilon - iR = 0$$
$$i = \frac{\varepsilon}{R}$$



- Travel counterclockwise from a:

$$\Delta V_{da} = V_d - V_a = 0$$
$$\Delta V_{cd} = V_c - V_d = iR$$
$$\Delta V_{bc} = V_b - V_c = 0$$
$$\Delta V_{ab} = V_a - V_b = -\varepsilon$$
$$\sum_{closedloop} \Delta V = 0 + iR + 0 - \varepsilon = 0$$

$$iR - \varepsilon = 0$$
$$i = \frac{\varepsilon}{R}$$



Resistances in Series

- Junction Rule: When a potential difference V is applied across resistances connected in series, the resistances have identical currents i:

$$\boxed{i = i_1 = i_2 = i_3}$$

- Loop Rule: The sum of the potential differences across resistances is equal to the applied potential difference V:

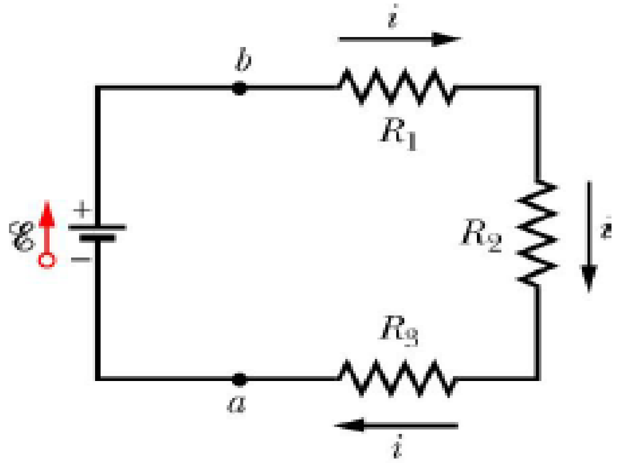
(a) $\varepsilon - iR_1 - iR_2 - iR_3 = 0$ $\quad i = \dfrac{\varepsilon}{R_1 + R_2 + R_3}$

(b) $\varepsilon - iR_{eq} = 0$ $\quad i = \dfrac{\varepsilon}{R_{eq}}$

$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

- The equivalent resistance of a series combination of resistors is the numerical sum of the individual resistances and is always greater than any individual resistance.

$$R_{eq} = \sum_{i=1}^{n} R_i$$



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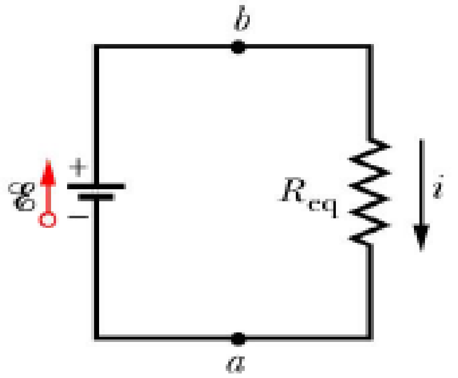


(b)

Resistances in Parallel

- When a potential difference V is applied across resistances connected in parallel, the resistances all have that same potential difference V.

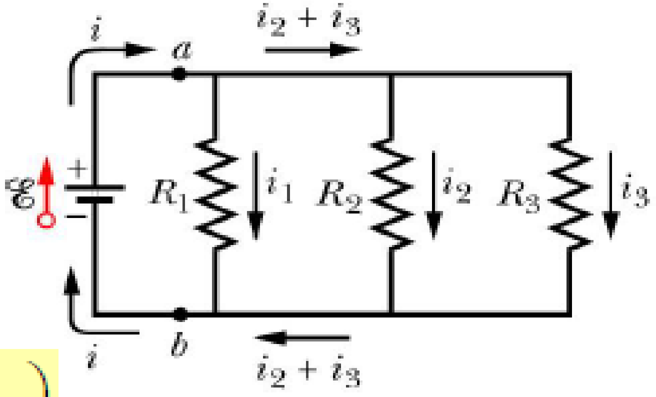
$$\boxed{V = V_1 = V_2 = V_3}$$

(a) Junction Rule:

$$i_1 = \frac{V}{R_1}, i_2 = \frac{V}{R_2}, i_3 = \frac{V}{R_3} \qquad i = i_1 + i_2 + i_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

(b) Loop Rule:

$$V - iR_{eq} = 0 \qquad i = \frac{V}{R_{eq}} \qquad \boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$



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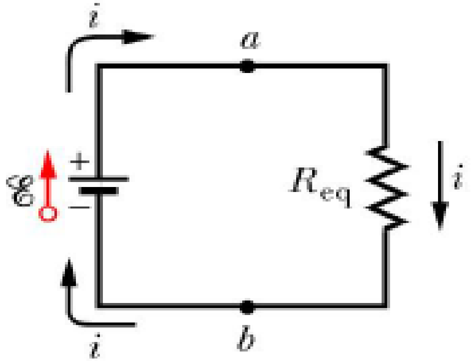


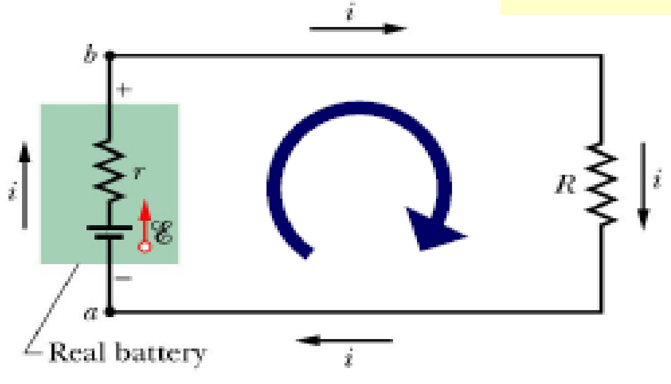
(b)

- The inverse of the equivalent resistance of two or more resistors in a parallel combination is the sum of the inverse of the individual resistances. Furthermore, the equivalent resistance is always less than the smallest resistance in the group.

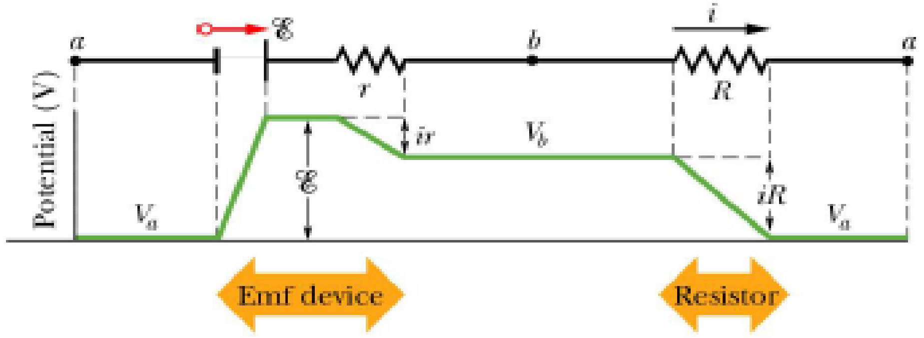
$$\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}$$

Example: Real Battery

- Real battery has internal resistance to the internal movement of charge.
- Current: $\varepsilon - ir - iR = 0$ $\quad i = \dfrac{\varepsilon}{R+r}$
- Potential difference:
  clockwise: $V_a + \varepsilon - ir = V_b \quad V_b - V_a = \varepsilon - ir = \varepsilon - \dfrac{\varepsilon}{R+r} r = \dfrac{\varepsilon}{R+r} R$
- Power $\quad P = iV = i(\varepsilon - ir) = i\varepsilon - i^2 r$



(a)



(b)

Example: Multiple Batteries

- What is the potential difference and power between the terminals of battery 1 and 2?
- Current i in this single-loop: (counterclockwise)

$$-\varepsilon_1 + ir_1 + iR + ir_2 + \varepsilon_2 = 0$$

$$i = \frac{\varepsilon_1 - \varepsilon_2}{r_1 + R + r_2} = 0.2396\ A$$

- Potential difference: (clockwise)

$$V_b - ir_1 + \varepsilon_1 = V_a$$
$$V_a - V_b = -ir_1 + \varepsilon_1 = +3.84V$$

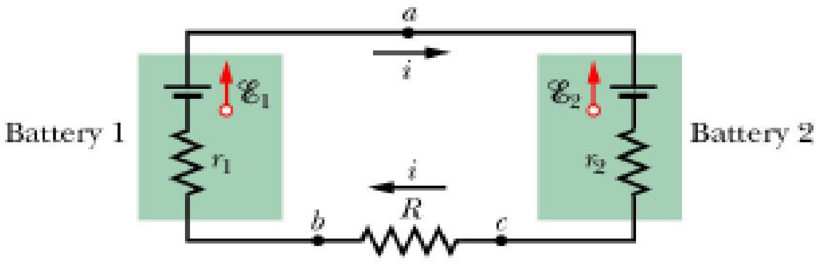
$$V_a - \varepsilon_2 - ir_2 = V_c$$
$$V_a - V_c = \varepsilon_2 + ir_2 = +2.53V$$

- Power: $P_{\varepsilon 1} = iV_{ab} = 0.92W$

$P_{\varepsilon 2} = iV_{ac} = 0.60W$ $\quad P_{\varepsilon 1} = P_{\varepsilon 2} + P_R$

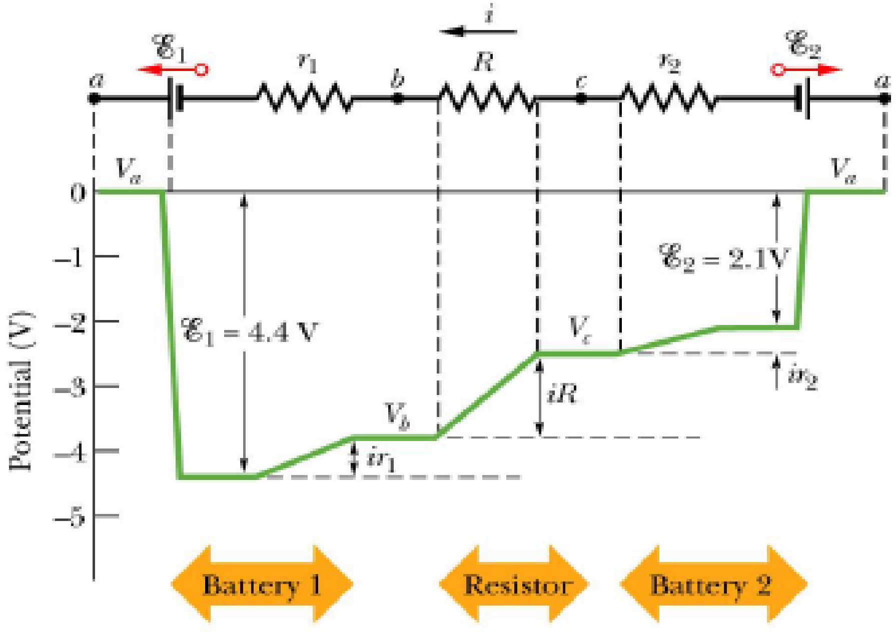
$i \cdot \varepsilon$ $\quad P_R = i^2 R = 0.32W$

A battery (EMF) absorbs power (charges up) when *i* is opposite to ε.



(a)

$$\varepsilon_1 = 4.4V, \varepsilon_2 = 2.1V, r_1 = 2.3\Omega, r_2 = 1.8\Omega, R = 5.5\Omega$$



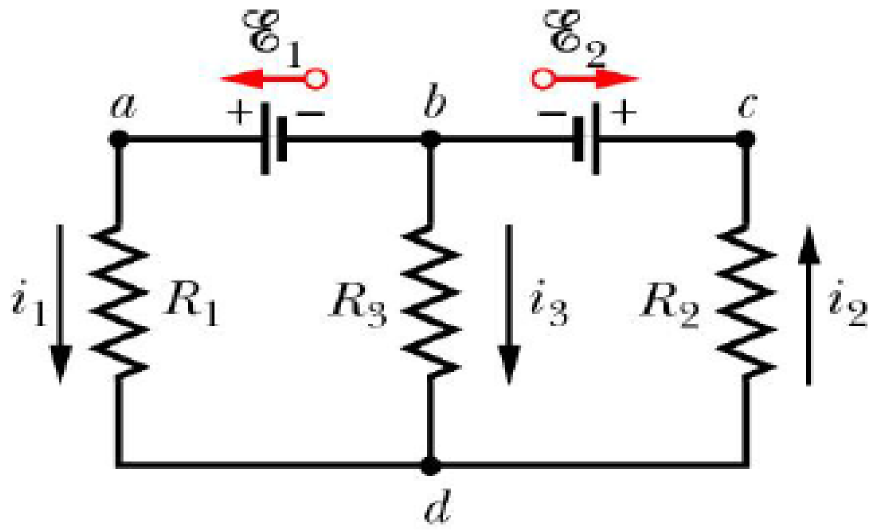
(b)

Multiloop Circuits

- Determine junctions, branches and loops.
- Label arbitrarily the currents for each branch. Assign same current to all element in series in a branch.
- The directions of the currents are assumed arbitrarily; negative current result means opposite direction.
- Junction rule:

$$i_1 + i_3 = i_2$$

- You can use the junction rule as often as you need. In general, the number of times you can use the junction rule is one fewer than the number of junction points in the circuit.



Multiloop Circuits

- Determine loop and choose moving direction arbitrarily.
- When following the assumed current direction, ir is negative and voltage drops; Reverse when going against the assumed current; Emf is positive when traversed from – to +, negative otherwise.
- Loop rule:

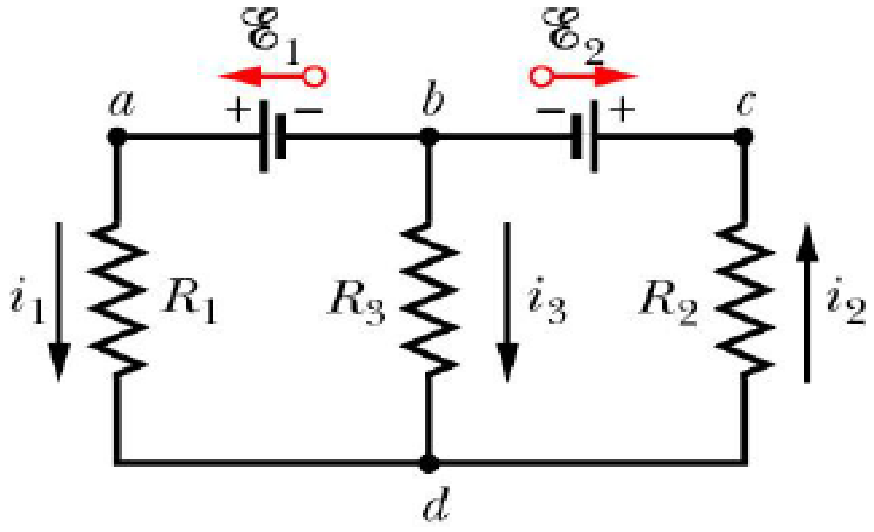
  badb: left-hand loop in counterclockwise

  $$\varepsilon_1 - i_1 R_1 + i_3 R_3 = 0$$

  bdcb: right-hand loop in counterclockwise

  $$-i_3 R_3 - i_2 R_2 - \varepsilon_2 = 0$$

- You can apply the loop rule as often as needed as long as a new circuit element or a new current appears in each new equation.



- Equivalent loop and wise

  badcb: big loop in counterclockwise

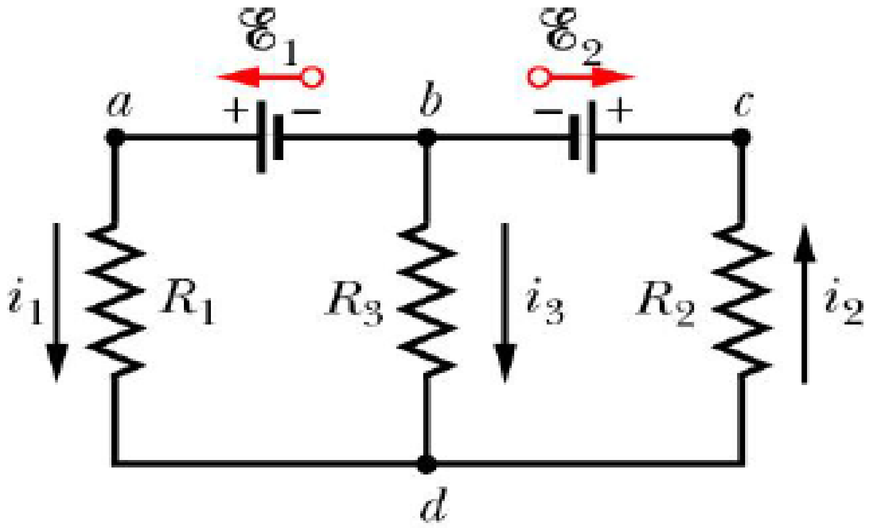
  $$\varepsilon_1 - i_1 R_1 - i_2 R_2 - \varepsilon_2 = 0$$

  bcdb: right-hand loop in clockwise

  $$\varepsilon_2 + i_2 R_2 + i_3 R_3 = 0$$

Multiloop Circuits

- In general, to solve a particular circuit problem, the number of independent equations you need to obtain from the two rules equals the number of unknown currents.



$$i_1 + i_3 = i_2$$

$$\varepsilon_1 - i_1 R_1 + i_3 R_3 = 0$$

$$-i_3 R_3 - i_2 R_2 - \varepsilon_2 = 0$$

- Solution:

$$i_1 = \frac{\varepsilon_1 R_2 + \varepsilon_1 R_3 - \varepsilon_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

$$i_2 = \frac{\varepsilon_1 R_2 - \varepsilon_2 R_3 - \varepsilon_2 R_1}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

$$i_3 = \frac{-\varepsilon_2 R_1 - \varepsilon_1 R_2}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

RC Circuits – Charging a Capacitor

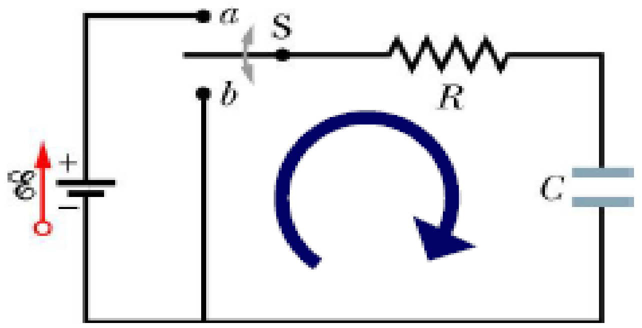
- RC circuits: time-varying currents, switch to a
- Start with,
  Loop rule $$\varepsilon - iR - \frac{q}{C} = 0$$
- Then, $$i = \frac{dq}{dt}$$
- Substituting and rearranging, $$\boxed{R\frac{dq}{dt} + \frac{q}{C} = \varepsilon}$$
- Boundary condition,

$$q(t=0) = 0; \quad i(t=0) = \frac{\varepsilon}{R}; \qquad q(\max) = C\varepsilon; \quad i = 0;$$

- Therefore,

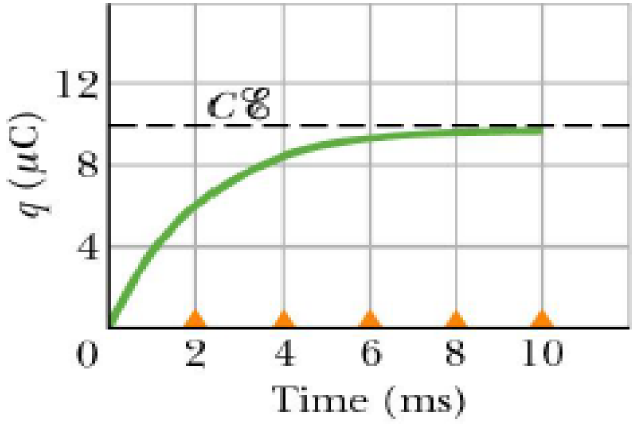
$$\frac{dq}{dt} = \frac{\varepsilon}{R} - \frac{q}{RC} \qquad \frac{dq}{dt} = \frac{C\varepsilon}{RC} - \frac{q}{RC} = -\frac{q - C\varepsilon}{RC} \qquad \frac{dq}{q - C\varepsilon} = -\frac{1}{RC}dt$$

- Integrating, $$\int_0^q \frac{dq}{q - C\varepsilon} = -\frac{1}{RC}\int_0^t dt \qquad \ln\left(\frac{q - C\varepsilon}{-C\varepsilon}\right) = -\frac{t}{RC}$$



RC Circuits – Charging a Capacitor

- Charge, $\boxed{q(t) = C\varepsilon(1 - e^{-t/RC})}$
- For charging current, $\boxed{i(t) = \dfrac{dq(t)}{dt} = \dfrac{\varepsilon}{R} e^{-t/RC}}$
- A capacitor that is being charged initially acts like ordinary connecting wire relative to the charging current. A long time later, it acts like a broken wire.
- Potential difference, $\boxed{V_C(t) = \dfrac{q(t)}{C} = \varepsilon(1 - e^{-t/RC})}$
- $t = 0$: $q = 0$, $V_c = 0$, $i = \varepsilon/R$;
- $t \Rightarrow \infty$: $q = c\varepsilon$, $V_c = \varepsilon$, $i = 0$;
- $t = RC$: $q = c\varepsilon(1-e^{-1}) = 0.632c\varepsilon$; $i = \varepsilon/Re^{-1} = 0.368\ \varepsilon/R$



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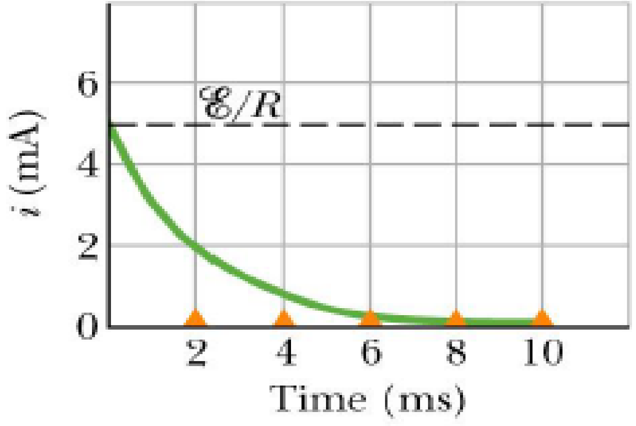
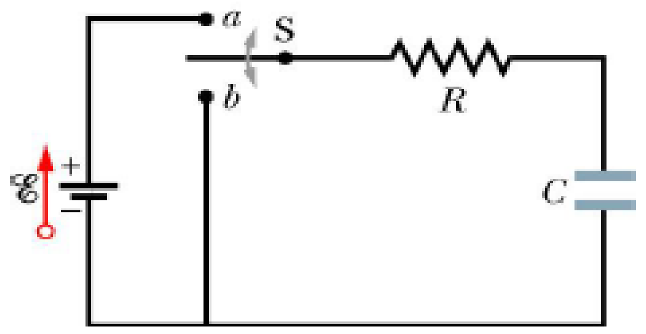


(b)

RC Circuits – Discharging a Capacitor

- RC circuits: time-varying currents, switch to b
- Start with, Loop rule $\boxed{-\dfrac{q}{C} - iR = 0}$
- Then, $-R\dfrac{dq}{dt} = \dfrac{q}{C}$ $\qquad \dfrac{dq}{q} = -\dfrac{1}{RC}dt$
- Boundary condition, $q(t=0) = q_0$
- Therefore, $\int_{q_0}^{q} \dfrac{dq}{q} = -\dfrac{1}{RC}\int_0^t dt$ $\qquad \ln\left(\dfrac{q}{q_0}\right) = -\dfrac{t}{RC}$
- Hence, $\boxed{q(t) = q_0 e^{-t/RC}}$ $\qquad \boxed{i(t) = \dfrac{dq(t)}{dt} = -\dfrac{q_0}{RC}e^{-t/RC}}$
- $t = 0$: $q = q_0 = CV_0$, $i = q_0/RC$;
- $t \Rightarrow \infty$: $q = 0$, $i = 0$;



Summary

- An emf device does work on charges to maintain a potential difference between its output terminals. $\varepsilon = \frac{dW}{dq}$
- Kirchhoff's rules:

  Loop rule. The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

  Junction rule. The sum of the current entering any junction must be equal to the sum of the currents leaving that junction.
- Series resistances: when resistances are in series, they have the same current. The equivalent resistance that can replace a series combination of resistance is $R_{eq} = \sum_{i=1}^{n} R_i$
- Parallel resistance: when resistances are in parallel, they have the same potential difference. The equivalent resistance that can replace a parallel combination of resistance is, $\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}$
- Single loop circuits: the current in a single loop circuit is given by $i = \frac{\varepsilon}{R+r}$
- Power: when a real battery of emf and internal resistance r does work on the charges in a current I through the battery, $P = iV = i(\varepsilon - ir) = i\varepsilon - i^2 r$
- RC Circuits: when an emf is applied to a resistance R and capacitor C in series, $q(t) = C\varepsilon(1 - e^{-t/RC})$ $i(t) = \frac{dq(t)}{dt} = \frac{\varepsilon}{R} e^{-t/RC}$
- RC Circuits: when a capacitor discharges through a resistance R, the charge decays according to $q(t) = q_0 e^{-t/RC}$
- And the current is $i(t) = \frac{dq(t)}{dt} = \frac{q_0}{RC} e^{-t/RC}$